SMALL SCALE EFFECT AND VIBRATION OF GRAPHENE SHEETS WITH VARIOUS BOUNDARY CONDITIONS

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Abstract

Elastic theory of graphene sheets is reformulated using the nonlocal differential constitutive relations of Eringen. The equations of motion of the nonlocal theories are derived. Levy's approach has been employed to solve the governing differential equations for various boundary conditions. Nonlocal theories are employed to bring out the small scale effect of the nonlocal parameter on the natural frequencies of the graphene sheets. Present vibration results associated with various boundary conditions are in good agreement with those available in literature. Further, effects of (i) nonlocal parameter, (ii) size of the graphene sheets and (iii) boundary conditions on nondimensional vibration frequencies are investigated. The theoretical development as well as numerical solutions presented here in should serve as reference for nonlocal theories of nanoplates and nanoshells.

***Keywords**: Vibration, Graphene Sheets, Nonlocal Elasticity, Levy's Solution, Boundary Conditions*

Nomenclature

a, b	= Length and breadth of the graphene sheet
D	= Bending rigidity of the graphene sheet
E	= Young's modulus of the graphene sheet material
h	= Thickness of the graphene sheet
L	= Length (or breadth) of a square graphene sheet
M_1^{xx}, M_1^{yy}, M_1^{xy}	= Moment resultants
N_0^{xx}, N_0^{yy}, N_0^{xy}	= In-plane force resultants
S(x)	= Fourth order elasticity tensor
σ^l	= Macroscopic local stress tensor
u, v	= Displacement of the point (x, y, 0) of graphene sheet along x and y axis, respectively
V_0^{xx}, V_0^{yy},	= Transverse force resultants
ε_{xx}, ε_{yy}, ε_{zz}, ε_{xy}, ε_{yz}, ε_{xzy}	= Strain tensors
M	= Nonlocal parameter
N	= Poisson's ratio of the graphene sheet material
P	= Density of the graphene sheet material
σ_{xx}^{nl}, σ_{yy}^{nl}, σ_{zz}^{nl}, σ_{xy}^{nl}, σ_{yz}^{nl}, σ_{xz}^{nl}	= Nonlocal stress tensors
∇^2	= Laplacian operator in two dimensional Cartesian coordinate system
P	= Density of graphene sheet material
w^c	= Deflections of the single layered graphene sheet at point (x, y) calculated using CLPT

Introduction

Nano-structured elements have attracted attention of scientific community due to their superior properties.

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Conducting experiments with nanoscale size specimens is found to be difficult and expensive. Therefore, development of appropriate mathematical models for nanostructures is an important issue concerning application of nano-structures. Generally, three approaches have been developed to model nanostructures. These approaches are (a) atomistic [1, 2] (b) hybrid atomistic-continuum mechanics [3-6] and (c) continuum mechanics. Both atomistic and hybrid atomistic-continuum mechanics are computationally expensive and are not suitable for analyzing large scale systems. Continuum mechanics approach is less computationally expensive than the former two approaches. It has been found that continuum mechanics results are in good agreement with atomistic and hybrid approaches.

Vibration of nanostructures is of great importance in nanotechnology. Understanding vibration behavior of nanostructures is the key step for many NEMS devices like oscillators, clocks and sensor devices. There are already exploratory studies on the continuum models for vibration of Carbon nanotubes (CNTs) or similar micro or nanobeam like elements [7-12]. A review related to the importance and modeling of vibration behavior of various nanostructures can be found in Gibson's et. al. [13]. In these works it has been suggested that Nonlocal Elasticity Theory developed by Eringen [14-15] should be used in the continuum models for accurate prediction of vibration behaviors. This is due to the scale effect of the nanostructures. Importance of accurate prediction of nanostructures' vibration characteristics have been discussed by Gibson et. al. [13]. A relevant reference concerning nonlocal theories for bending, buckling and vibration analysis of beams is reported by Reddy [16].

Similar to CNTs, graphene sheets possess superior mechanical properties [17-18]. But in contrast to one dimensional structures, limited work have been found on vibration analysis of two dimensional graphene sheets [18-21]. In the continuum models used in [18-21] only classical plate theory (CLPT) has been considered for modeling the graphene sheets. These mathematical models do not take scale effect into account. It is importance to incorporate nonlocal elasticity theories in the vibration analysis of graphene sheets. In the present paper attempt is made to study the vibration of the graphene sheets using nonlocal elasticity theory. Classical plate theory have been incorporated in the analysis. Levy approach has been used to solve the governing equations for different boundary conditions. Effect of (i) nonlocal parameter, (ii) size of the graphene sheets and (iii) boundary conditions on nondimensional vibration frequencies are investigated.

Formulation

The coordinate system used for the graphene sheet is shown in Fig.1. Origin is chosen at one corner of the midplane of the plate. The *x, y* coordinates of the axes are taken along the length and width of the plate. *z* coordinate is taken along the thickness of the plate. Following stress resultants are used in the present formulation

$$N_0^{xx} = \int_{-h/2}^{h/2} \sigma_{xx}^{nl}\, dz \ , \quad N_0^{yy} = \int_{-h/2}^{h/2} \sigma_{yy}^{nl}\, dz \ , \quad N_0^{xy} = \int_{-h/2}^{h/2} \sigma_{xy}^{nl}\, dz$$

$$V_0^{xx} = \int_{-h/2}^{h/2} \sigma_{xz}^{nl}\, dz \ , \quad V_0^{yy} = \int_{-h/2}^{h/2} \sigma_{yz}^{nl}\, dz \ , \quad M_1^{xx} = \int_{-h/2}^{h/2} z\, \sigma_{xx}^{nl}\, dz$$

$$M_1^{yy} = \int_{-h/2}^{h/2} z\, \sigma_{yy}^{nl}\, dz \ , \quad M_1^{xy} = \int_{-h/2}^{h/2} z\, \sigma_{xy}^{nl}\, dz \qquad (1)$$

Here h denotes the height of the plate. σ_{xx}^{nl}, σ_{yy}^{nl}, σ_{zz}^{nl}, σ_{xy}^{nl}, σ_{yz}^{nl} and σ_{xz}^{nl} represent the nonlocal stress tensors. In classical local elasticity theories, stress at a point depends only on the strain at that point. While in nonlocal elasticity theories it is assumed that the stress at a point depends on the strains at all the points of the continuum. In other words, according to this nonlocal theory strain at a point depends on both stress and spatial derivatives of the stress at that point. According to Eringen [14] the nonlocal constitutive behavior of a Hookean solid is represented by the following differential constitutive relation

$$(1 - \mu \nabla^2)\, \sigma^{nl} = \sigma^{l} \qquad (2)$$

Here μ is the nonlocal parameter; σ^l is the local stress tensor at a point which is related to strain by generalized Hooke's law

$$\sigma^l(x) = S(x) : \varepsilon(x) \qquad (3)$$

Where S is the fourth order elasticity tensor and ':' denotes the double dot product.

Single Layered Graphene Sheet

A typical single layer graphene sheet is shown in Fig.1. Classical plate theory for the single layered graphene sheet is based on the following displacement field

$$u_x = u\,(x\,,y\,,t\,) - z\,\frac{\partial w^c}{\partial x}\,,\; u_y = v\,(x\,,y\,,t\,) - z\,\frac{\partial w^c}{\partial y}$$

and

$$u_z = w^c\,(x\,,y\,,t\,) \tag{4}$$

Here *u, v* and w^c denote displacement of the point (*x, y, 0*) along *x, y* and *z* directions, respectively.

The strains are expressed as

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\,\frac{\partial^2 w^c}{\partial x^2}\,,\; \varepsilon_{yy} = \frac{\partial v}{\partial x} - z\,\frac{\partial^2 w^c}{\partial y^2}\,,$$

$$\varepsilon_{zz} = 0\,,\; \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\,\frac{\partial^2 w^c}{\partial xy}\right),\; \varepsilon_{xz} = 0\,,\; \varepsilon_{yz} = 0 \tag{5}$$

ItcanbeseenfromEq.(2)thatnonlocalbehaviorenters intotheproblemthroughtheconstitutiverelations.Principal of virtual work is independent of constitutive relations. So this can be applied to derive the equilibrium equations of the nonlocal graphene sheets. Using the principle of virtual displacements, following governing equations can be obtained [22] :

$$\frac{\partial N_0^{xx}}{\partial x} + \frac{\partial N_0^{xy}}{\partial y} = m_0\,\frac{\partial^2 u}{\partial t^2} \tag{6.1}$$

$$\frac{\partial N_0^{yy}}{\partial y} + \frac{\partial N_0^{xy}}{\partial x} = m_0\,\frac{\partial^2 v}{\partial t^2} \tag{6.2}$$

$$\frac{\partial^2 M_1^{xx}}{\partial x^2} + 2\,\frac{\partial^2 M_1^{xy}}{\partial x\,\partial y} + \frac{\partial^2 M_1^{yy}}{\partial y^2} + q + \frac{\partial}{\partial x}\left(N_0^{xx}\,\frac{\partial w^c}{\partial x}\right) +$$

$$\frac{\partial}{\partial y}\left(N_0^{yy}\,\frac{\partial w^c}{\partial y}\right) + \frac{\partial}{\partial x}\left(N_0^{xy}\,\frac{\partial w^c}{\partial y}\right)$$

$$+ \frac{\partial}{\partial y}\left(N_0^{xy}\,\frac{\partial w^c}{\partial x}\right) = m_0\,\frac{\partial^2 w^c}{\partial t^2} - m_2\left(\frac{\partial^4 w^c}{\partial x^2\,\partial t^2} + \frac{\partial^4 w^c}{\partial y^2\,\partial t^2}\right) \tag{6.3}$$

m_0 and m_2 are mass moments of inertia and are defined as follows :

$$m_0 = \int_{-h/2}^{h/2} \rho\, d\,z\;,\quad m_2 = \int_{-h/2}^{h/2} \rho\, h^2\, d\,z \tag{7}$$

Here ρ denotes the density of the material. In classical plate theory, transverse shear stresses are neglected. Using Eq. (2), the plane stress constitutive relation for a nonlocal graphene sheet is written as

$$\begin{Bmatrix} \sigma_{xx}^{nl} \\ \sigma_{yy}^{nl} \\ \sigma_{xy}^{nl} \end{Bmatrix} - \mu\nabla^2 \begin{Bmatrix} \sigma_{xx}^{nl} \\ \sigma_{yy}^{nl} \\ \sigma_{xy}^{nl} \end{Bmatrix} = \begin{bmatrix} E/(1-\upsilon^2) & \upsilon E/(1-\upsilon^2) & 0 \\ \upsilon\, E/(1-\upsilon^2) & E/(1-\upsilon^2) & 0 \\ 0 & 0 & 2G \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix} \tag{8}$$

E, G and *v* denote elastic modulus, shear modulus and Poisson's ratio, respectively. Using strain displacement relationship (Eq. (5)), stress-strain relationship (Eq. (8)) and stress resultants definition (Eq. (1)), we can express stress resultants in terms of displacements as follows :

$$M_1^{xx} - \mu\nabla^2 M_1^{xx} = -D\left(\frac{\partial^2 w^c}{\partial x^2} + \upsilon\,\frac{\partial^2 w^c}{\partial y^2}\right) \tag{9.1}$$

$$M_1^{yy} - \mu\nabla^2 M_1^{yy} = -D\left(\frac{\partial^2 w^c}{\partial y^2} + \upsilon\,\frac{\partial^2 w^c}{\partial x^2}\right) \tag{9.2}$$

$$M_1^{xy} - \mu\nabla^2 M_1^{xy} = -D\;(1-\upsilon)\;\frac{\partial^2 w^c}{\partial x\,y} \tag{9.3}$$

Here $D = \dfrac{Eh^3}{12\,(1-\upsilon^2)}$ denotes the bending rigidity of the graphene sheet. Using Eq. (6.3) and Eq. (9) we get the following governing equations in terms of the displacements [31].

$$-D\,\nabla^4 w^c + \mu\nabla^2 \Bigg[-q - \frac{\partial}{\partial x}\left(N_0^{xx}\frac{\partial w^c}{\partial x}\right) - \frac{\partial}{\partial y}\left(N_0^{yy}\frac{\partial w^c}{\partial y}\right) - \frac{\partial}{\partial x}\left(N_0^{xy}\frac{\partial w^c}{\partial y}\right)$$

$$- \frac{\partial}{\partial y}\left(N_0^{xy}\frac{\partial w^c}{\partial x}\right) + m_0\frac{\partial^2 w^c}{\partial t^2} - m_2\left(\frac{\partial^4 w^c}{\partial x^2\,\partial t^2} + \frac{\partial^4 w^c}{\partial y^2\,\partial t^2}\right)\Bigg]$$

$$+ q + \frac{\partial}{\partial x}\left(N_0^{xx}\frac{\partial w^c}{\partial x}\right) + \frac{\partial}{\partial y}\left(N_0^{yy}\frac{\partial w^c}{\partial y}\right) + \frac{\partial}{\partial x}\left(N_0^{xy}\frac{\partial w^c}{\partial y}\right)$$

$$+ \frac{\partial}{\partial y}\left(N_0^{xy}\frac{\partial w^c}{\partial x}\right) = m_0\frac{\partial^2 w^c}{\partial t^2} - m_2\left(\frac{\partial^4 w^c}{\partial x^2\,\partial t^2} + \frac{\partial^4 w^c}{\partial y^2\,\partial t^2}\right) \tag{10}$$

It can be noted that governing equation for traditional local classical plate theory can be obtained by setting $\mu = 0$ in Eq. (10).

Levy's Solution

Consider an isotropic and homogeneous rectangular graphene sheet with uniform thickness h, length a, width b, modulus of elasticity E, Poisson's ratio v and the rectangular Cartesian coordinate system as shown in Fig.1(b). This paper is concerned with the graphene sheets which are simply supported on two opposite edges with any combination of simply supported, clamped conditions at the remaining two edges.

Governing differential equation of graphene sheet is given by :

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right)$$

$$+\rho\, h\,\frac{\partial^2 w}{\partial t^4} - \mu\,\rho\, h\left(\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\partial^4 w}{\partial y^2 \partial t^2}\right) = 0 \qquad (11)$$

Substituting

$$w = W(x, y)\, e^{i\omega t}$$

We get

$$D\left(\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4}\right)$$

$$-\rho\, h\,\omega^2\left[W - \mu\frac{\partial^2 W}{\partial x^2} - \mu\frac{\partial^2 W}{\partial y^2}\right] = 0 \qquad (12)$$

Substituting

$$W = Y(y)\,\sin\left(\frac{m\pi}{a}x\right)$$

We get

$$\frac{\partial^4 Y}{\partial y^4} + \frac{\partial^2 Y}{\partial y^2}\left\{\frac{\mu\,\rho h\,\omega^2}{D} - 2\left(\frac{m\pi}{a}\right)^2\right\}$$

$$+ Y\left[\left(\frac{m\pi}{a}\right)^4 - \frac{\mu\,\rho h\,\omega^2}{D}\left(\frac{m\pi}{a}\right)^2 - \frac{\rho\, h\,\omega^2}{D}\right] = 0 \qquad (13)$$

Substituting

$$Y = e^{\frac{\psi y}{b}}$$

We get

$$\left(\frac{\psi}{b}\right)^4 + \left(\frac{\psi}{b}\right)^2\left\{\frac{\mu\,\rho h\,\omega^2}{D} - 2\left(\frac{m\pi}{a}\right)^2\right\}$$

$$+ \left[\left(\frac{m\pi}{a}\right)^4 - \frac{\mu\,\rho h\,\omega^2}{D}\left(\frac{m\pi}{a}\right)^2 - \frac{\rho\, h\,\omega^2}{D}\right] = 0 \qquad (14)$$

$$\left(\frac{\psi}{b}\right)^4 = \left(\frac{m\pi}{a}\right)^2 - \frac{\mu\,\rho h\,\omega^2}{2D} \pm \sqrt{\left\{\frac{\rho\, h\,\omega^2}{D} + \frac{\rho^2\, h^2\,\mu^2\,\omega^2}{4D^2}\right\}}$$

$$\psi_1 = b\sqrt{\left\{\left\{\frac{m\pi}{a}\right\}^2 - \frac{\mu\,\rho h\,\omega^2}{2D} + \sqrt{\left\{\frac{\rho\, h\,\omega^2}{D} + \frac{\mu^2\,\rho^2\,h^2\,\omega^4}{4D^2}\right\}}\right\}}$$

$$\psi_2 = b\sqrt{\left\{-\left\{\frac{m\pi}{a}\right\}^2 + \frac{\mu\,\rho h\,\omega^2}{2D} + \sqrt{\left\{\frac{\rho\, h\,\omega^2}{D} + \frac{\mu^2\,\rho^2\,h^2\,\omega^4}{4D^2}\right\}}\right\}}$$

Assuming

$$W(x, y) = X(x) \,.\, Y(y)$$

Where,

$$X = \sin\left(\frac{m\,\pi\, x}{a}\right)$$

$$Y = A\cosh\left(\frac{\psi_1 y}{b}\right) + B\sinh\left(\frac{\psi_1 y}{b}\right) + C\cos\left(\frac{\psi_2 y}{b}\right) + D\sin\left(\frac{\psi_2 y}{b}\right)$$

(a) SSSS : The graphene sheets which are simply supported on all four edges of the graphene sheet.

$Y(0) = 0$

$Y(b) = 0$

$$\frac{\partial^2 Y}{\partial y^2} = 0 \text{ at } y = 0, b$$

On applying the boundary condition we get

$$A + C = 0$$

$$A\frac{\psi_1^2}{b^2}-C\frac{\psi_2^2}{b^2}=0$$

$$A\cosh(\psi_1)+B\sinh(\psi_1)+C\cos(\psi_2)+D\sin(\psi_2)=0$$

$$A\frac{\psi_1^2\cosh(\psi_1)}{b^2}+B\frac{\psi_1^2\sinh(\psi_1)}{b^2}-C\frac{\psi_2^2\cos(\psi_2)}{b^2}-D\frac{\psi_2^2\sin(\psi_2)}{b^2}=0$$

$$\left\{\begin{matrix} 1 & 0 & 1 & 0 \\ \frac{\psi_1^2}{b^2} & 0 & \frac{-\psi_2^2}{b^2} & 0 \\ \cosh(\psi_1) & \sinh(\psi_1) & \cos(\psi_2) & \sinh(\psi_2) \\ \frac{\psi_1^2\cosh(\psi_1)}{b^2} & \frac{\psi_1^2\sinh(\psi_1)}{b^2} & \frac{-\psi_2^2\cos(\psi_2)}{b^2} & \frac{-\psi_2^2\sin(\psi_2)}{b^2} \end{matrix}\right\}\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}=0$$

For nontrivial case

$$\left\{\begin{matrix} 1 & 0 & 1 & 0 \\ \frac{\psi_1^2}{b^2} & 0 & \frac{-\psi_2^2}{b^2} & 0 \\ \cosh(\psi_1) & \sinh(\psi_1) & \cos(\psi_2) & \sinh(\psi_2) \\ \frac{\psi_1^2\cosh(\psi_1)}{b^2} & \frac{\psi_1^2\sinh(\psi_1)}{b^2} & \frac{-\psi_2^2\cos(\psi_2)}{b^2} & \frac{-\psi_2^2\sin(\psi_2)}{b^2} \end{matrix}\right\}=0 \tag{15}$$

We get ω for SSSS Graphene sheets for different modes.

(b) SSSC : The graphene sheets which are simply supported on two opposite edges with simply supported and clamped conditions at the remaining two edges.

$Y(0)=0$

$$\frac{\partial^2 Y}{\partial y^2} = 0 \text{ at } y=0$$

$Y(b)=0$

$$\frac{\partial Y}{\partial y} = 0 \text{ at } y=b$$

On applying the boundary condition we get

$$A+C=0$$

$$A\frac{\psi_1^2}{b^2}-C\frac{\psi_2^2}{b^2}=0$$

$$A\cosh(\psi_1)+B\sinh(\psi_1)+C\cos(\psi_2)+D\sin(\psi_2)=0$$

$$A\frac{\psi_1\sinh(\psi_1)}{b}+B\frac{\psi_1\cosh(\psi_1)}{b}-C\frac{\psi_2\sin(\psi_2)}{b}+D\frac{\psi_2\cos(\psi_2)}{b^2}=0$$

$$\left\{\begin{matrix} 1 & 0 & 1 & 0 \\ \frac{\psi_1^2}{b^2} & 0 & \frac{-\psi_2^2}{b^2} & 0 \\ \cosh(\psi_1) & \sinh(\psi_1) & \cos(\psi_2) & \sinh(\psi_2) \\ \frac{\psi_1\sinh(\psi_1)}{b} & \frac{\psi_1\cosh(\psi_1)}{b} & \frac{-\psi_2\sin(\psi_2)}{b} & \frac{\psi_2\cos(\psi_2)}{b} \end{matrix}\right\}\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}=0$$

For nontrivial solution

$$\left\{\begin{matrix} 1 & 0 & 1 & 0 \\ \frac{\psi_1^2}{b^2} & 0 & \frac{-\psi_2^2}{b^2} & 0 \\ \cosh(\psi_1) & \sinh(\psi_1) & \cos(\psi_2) & \sinh(\psi_2) \\ \frac{\psi_1\sinh(\psi_1)}{b} & \frac{\psi_1\cosh(\psi_1)}{b} & \frac{-\psi_2\sin(\psi_2)}{b} & \frac{\psi_2\cos(\psi_2)}{b} \end{matrix}\right\}=0 \tag{16}$$

We get ω for SSSC Graphene sheets for different modes.

(b) SCSC : The graphene sheets which are simply supported on two opposite edges with clamped conditions at the remaining two edges.

$Y(0) = 0$

$Y(b) = 0$

$$\frac{\partial Y}{\partial y} = 0 \text{ at } y = 0, b$$

On applying the boundary condition we get

$$A + C = 0$$

$$B\frac{\psi_1}{b} + D\frac{\psi_2}{b} = 0$$

$$A\cosh(\psi_1) + B\sinh(\psi_1) + C\cos(\psi_2) + D\sin(\psi_2) = 0$$

$$A\frac{\psi_1 \sinh(\psi_1)}{b} + B\frac{\psi_1 \cosh(\psi_1)}{b} - C\frac{\psi_2 \sin(\psi_2)}{b} + D\frac{\psi_2 \cos(\psi_2)}{b^2} = 0$$

$$\begin{Bmatrix} 1 & 0 & 1 & 0 \\ 0 & \frac{\psi_1}{b} & 0 & \frac{\psi_2}{b} \\ \cosh(\psi_1) & \sinh(\psi_1) & \cos(\psi_2) & \sinh(\psi_2) \\ \frac{\psi_1 \sinh(\psi_1)}{b} & \frac{\psi_1 \cosh(\psi_1)}{b} & \frac{-\psi_2 \sin(\psi_2)}{b} & \frac{\psi_2 \cos(\psi_2)}{b} \end{Bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = 0$$

For nontrivial solution

$$\begin{Bmatrix} 1 & 0 & 1 & 0 \\ 0 & \frac{\psi_1}{b} & 0 & \frac{\psi_2}{b} \\ \cosh(\psi_1) & \sinh(\psi_1) & \cos(\psi_2) & \sinh(\psi_2) \\ \frac{\psi_1 \sinh(\psi_1)}{b} & \frac{\psi_1 \cosh(\psi_1)}{b} & \frac{-\psi_2 \sin(\psi_2)}{b} & \frac{\psi_2 \cos(\psi_2)}{b} \end{Bmatrix} = 0 \quad (17)$$

We get ω for SCSC Graphene sheets for different modes.

$$\lambda = \omega a^2 \sqrt{\frac{\rho h}{D}} \quad \text{[Non-dimensional frequency]}$$

Results and Discussions

The governing equations for vibration of nonlocal graphene sheets are written in Eq. (10). It can be seen that putting $\mu = 0$ in these equations traditional local elastic graphene sheet vibration equations are obtained. These governing equations for local elasticity theory are same as expressed in Reddy [22]. Natural frequencies are nondimensionalized as $\lambda = \omega a^2 \sqrt{\frac{\rho h}{D}}$.

Nondimensional natural frequencies are computed for the graphene sheets with various boundary conditions. These results for SSSS, SSSC and SCSC boundary conditions using numerical approximation are listed in Tables-1 to 3, respectively. Further computation for a graphene sheet is carried out using finite element method (FEM) and results are listed in Tables-1 to 3. In the finite element model a convergence study is carried out and 900 of 8 noded isoparametric elements are employed. Here one could observe that the present results are in good agreement with the results reported by Leissa [33] and the FEM computations [32].

Further, putting $b = \infty$ in the Levy's solution of the governing differential equation nonlocal solutions for free vibration of beam are obtained. These derived equations do match with the nonlocal equations associated with free vibration of nanobeam (Reddy [16]). Euler-Bernoulli theory (EBT) is considered in the analysis [16]. A beam with elastic modulus E_b = 30GPa, length L_b = 10 m, height h_b = varied, density ρb = 1 kg/m^3 is considered. Non-dimensional natural frequencies are expressed as $\overline{\omega}_b = \omega_b \times L_b^2 \sqrt{\frac{\rho_b h_b}{E_b I_b}}$. Comparison of these beam results with those available in the literature are presented in Table-4. Present results for the nanobeams are in good agreement with those reported in the literature [16].

Table-1 : Nondimensional Natural Frequencies for SSSS Graphene Sheet

m	n	Leissa [33]	FEM [32]	Present
1	1	19.739	19.739	20.016
	2	49.348	49.373	50.041
	3	98.696	98.821	100.084
2	1	49.348	49.368	50.041
	2	78.956	78.924	80.067
	3	128.304	128.372	130.108

Table-2 : Nondimensional Natural Frequencies for SSSC Graphene Sheet

m	n	Leissa [33]	FEM [32]	Present
1	1	25.290	23.642	23.998
	2	59.834	58.618	59.471
	3	114.117	113.273	114.821
2	1	54.899	51.651	52.400
	2	89.443	86.126	87.348
	3	148.726	133.810	142.825

Table-3 : Nondimensional Natural Frequencies for SCSC Graphene Sheet

m	n	Leissa [33]	FEM [32]	Present
1	1	32.076	29.938	29.357
	2	71.554	69.309	70.299
	3	130.772	129.141	130.909
2	1	61.685	54.654	55.188
	2	101.163	94.570	95.354
	3	160.381	140.180	156..036

For various nonlocal parameters and sizes of the graphene sheets the frequency ratios are plotted in Figs.2-8. Frequency ratios is defined as the ratio of the frequency obtained using nonlocal elasticity theory to the frequency obtained using local elasticity theory ($\mu = 0$). From these Figs.2-8, one could observe that for single layered graphene sheets, frequency ratios calculated using local and nonlocal elasticity theory depend on (i) size (length or breadth) of the graphene sheet (ii) mode of the vibration (iii) nonlocal parameter and (iv) boundary conditions. In the present work the graphene sheet is considered to be square shape. From these Figs.2-8, one could note that profound scale effect for smaller size graphene sheet and higher values of nonlocal parameter. Further, from these figures it can be observed that lower frequency ratio is obtained at higher values of nonlocal parameter. Furthermore it can be observed that as length increases, frequency ratio increases. This observation is attributed to the fact that nonlocal effect is more profound in the case of small nano lengths graphene sheets. As length increases, frequency ratio also increases. At length > 30nm effect of nonlocal parameter can be neglected. Frequency ratios for various lengths of the graphene sheet and various modes

Table-4 : Nondimensional Natural Frequencies of Beams Using EBT

L/h	μ	Nondimensional Natural Frequency from EBT [16]	Nondimensional Natural Frequency from EBT [Present]
100	0.0	9.8696	9.8696
	0.5	9.6347	9.6347
	1.0	9.4159	9.4159
	1.5	9.2113	9.2113
	2.0	9.0195	9.0195
20	0.0	9.8696	9.8696
	0.5	9.6347	9.6347
	1.0	9.4159	9.4158
	1.5	9.2113	9.2112
	2.0	9.0195	9.0194

of vibration are plotted in Figs.3,5,7. The value of the nonlocal parameter (μ) is assumed to be 2 nm^2. It can also be observed that the frequency ratios decrease with increase in vibration modes. This reveals that nonlocal parameter is more prominent in higher vibration modes and one should include the nonlocal elasticity theory.

Conclusions

Based on Eringen's differential constitutive equations of nonlocal elasticity equations of motion of the graphene sheets are derived. The equations of motion are then solved by Levy's approach to obtain closed form solution for the free vibration of graphene sheets with various boundary conditions. Results obtained using present methodology are in good agreement with the results available in the literature. Effects of (i) nonlocal parameter, (ii) size of the graphene sheets and (iii) boundary conditions on vibration response are investigated. Nondimensional frequencies decrease with increase in mode number. As the size of the graphene sheets decreases the effect of nonlocal theory becomes more significant and predicts smaller nondimensional natural frequencies.

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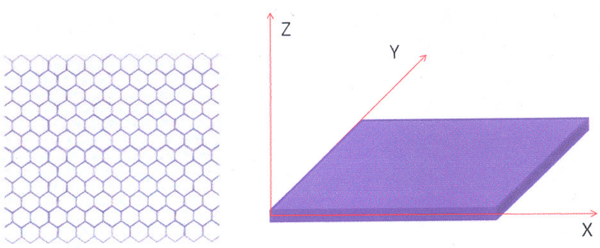

Fig.1 Model of Single Layered Graphene Sheet

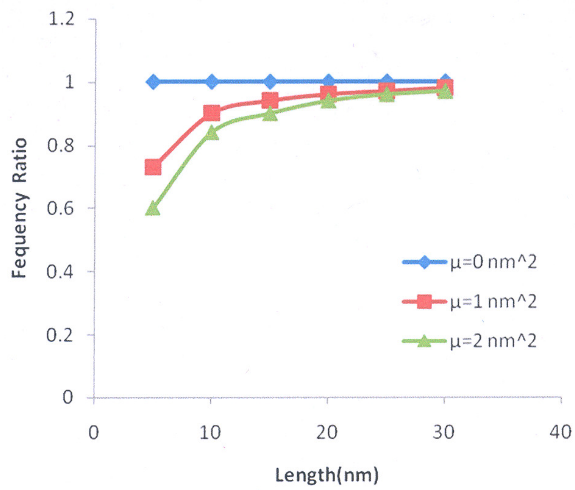

Fig.2 Variation of Natural Frequencies Ratio with the Length of a Square Graphene Sheet (SSSS) for Various Nonlocal Parameter

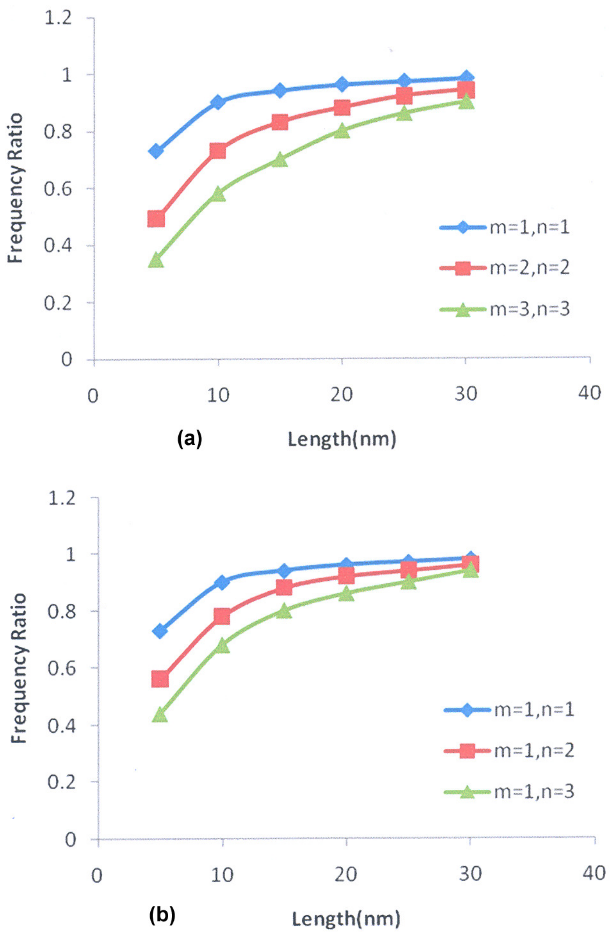

Fig.3 Variation of Natural Frequencies Ratio with Length of a Square Graphene Sheet (SSSS) for Various Modes of Vibration : (a) m = n (b) m ≠ n

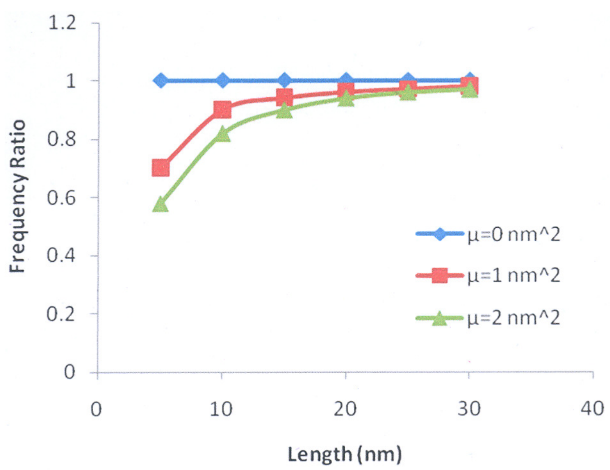

Fig.4 Variation of Natural Frequencies Ratio with the Length of a Square Graphene Sheet (SSSC) for Various Nonlocal Parameter

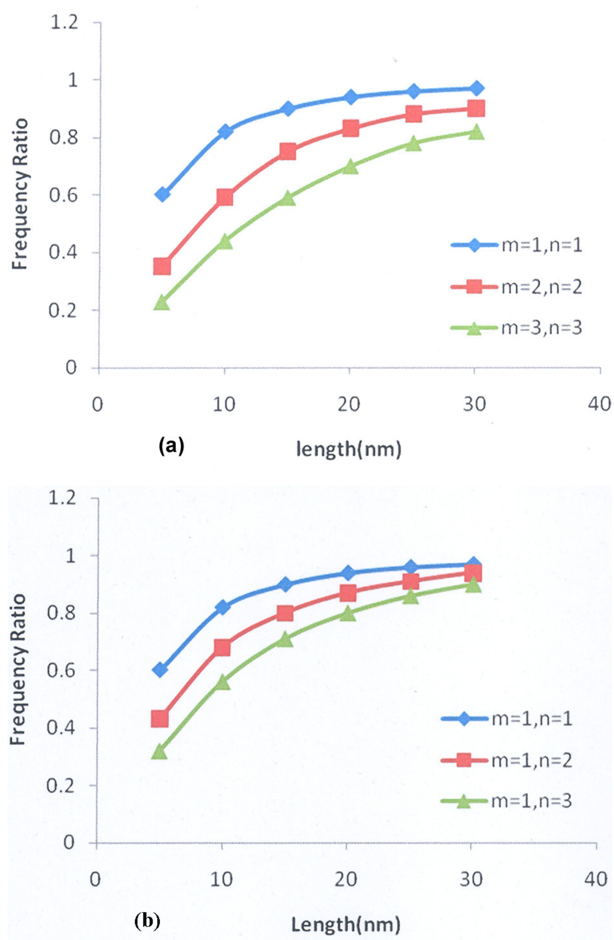

Fig.5 Variation of Natural Frequencies Ratio with Length of a Square Graphene Sheet (SSSC) for Various Modes of Vibration : (a) m = n (b) m ≠ n

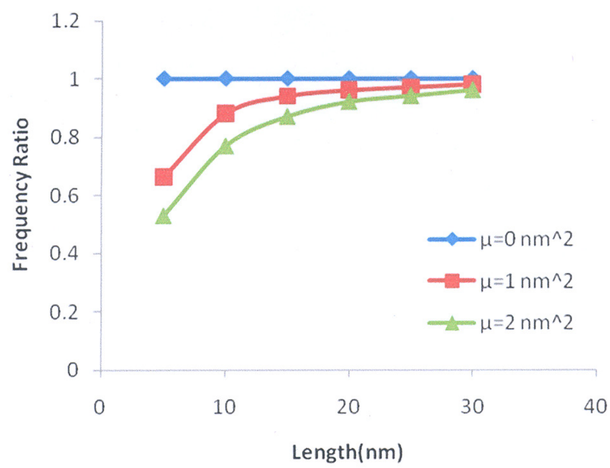

Fig.6 Variation of Natural Frequencies Ratio with the Length of a Square Graphene Sheet (SCSC) for Various Nonlocal Parameter

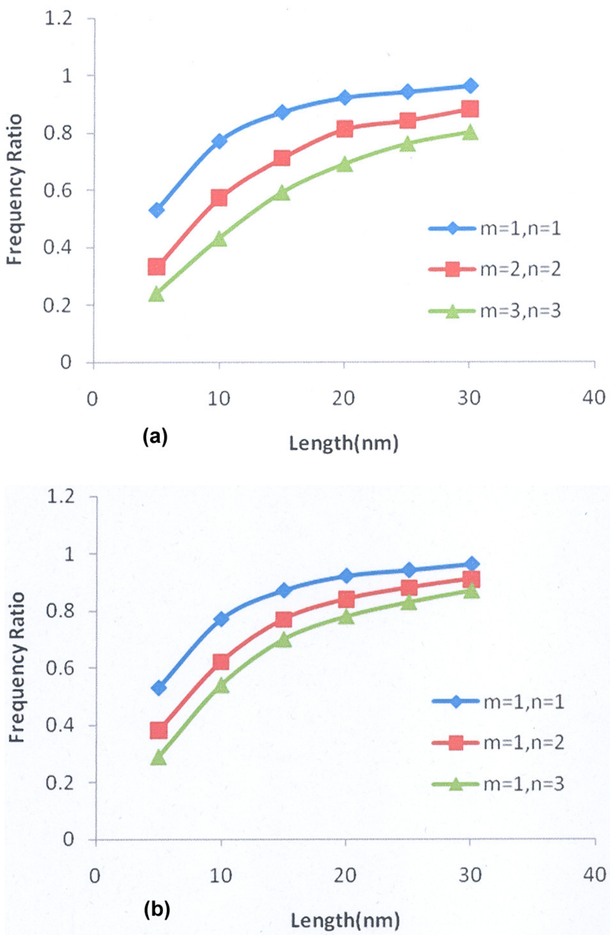

Fig.7 Variation of Natural Frequencies Ratio with Length of a Square Graphene Sheet (SCSC) for Various Modes of Vibration : (a) $m = n$ (b) $m \neq n$

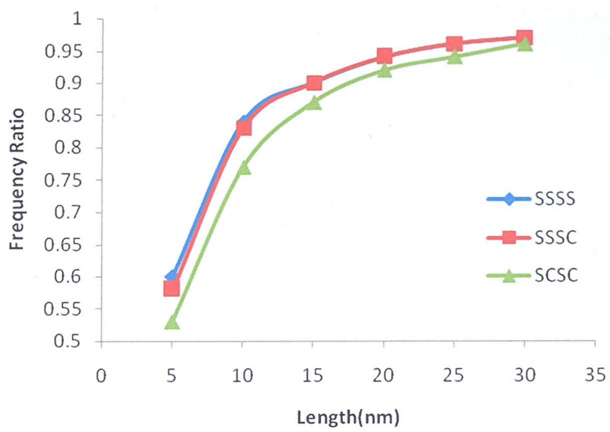

Fig.8 Variation of Natural Frequencies Ratio with Length of a Square Graphene Sheet for Various Boundary Condition (SSSS, SSSC, SCSC)