TECHNICAL NOTE

SMOOTHENED ABSOLUTE KINEMATIC POSITION ESTIMATION OF LEO SATELLITE USING GPS OBSERVABLES

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Abstract

In this paper, our objective is to use the navigation signals (pseudorange and carrier phase measurements) from GPS to kinematically estimate the absolute position of a low Earth orbit satellite and its receiver clock biases. GPS observables can estimate the precise position of a satellite in near real time. The GPS satellite navigation file is downloaded from Scripps Orbit and Permanent Array Centre (SOPAC) website for starting epoch of 01-01-2016 00:00 UT. The observables are then generated for a Low-Earth-Orbit (LEO) satellite at an altitude of 901.4 km. The absolute position is estimated using pseudorange measurements. The accuracies obtained are (5.68 8.04 6.17) m in ECEF frame. This estimated absolute position is then smoothened with the time-differenced carrier phase measurements. The Kalman filter gains are computed in near real time. The smoothened accuracies obtained are (2.24 2.60 2.6982) m in ECEF frame, thus showing improved estimated absolute position accuracy.

Nomenclature

$\rho_m^{(k)}$ = Measured pseudorange between LEO satellite and k^{th} GPS satellite
$\phi_m^{(k)}$ = Measured carrier phase of signal received by LEO satellite from k^{th} GPS satellite
$\vec{r}$ = Position vector of LEO satellite
$\vec{r}_{gps}^{(k)}$ = Position vector of k^{th} GPS satellite
b = Clock bias of satellite clock
λ = Wavelength of the carrier signal
$I_{\phi\ or\ \rho}$ = Ionospheric delay in carrier phase or pseudorange
$T_{\phi\ or\ \rho}$ = Tropospheric delay in carrier phase or pseudorange
N = Integer ambiguity
$\rho_m^{*(k)}$ = Ionosphere free pseudorange
$\varepsilon_{\phi\ or\ \rho}^{(k)}$ = Unmodeled effets, modelling errorsand measurement error in carrier phase or pseudorange
f_1/f_2 = Centre frequency of L1/L2 carrier frequencies at 1575.42/1227.60 MHz
G = Geometry matrix
σ_ρ = Standard deviation of receiver code phase noise
σ_{cp} = Standard deviation of receiver carrier phase noise
Q = Process noise covariance matrix
R = Measurement noise covariance matrix

Introduction

The Global Positioning System (GPS) based instantaneous position determination has been studied in literature extensively since early 1980's and has been used on-board low-earth-orbit satellites by NASA, ESA, and DLR for instant position estimation [1, 2]. GPS receivers are flown on Indian Space Research Organisation (ISRO) low-earth-orbit remote sensing satellites for position determination and propagation at ground. The first successful ISRO mission was in 1999 [3] when a low-earth satellite was fitted with a GPS receiver, and its measurements were used to propagate the orbit. The world today is witnessing an ever-increasing demand for satellites for various applications like scientific missions, remote sensing, oceanography, weather forecasting, communication, internet etc.

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The demand for satellite launch continues to rise. As a result, the burden on the ground support to maintain has also been escalating. Hence, our objective in this paper is to develop an autonomous on-board absolute navigation of LEO satellites. This autonomy requires precise knowledge of the absolute state (position, velocity) and time of spacecraft. In this paper we present an algorithm for estimating absolute kinematic states of the satellites in LEO using GPS navigation signals. The precise determination of the position of satellite using Global Navigation Satellite Systems (GNSS) can be major contributor towards achieving autonomy. It can also be harnessed for autonomous formation keeping applications [4-8].

Presently, three satellite navigation systems with global coverage are operational, namely, GPS, GLONASS and BeiDou. India has developed its own regional navigation system, i.e. NavIC (Navigation by Indian Constellation) which is a constellation of 7 satellites at GEO. The algorithm described here can be used for position determination using NavIC observables for kinematic absolute navigation of LEO satellites.

The GPS observables used in this paper are pseudorange and carrier phase measurements. The dual frequency, GPS signals of interest are centred on 1575.42 MHz (L1) and 1227.6MHz (L2 signal). Each GPS satellite modulates the carrier with a unique pseudo-random noise code known as the C/A code, at 1.023 MHz chipping rate. The C/A code identifies the satellite, and enables the receiver to measure the transit time of the signal from the satellite to the receiver. This observed range based on transit time, known as pseudorange is used to solve for the position of the observer. The difference of carrier phase measurements at two time epochs can be used to reduce the position noise level in the pseudorange measurements.

The three reference frames used in this study are: earth-centered inertial (ECI), earth-centered-earth-fixed (ECEF) and radial-transverse-normal (R-T-N) frames. The ECI frame has its origin at the centre of mass of the Earth with x_{eci} -axis along the Vernal Equinox, z_{eci} -axis parallel to the rotation axis (i.e. polar axis) of the Earth, and y_{eci} -axis renders this a right-handed orthogonal system. Like ECI frame, the ECEF frame also has its origin at the Earth centre of mass and the third axis, z_{ecef} (i.e. polar axis), coincides with z_{eci}. The x_{ecef} -axis and y_{ecef} -axis lie in the equatorial plane and rotates with the Earth, with positive x_{ecef} passing through the prime meridian, and y_{ecef} completing a right-handed triad. The RTN frame is used to represent the position of the spacecraft orbiting the Earth; it is aligned with the local radial (R) pointing up from the Earth centre, along-track (transverse, T) and cross-track (normal, N) directions.

This paper consists the following sections:

- Measurement models used for observable generation
- GPS based LEO satellite position estimation
- Kalman Smoothing of Pseudorange
- Simulation results
- Conclusion and future work

Measurement Models Used for Observable Generation

The GPS satellite orbits are derived from the navigation file to find out their position in ECEF frame. The GPS observables/navigation files that are used for this purpose are taken from Scripps Orbit and Permanent Array Centre (SOPAC) website, received at Indian Institute of Science. From the navigation message, the position of all the navigation satellites can be determined in the ECEF frame. These measurements are in terms of orbital parameters, and by using equations based on Kepler's laws the coordinates of the navigation satellites are obtained. This helps in finding the direction of the unit vector of each satellite with respect to the receiver if the position of receiver (reference station) is known. The orbit of LEO satellite is simulated in STK. In absence of access to observables at LEO, they are obtained using mathematical models. Using the respective orbital positions of the visible GPS satellites and that of the LEO satellite, the measurements are emulated using following set of equations, assuming that the receiver is mounted on the LEO satellite.

Pseudorange Measurement Model

Assuming the receiver to be located at a known position, we can calculate the pseudorange $\rho_m^{(k)}$ from the k^{th} GPS satellite using the following relation,

$$\rho_m^{(k)} = | \vec{r} - \vec{r}_{gps}^{\,(k)} | + (b - b^{(k)}) + I_\rho + T_\rho + \varepsilon_\rho^{(k)} \tag{1}$$

Carrier Phase Measurement Model

The carrier phase measurement is the difference between the phases of the receiver-generated carrier signal and the carrier wave received from the navigation satellite at the instant of the measurement. The carrier phase measurement ϕ, in units of cycle is given by:

$$\phi_m^{(k)} = \lambda^{-1}\left[|\vec{r} - \vec{r}_{gps}^{(k)}| + (b - b^{(k)}) - I_\phi + T_\phi \right] + N^{(k)} + \varepsilon_\phi^{(k)} \tag{2}$$

The GPS signal travels through Ionosphere and troposphere, thus affecting the travel time of the signal. The impact of Ionospheric and Tropospheric delay is modelled [9] and accounted for in the measurements Eq.(1) and (2). The measurements thus obtained are used in the algorithm.

GPS Based LEO Satellite Position Estimation

The absolute state vector estimation is carried out by the formulations given in [8]. To account for errors introduced in the observables by the atmosphere, we need high fidelity models for Ionosphere and Troposphere. The first-order Ionospheric error that contributes more of the total Ionospheric delay can be modelled as in Eq.(3). This provides Ionosphere-Free (IF) combination, when dual-frequency observations are available.

$$\rho_m^{*(k)} = \frac{f_1^2 * \rho_{m\,L1} - f_2^2 * \rho_{m\,L2}}{f_1^2 - f_2^2} \tag{3}$$

To begin the estimation, the pseudorange measurement equation is linearized about an initial rough estimate of the position. The linearized model gives incremental corrections to the estimated position. The improved estimate from each iteration is used as the point about which linearization is done in the subsequent iteration. The least-squares solution for the incremental corrections $\delta\hat{x}$ to the estimates can be written as

$$\delta\hat{x} = (G^T R^{-1} G)^{-1} G^T R^{-1} \Delta\rho \tag{4}$$

The corresponding geometry matrix is given by:

$$G = \begin{bmatrix} (e^{(1)})^T & 1 \\ (e^{(2)})^T & 1 \\ . & . \\ . & . \\ (e^{(k)})^T & 1 \end{bmatrix}_{k\,x\,4} \tag{5}$$

where $e^{(k)}$ is the unit position vector from k^{th} GPS satellite to the LEO satellite. R is the pseudorange measurement noise covariance matrix. The noise is assumed to be uncorrelated for simplicity. Hence, it is a diagonal matrix with each diagonal entry equal to σ_ρ^2.

The new improved estimates for the user position and clock bias are as given below. This solution is iterated until the change in the estimates is sufficiently small.

$$\hat{x}(j) = \hat{x}(j-1) + \delta\hat{x} \tag{6}$$

where $\hat{x} = [\hat{\vec{r}} \;\; \hat{b}]^T$ is a 4x1 vector consisting of the estimated position vector of the LEO and its receiver clock bias. Details can be referred to in [6].

Kalman Smoothing of Pseudorange

The pseudorange is measured with meter level accuracy, whereas the carrier phase is measured typically with an accuracy level of a few millimetres. The difference of carrier phase measurements at two time epochs can be used to reduce the position noise level in the pseudorange measurements [9]. A recursive Kalman filter [10] is used to smoothen the absolute position estimate, developed in the last subsection. The smoothing filter flow is shown in Flowchart-1. At the acquisition of carrier phase, the pseudorange estimate is initialized with the pseudorange measurement at that instant. P is the error covariance. P^- is initialised and the Kalman gain K is calculated. The pseudorange is time updated using the differential carrier phase. The pseudorange measurements are smoothened using time updated differential carrier phase measurements. The lower limit of Kalman Gain K is the ratio of variances of carrier phase measurement noise and pseudorange measurement noise. The filtered pseudorange (ρ_m) thus obtained is used for position estimation. The covariance is calculated and the filtered pseudorange is time updated in the next recursion of the filter.

Simulation Results

The Keplerian orbital elements of the LEO satellite are: semi-major axis (7279.8 km), inclination (-81.2277°) and eccentricity (0.0017°). The GPS parameters used for observables generation are as listed in Table-1.

The GPS satellites visible to the LEO are derived assuming 60° Field of View (FOV) from LEO to GPS orbit. The number of navigation satellites visible to the LEO satellites for 24 hours are shown in Fig.1. The estimated LEO satellite orbit is shown in Fig.2 in ECI frame.

The simulation results with and without smoothing are tabulated in Table-2. The mean errors achieved without smoothing are (-0.0755; -0.0412; 0.0230) m with a standard deviation of (3.5514; 3.6269; 4.1590) m about the three axes. While smoothing the pseudorange measurements using carrier phase, has shown improvement in the position estimation. The mean errors achieved with smoothing are (-0.0908; 0.0138; 0.0624) m with a standard deviation of (1.9891; 2.0679; 2.4544) m. Left column of the Fig.3 shows the estimation error without smoothing whereas the right column shows the estimation error with smoothing. As can be seen in Fig.3, the performance of the estimation degrades at some instances. This can be attributed to poor DOP (Dilution of Precision) at that instant. When the navigation satellites are bunched closely in the sky view, the geometric configuration is termed weak and the DOP value is high and vice versa. The DOP values are plotted in Fig.4. A direct correspondence can be observed between error in Fig.3 and DOP in Fig.4. The GDOP (Geometric DOP) is an overall indicator of the goodness of precision obtained which considers the DOP in all three axes. Some users specify an upper limit on the DOP, beyond which the collected data is not considered suitable for estimation.

Table-1 : Parameters for Measurement Generation

Sl. No.	Parameter	Assumed Value
1	Propagation Frequency	L1(1575.42 MHz), L2 (1227.6 MHz)
2	Receiver Code Phase Noise	0.2 m
3	Receiver Carrier Phase Noise	0.01 Cycles
4	Receiver Clock Bias	1e-3 ms
5	Navigation Sampling Rates	2 Hours

Conclusion and Future Works

In this study we have estimated the position of LEO satellite using GPS observables. The position is estimated using pseudorange measurements. This estimated position is smoothened using carrier phase measurement, in a linear Kalman filter. The simulation results have shown the 3-sigma worst case position error of [93.5514; 3.6269; 4.15900] m; without smoothing, whereas with smoothing these errors are (2.2864; 2.2864; 2.6261) m in ECEF frame. Also, it is noticed that for the achieved estimation errors depend on the dilution of precision (DOP) of GPS satellites. So, high errors at some instants can be attributed to poor geometry of GPS satellites at those instant. This accurate kinematic position estimation is suitable for precise orbit determination and related payload pointing operations. This work can be extended to state estimation of GEO satellites, using the signals from the GPS satellites on the other side of the earth.

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Table-2 : Worst Error Obtained in Each Axis in ECEF

	X (m)		Y (m)		Z (m)	
	μ	3σ	μ	3σ	μ	3σ
Without Smoothing	-0.0755	3.5514	-0.0412	3.6269	0.0230	4.1590
With Smoothing	-0.0908	1.9891	0.0138	2.0679	0.0624	2.4544

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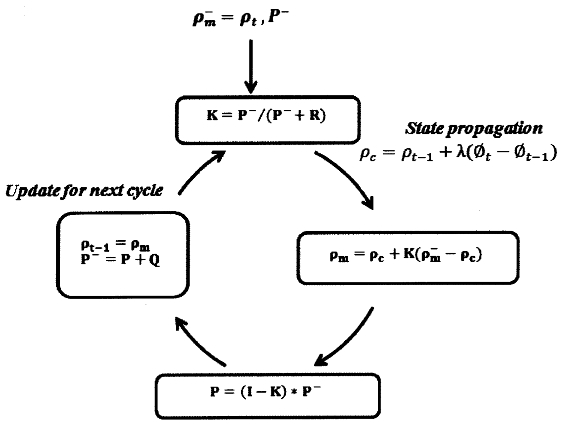

Flowchart 1. Kalman Smoothing of Pseudorange (Simplified)

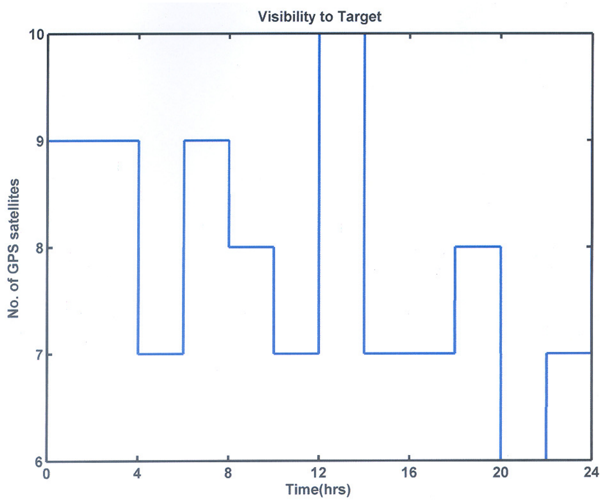

Fig.1 No. of GPS Satellites Visible

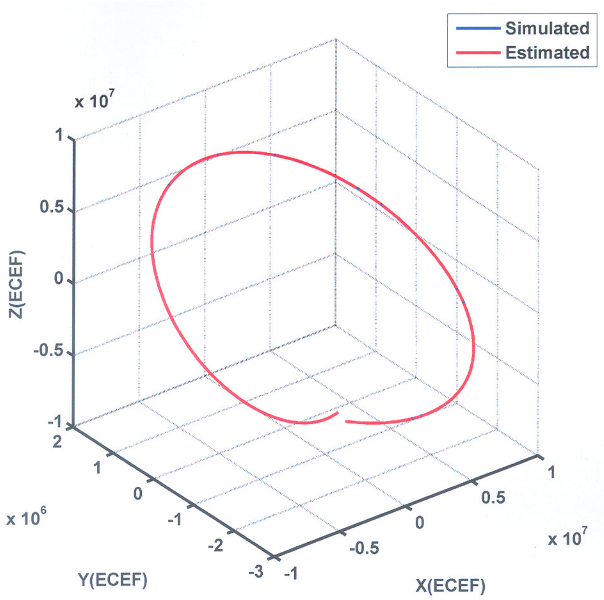

Fig.2 LEO Orbit

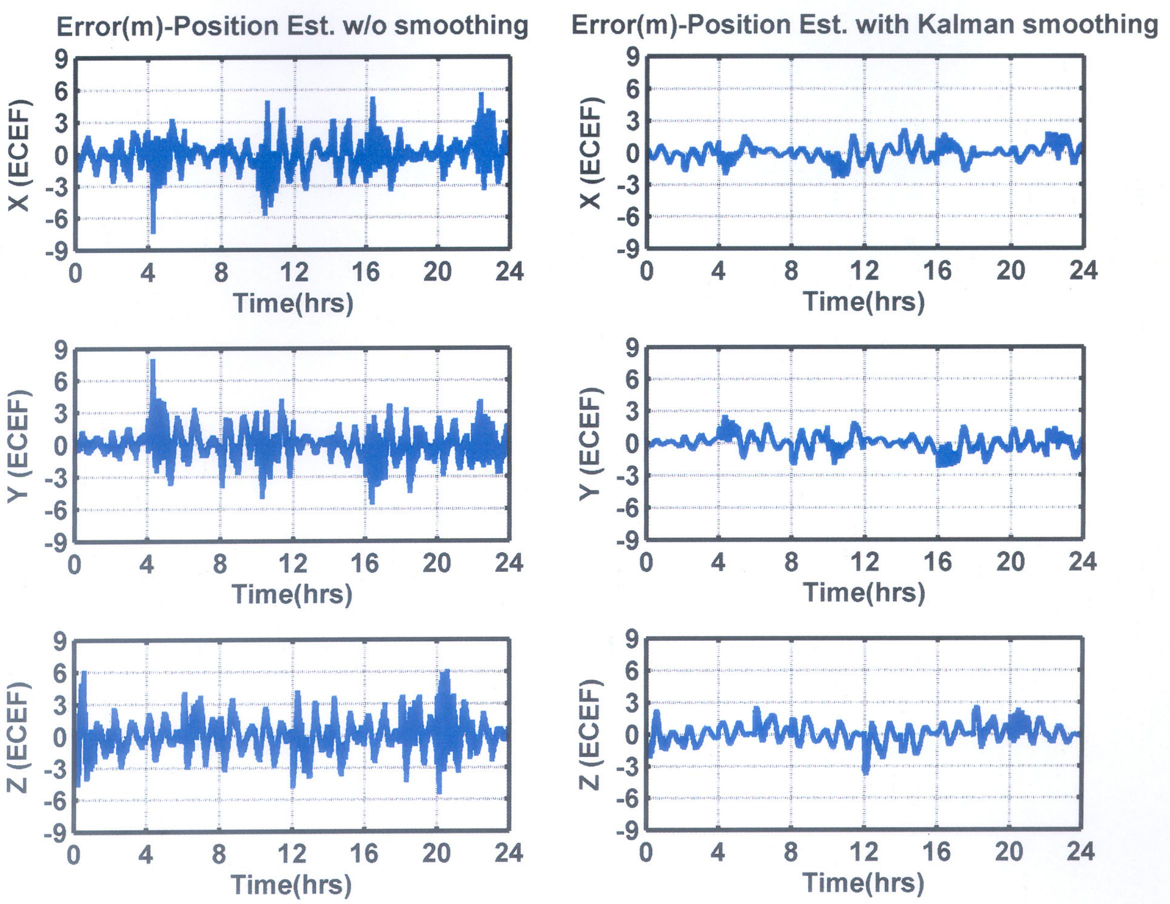

Fig.3 Error in Position Estimation : Without and With Kalman Smoothing

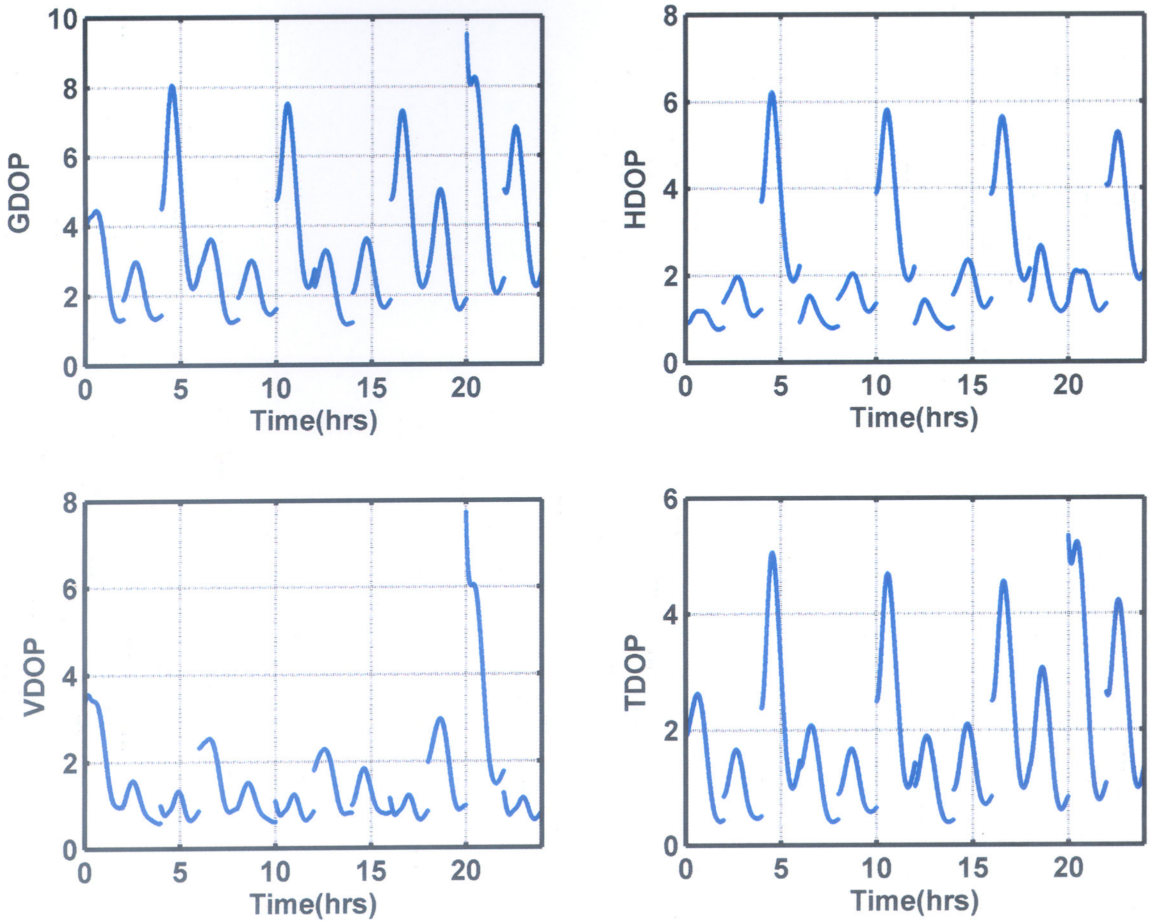

Fig.4 DOP of Visible Satellites