TECHNICAL NOTE

STUDY OF ACTUATION OF PLATE STRUCTURES BY PIEZOELECTRIC ACTUATORS THROUGH MODAL AND HARMONIC ANALYSES

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Abstract

The concept of intelligent structure or smart structure can be used for abatement or elimination of vibration of structures used in aerospace industry. Through active vibration control the performance of structures can be improved significantly. A three-dimensional finite element analysis of a cantilever plate with bonded piezoelectric actuators is carried out, with a view to seeking the optimal location of actuators on the plate. ANSYS software package, finite element program is used to carry out modal analysis and harmonic response analysis. The analysis points up the fact that active damping of structural vibration of the cantilever plate can be achieved by bonded piezoelectric actuators with optimal location.

Keywords: *Piezoelectric actuator, Flexural displacement, Modal analysis, Harmonic response analysis*

Introduction

Structures used in space applications, aircrafts, and so forth need to be light in order to bring down the cost of carriage by flight. Vibration of large amplitude occurs in these structures that are caused by low inherent damping of the construction materials and the flexibility of the structure. Vibration can cause structural instability. This can adversely affect precision and operational performance. Vibrations can be the cause of unpleasant motions and dynamic stresses leading to fatigue-failure of the structure. Energy loss, under-performance and noise production will result, [1, 2 and 3]. Vibration analysis of structures is, therefore, necessary for the elimination of vibration and noise. Vibration problems continue to affect the performance of many structures. Vibration restricts the reliability and maintainability of structures. Large aerospace structures have shells and plates. The dynamic response is complex, and can differ from the measurements made on the ground or from analytical results. Thus there is need to control and stabilize the structure during operation. In this context the concept of "Intelligent Structures" is invoked as a solution option. Flexible structures with either bonded or embedded sensors and actuators - made of piezoelectric materials - are termed intelligent structures. They have self-monitoring and self-controlling capabilities. They can detect and generate a number of modes of vibration. An active control system associated with intelligent structures can effectively control and suppress vibration of such structures. By virtue of two characteristics of piezoelectric materials, structures can be made smart or intelligent. The first characteristic is the direct piezoelectric effect and the second is the ability to be used as sensor or actuator in active vibration control systems. Use of piezoelectric actuators is a good solution to the problem of vibration damping where they are patched onto the flexible structure as in aerospace industry, [4, 5]. The study investigates the influence of piezoelectric patches symmetrically bonded to the opposite plate surface, on the dynamic behaviour of structures, [6, 7, 8, 9, 10]. Finite element approach is made use of to simulate the dynamic behaviour of a cantilevered plate with bonded piezoelectric actuator receiving a time-varying voltage input. The optimal positions and configuration of the piezoelectric patches can also be understood.

The analytical behaviour of planar patches of piezoelectric material symmetrically bonded at the top and bottom surfaces of elastic structures was studied by

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Dimitriadis and others, [11]. An analytical model and an experimental study were made by Charette and others for the response of plates actuated by piezoceramic elements, [12]. Vibration control of composite structures with bonded piezoelectric sensors and actuators was investigated by Lin and Huang, [13]. They also worked out a formulation methodology for vibration suppression. Hwang and Park presented a finite element formulation for vibration control of a laminated plate with piezoelectric sensors and actuators, [14] Yaqoob Yasin and others made a finite element analysis of actively controlled smart plate with patched actuators and sensors, [15]. Kumar and Narayanan studied for the optimal location of piezoelectric sensors and actuators for vibration abatement, [16]. Karaguille and others simulated active vibration control in smart structures using ANSYS code, [17].

This work is on finite element analysis of cantilevered plate structures with bonded piezoelectric actuators in order to seek the influence of actuator placement and configuration piezoelectric actuators bonded to the surface of plate so that optimal location and configuration of the piezoelectric actuators may be known for selective modal excitation for the purpose of vibration suppression.

In this study the finite element code ANSYS is used to model and analyze smart structure which consists of a structure having bonded piezoelectric actuators. Vibration displacement profile of the plate that has been excited by piezoelectric actuators is realized. The procedure is validated by running an analysis in ANSYS of a piezoelectric beam, (bimorph beam) electrically excited, and comparing the results with the analytical result for the same, viz,, bimorph beam.

Piezoelectric Effect

Piezoelectric material exhibits both direct and converse piezoelectric effects. Electric field generation as a response to mechanical strains is the direct effect which is used in piezoelectric sensors. In piezoelectric actuators the converse effect is exploited which is mechanical strain in response to electric field.

Modeling of the structure

The material behaviour of piezoelectric materials like sensors and actuators can be modeled by the following constitutive equations involving two mechanical variables and two electrical variables.

$$\{\sigma\} = [c]\{\varepsilon\} - [e]^T\{E\} \tag{1}$$

$$\{D\} = [e]\{\varepsilon\} - [g]\{E\} \tag{2}$$

In the above the mechanical variables σ and ε denote the stress and strain respectively. The electrical variables D and E denote the electric displacement and the electric field respectively. The material properties are denoted by the matrices $[c]$, $[e]$ and $[g]$ which are elasticity matrix, the piezoelectric matrix and the dielectric matrix respectively.

The piezoelectric matrix $[e]$ accounts for the piezoelectric effect, viz. the intrinsic coupling between mechanical and electric field. Eq.(1) characterizes the converse piezoelectric effect that fits piezoelectric materials to the use as actuators, while Eq.(2) characterizes the direct piezoelectric effect which makes piezoelectric materials useful as sensors.

The material behaviour of host structures made of conventional materials can be modeled by classical constitutive equations. Without the piezoelectric matrix $[e]$, Eq.(1) and (2) represent classical constitutive equations for structural and electrical fields, respectively.

Finite element equations for piezoelectric structures can be established by employing variational principles. Ref. [18] gives the global equation of motion governing a structure system with n degrees of freedom as:

$$\begin{pmatrix} M_{uu} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \ddot{u} \\ \ddot{\phi} \end{pmatrix} + \begin{pmatrix} C_{uu} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \ddot{u} \\ \ddot{\phi} \end{pmatrix} + \begin{pmatrix} K_{uu} & K_{u\phi} \\ K_{u\phi}^T & K_{\phi\phi} \end{pmatrix} \begin{pmatrix} u \\ \phi \end{pmatrix} = \begin{pmatrix} F_u \\ F_\phi \end{pmatrix} \tag{3}$$

where u denotes structural displacements, ϕ denotes electrical potential, and a dot above a variable denotes a time derivative; M_{uu} is the structural mass matrix, C_{uu} is the structural damping matrix, K_{uu} is the piezoelectric coupling matrix, $K_{\phi\phi}$ is the dielectric stiffness matrix, the superscript T denotes transpose of a matrix, F_u denotes the structural load and F_ϕ denotes the electrical load.

Validation

The finite element code ANSYS carries out the modal and harmonic response analyses in this work. The reliability of the results yielded by ANSYS is established by first running an analysis in ANSYS and then comparing the results with theoretical solution. A fairly good match between the two results will give confidence to accept the results from ANSYS of the analysis at hand.

A piezoelectric bimorph, as shown in Fig.1, is subjected to finite element analysis with ANSYS. The beam is made of two identical PVDF uniaxial beams with opposite polarities. The properties of PVDF are shown in Table-1.

Table-1

Material Properties	
Young's Modulus E_1	2 GPa
Poisson's ration ν_{12}	0.29
Shear Modulus G_{12}	0.775 GPa
Piezoelectric strain coefficients d_{31}	2.2e-11 C/N
d_{32}	0.3e-11 C/N
d_{33}	3.0e-.11 C/N
Relative permittivity at constant stress ε_{33}	12

Beam length, L = 100 mm,

Layer thickness, H =0.5 mm

The theoretical solution [19] is given by the following formula:

$$U_y = \frac{-3\,(d_{31})\,VL^2}{8\,H^2}$$

where E is Young's modulus, V is the applied voltage and H is the thickness of one layer of the beam.

When a unit voltage is applied across the thickness, the deflections at various location of the beam along the length are calculated by the theoretical solution and ANSYS. By applying various voltages the tip deflection is calculated with the exact solution and compared with those yielded by ANSYS. The results are presented in Table-2, Table-3 and Fig.2a and 2b. Comparing the results from Theory and that from ANSYS it is seen that there is close agreement between the two. The near agreement between the two results gives confidence to put reliability in the results yielded by further finite element analyses with ANSYS.

Table-2 : Deflection of PVDF Beam

Axial Distance (mm)	Theoretical Solution (µm)	ANSYS Solution (µm)
20	1.32	1.31
40	5.28	5.26
60	11.88	11.94
80	21.12	21.19
100	33	32.9

Table-3 : Tip Deflection of PVDF Beam Vs Voltage

Voltages (V)	Theoretical Solution (µm)	ANSYS Solution (µm)
100	33	32.9
150	49.5	49.35
200	66	65.8
250	82.5	82.25
300	99	98.7

Finite Element Analysis with ANSYS

A cantilevered steel plate is submitted to finite element analysis employing ANSYS, finite element code. The displacement of the plate when subjected to a varying voltage through bonded piezoelectric actuators is investigated. This analysis is termed harmonic response analysis. The harmonically varying voltage is applied at the natural frequency of the plate vibration. Therefore, in order that the natural frequency of vibration and vibration modes be known, it becomes necessary to carry out a modal analysis of the plate. The modal analysis will yield the modes of vibration and the natural frequencies of vibration. The finite element code, ANSYS, is made use of for the modal analysis.

Modal Analysis

The domain is a steel plate of the following dimensions:

width, (x),= 320 mm
thickness, (y),= 2mm
length, (z),= 360 mm, the unit metre is used in ANSYS.

The properties of the steel plate at hand are :

Young's Modulus = 207 GPa
Poisson's ratio = 0.3, and density = 7870 Kg/m^3.

The element solid45 is used to discretize the steel plate after modeling it as a rectangular block. The steel plate is clamped at z = 0. The Reduced Method is preferred for modal analysis for smart structures. Master Degrees of Freedom (MDOF), are required for Reduced Method. The analysis was carried out first with 100 MDOF and then again with 50 MDOF. Convergence occurred when the number of elements was increased from 286 to 1152, when the difference in the values of frequency was very small.

Result of Modal Analysis

The natural frequencies were identical in two cases and they are, for the six modes

(i) 13.22, (ii) 35.483, (iii) 81.824, (iv) 125.4, (v) 126.11, (vi) 229.87(Hz). The six mode shapes of vibration of plate of plate from Modal Analysis with ANSYS are are shown in Fig.3a - 3f. The first six natural frequencies have more effect than higher frequencies.

Harmonic Response Analysis

Harmonic response analysis was carried out on the same steel plate as the above. On the top surface and bottom surface of the plate piezoelectric actuators were bonded.

The element solid5 modeled the piezoelectric actuator in ANSYS. Finite element modeling of the plate with actuator is identical to that of modal analysis. Electrical load of 250 V was applied to the actuators as shown in Fig.5, and the frequency of excitation voltage was varied. Harmonic response analysis was done with three locations of piezoelectric actuators: (i) Pair of actuators near the free end as shown in Fig.4. Patch size (0.24 x 0.02 x 0.001). (ii) central location of actuators (0.16 x 0.04 x 0.001). (iii) actuators near the right edge of plate (0.2 x 0.02 x 0.001), (units: m). The voltage applied to the actuators was 250 V, Fig.5.

Result of Harmonic Response Analysis

Harmonic response analysis with ANSYS of the steel plate with a pair of actuators excited the plate into vibration displacement profile on applying voltage of 250 V at different frequencies, which were the natural frequencies of vibration, Fig.6. Three locations of actuator were selected, and at each location the frequency was varied:

- **Actuators near the free end of the cantilevered plate**: Vibration displacement profile precisely matching mode shape 6 of plate was excited at f = 234 Hz. Mode shape 4 of plate was excited at f = 20, 40, 86, 135, 130 Hz.
- **Actuator by the right edge of plate**: Except mode shape 4 of plate all other vibration profiles were excited at frequencies near the natural frequency for the relevant mode.
- **Actuator at the centre of plate**: Vibration displacement profiles that match mode shapes 3, 4 and 6 of plate were excited at frequencies neat the natural frequency for the relevant mode.

Fig.6a-f, show six vibration displacement modes yielded by harmonic response analysis.

When the pair of piezoelectric actuators was placed near the clamped edge the electrical load did not impel the steel plate into vibration. Therefore it is inferred that that location is not an optimal one. Fig.7 shows that the maximum deflection (2 mm) is obtained with central location of piezoactuator, at frequency of excitation near 130 Hz. Other two locations also yield the greatest deflection (less than 2 mm) at this frequency. Clearly, the central location of the actuator for a cantilevered steel plate is the preferred one for vibration suppression. For suppression of all the six modes of vibration of the plate it is necessary to bond piezoactuators at all the three locations.

Optimal Location of Piezoelectric Actuators from Harmonic Response Analysis

In harmonic response analysis three selected locations on the steel plate (flank, edge and centre) for the piezoelectric actuators resulted in vibration deflection of the plate, Other locations of the piezoelectric actuators, like ‘near clamped edge’ did not yield vibration deflection profile through harmonic response analysis. Such locations of the piezo-actuator as yield vibration displacement profiles with harmonic response analysis are optimal locations. Thus, by carrying out harmonic response analysis, as above, optimal locations of piezoactuators can be identified.

Effect of Bonding Piezoactuator on Natural Frequency of Vibration

In harmonic response analysis the steel cantilevered plate was excited into vibration at frequencies slightly different from the natural frequencies of vibration as obtained from the modal analysis. This small variation is attributed to the bonding of the piezoelectric actuators on the steel plate. The bonding increases the stiffness of the structure. In modal analysis the host structure, viz. the steel plate without the piezoelectric actuator, was subjected to analysis.

Sum and Conclusions

- A cantilevered steel plate was submitted to a finite element analysis with a view to finding the optimal location of piezoelectric actuators for the purpose of vibration suppression.
- FEA program, ANSYS, was used for the analysis preceded by validation.
- Modal analysis revealed the natural frequencies of vibration.
- Harmonic response analysis with variation in excitation frequency identified the optimal location of piezoelectric actuator.
- Three locations of piezoelectric actuator and variation of frequency at each location yielded vibration displacement profiles through harmonic response analysis. As these three locations yielded deflection profile they are identified as optimal location of actuator.
- Both location and frequency of excitation at each location influenced flexural displacement.
- The input excitation frequency affects the modal response of the smart structure where actuators are properly located.
- Vibration displacement profiles excited, by varying frequency in harmonic response analysis, are not exact replicas of the mode shapes that modal analysis reveals. This is due to the forces and moments that the piezoelectric actuators transfer to the plate through actuation mechanism.

It is important to select actuator location carefully in order to dampen the vibrations of the plate at these resonant frequencies. More than one pair of actuator or multipair actuators on the plate will provide larger deformation than single-pair actuator, i.e. all optimal locations are to be made use of for piezoelectric actuation. Vibration can be suppressed through energy suppression by placing actuators on optimal locations which have maximum strain energy. The use of piezoelectric actuators for decreasing the vibration of plates is efficient.

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Appendix

The properties of piezoelectric material (No.P1C151) are given below with the input ANSYS script in the form of a macro.

```
mp,dens,2,7800
mp,perx, 2,1.71e-8 ! Permittivity matrix
mp,pery, 2,1.71 e-8
```

```
mp,perz,2,1.71 e-8
tb,piez,2        ! Piezoelectric stress matrix
tbdata,3,-9.6
tbdata,6,-9.6
tbdata,9,15.1
tbdata,14,12
tbdata,16,12
tb,anel,2        ! Piezoelectric stiffness matrix
tbdata,1,10.8e10,6.31e10,6.38e10
tbdata,7,10.8e10,6.38e10
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tbdata,21,1.96e10
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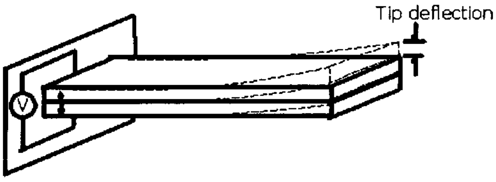


Fig.1 Piezoelectric Cantilever Beam of Two Piezoceramic Sheets. Deflection is produced when a voltage is applied. The arrow marks show the direction of polarization

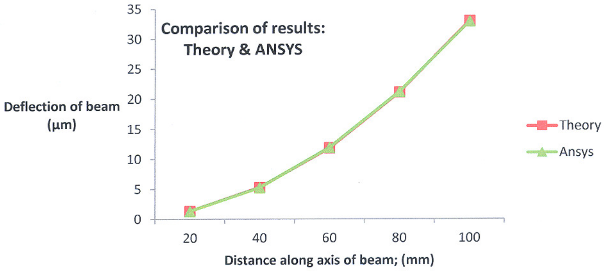


Fig.2a Graphical Presentation of Data of Table-2

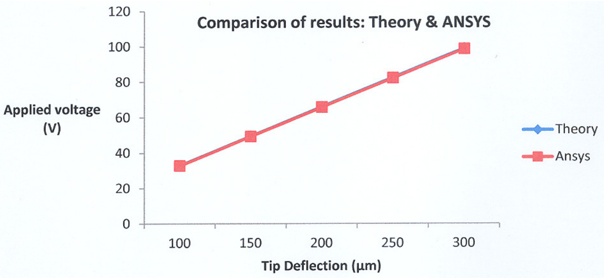
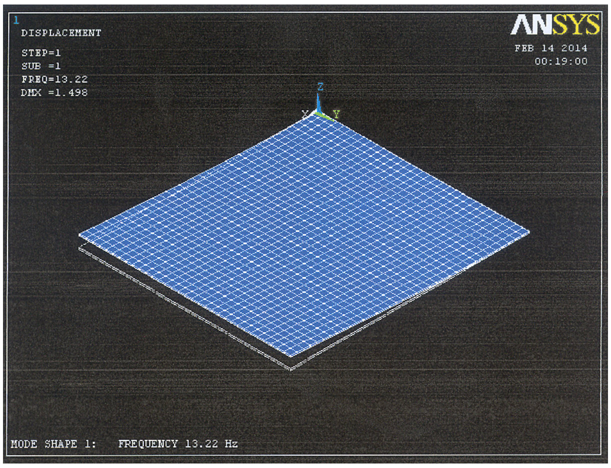


Fig.2b Graphical Presentation of Data of Table-3



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1
DISPLACEMENT
STEP=1
SUB =1
FREQ=13.22
DMX =1.498
FEB 14 2014
00:19:00
MODE SHAPE 1:    FREQUENCY 13.22 Hz
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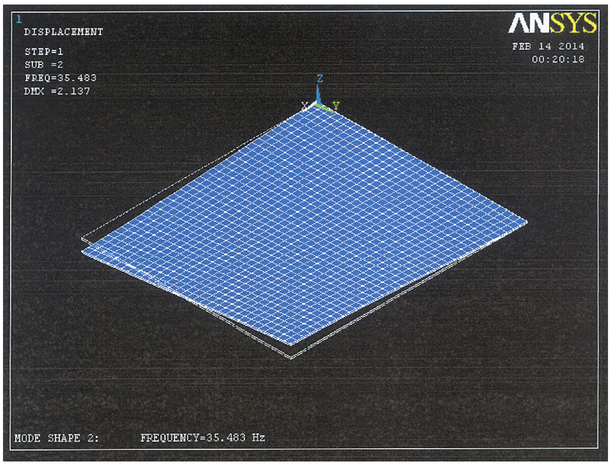
Fig.3a



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DISPLACEMENT
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SUB =2
FREQ=35.483
DMX =2.137
FEB 14 2014
00:20:18
MODE SHAPE 2:      FREQUENCY=35.483 Hz
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Fig.3b

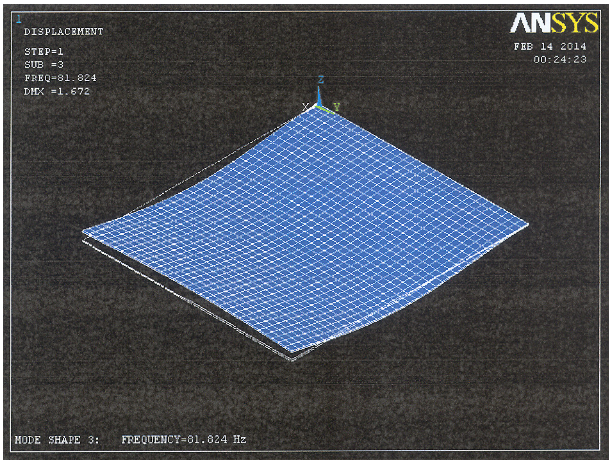

Fig.3c

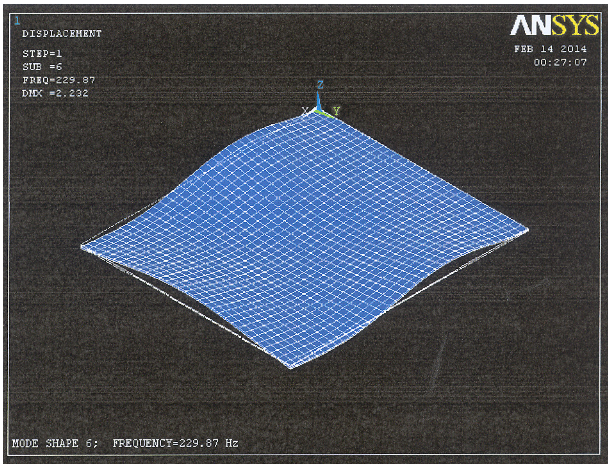

Fig.3f

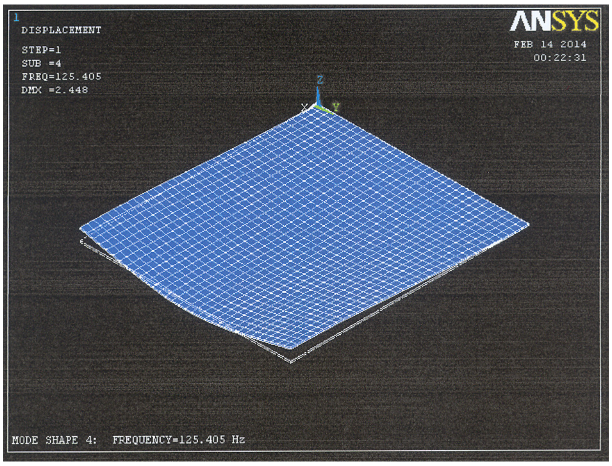

Fig.3d

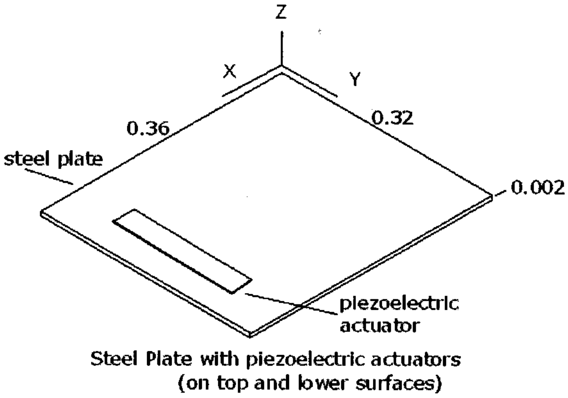

Fig.4 Location of Actuator Near the Free End of Cantilevered Plate

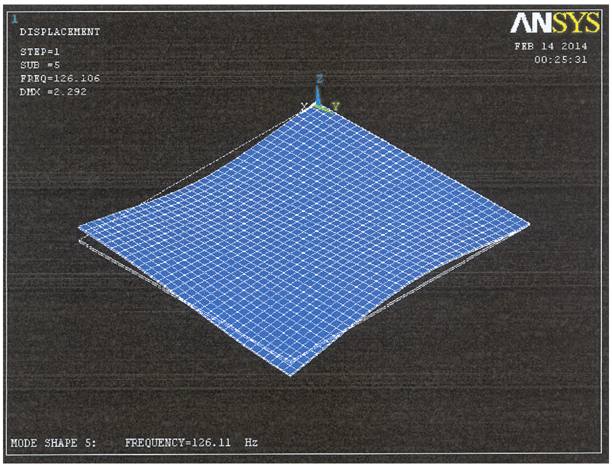

Fig.3e

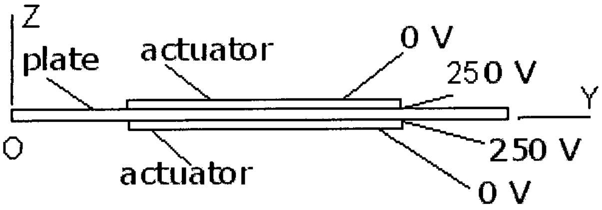

Fig.5 Voltage Applied to Actuators

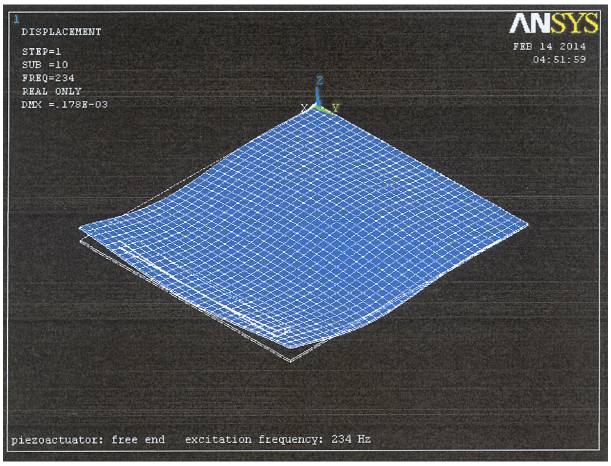

Fig.6a Piezoactuator, Free End, Mode Shape 6

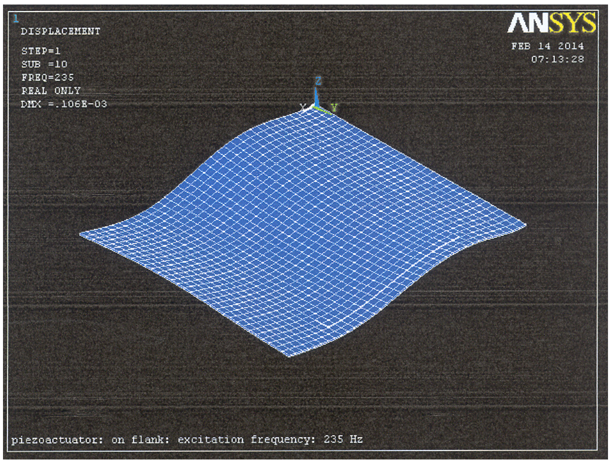

Fig.6d Piezoactuator, Flank, Mode Shape 6

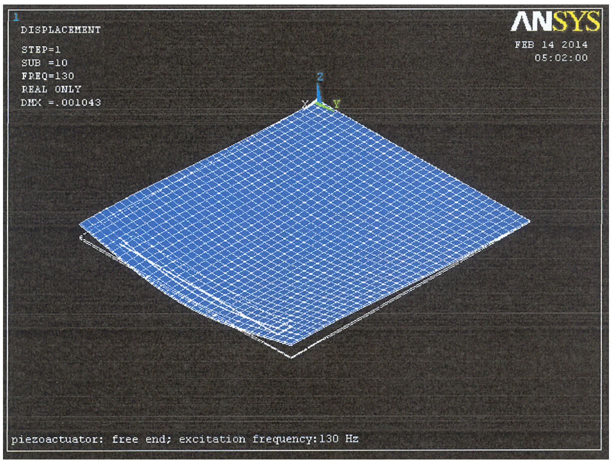

Fig.6b Piezoactuator, Free End, Mode Shape 4

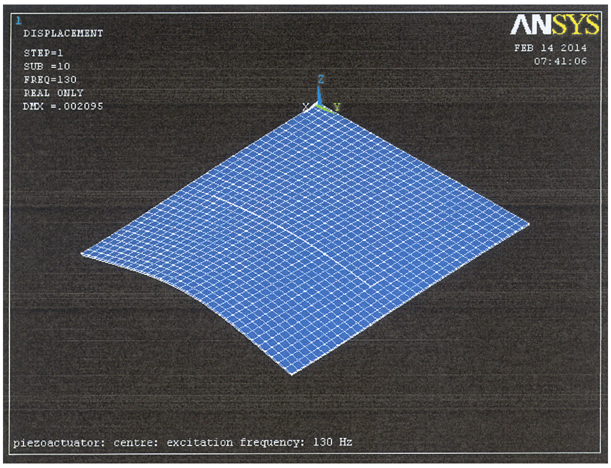

Fig.6e Piezoactuator, Centre, Mode Shape 4

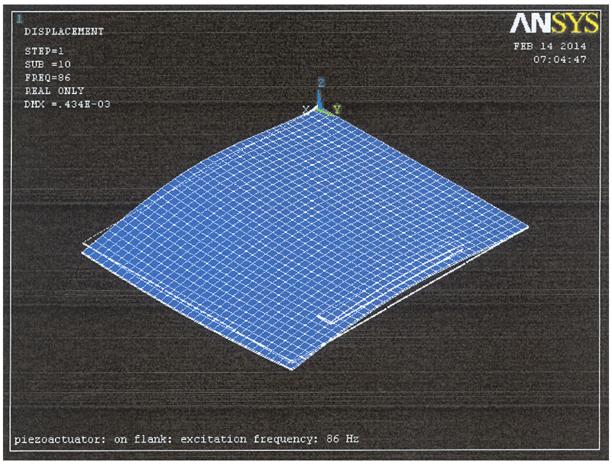

Fig.6c Piezoactuator, Flank, Mode Shape 3

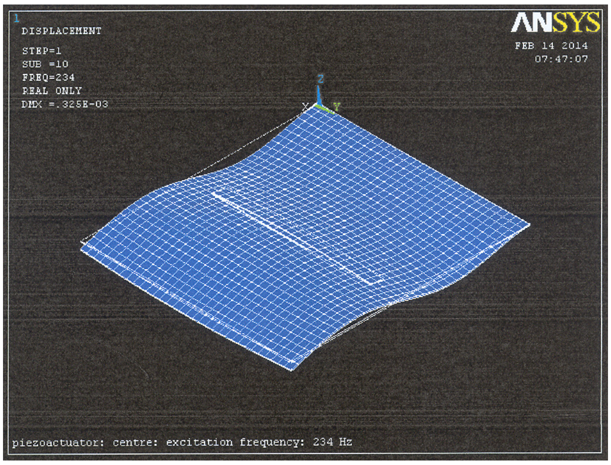

Fig.6f Piezoactuator, Centre, Mode Shape 6

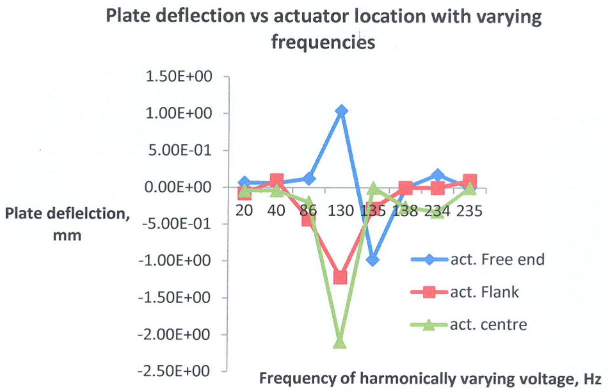

Fig.7 Deflection of Steel Plate for Different Location of Piezoactuator with Varying Frequencies