NUMERICAL AND EXPERIMENTAL ANALYSIS TO PREDICT THE COMPRESSIVE STRENGTH OF PRISTINE COMPOSITE LAMINATES

K.L. Singh, V.R. Ranganath and C.M. Manjunatha
Structural Technologies Division
CSIR National Aerospace Laboratories
HAL Airport Road, Post Box No. 1779
Bangalore-560 017, India

Abstract

In this work, the numerical and experimental work has been carried out to verify the compressive strength of pristine composite laminates. Theoretical computations have been carried out using MATLAB to compute the strength and buckling load. Finite Element Analysis has been carried out for the composite laminate using two and three dimensional elements. PATRAN as the Pre and Post processing package and NASTRAN as the solver have been used in this work. The various failure theories considered in this article are Tsai-Wu, Hill, Hoffman, Maximum Stress and Quadratic failure theory (YSFT). All the first four failure theories are available as a standard criterion in the PATRAN package. The fifth quadratic failure criterion has been implemented using PATRAN Command Language. Static analysis along with failure theories has been carried out to compute the strength of the laminate. Static and buckling analysis has been presented for both in two-and three-dimensional models. Experiments have been carried out on number of pristine laminates and the strain values obtained from FEA have been compared with the experimental values. The theoretical computation provides a load value which is about the elastic limit of the specimen. It is generally observed that quadratic failure theory (YSFT) provides least failure indices and may be employed for conservative design.

Keywords*: Finite element analysis; Buckling; Failure theories; Composite; Compression*

Introduction

The topic of numerical analysis and experimental work comprises the study of a composite laminate under compression loading. The consideration of compression loading for composite is important as their strength in compression is half that of their strength in tension. Secondly the composite laminates are prone to delamination under compressive load. It is important to know the laminate strength and its behavior for their use in critical structures like an aircraft. An in-depth understanding of their structural behavior can help us to exploit the composites to the maximum possible extent.

One of the persistent difficulties in the design and analysis of composites has been prediction of laminate fracture under uni-axial and or combined loading by using either unidirectional composite (ply) data or micromechanics with pristine constituent material properties. The difficulty has been compounded many times over by the availability of many and diverse failure criteria. It is apparent, therefore, that some kind of a formalized comparison among the various failure theories with measured data would be instructive and very useful. The results of Parts A, B and C of an international exercise the 'World-Wide Failure Exercise' (WWFE) to assess the accuracy of current theoretical methods of failure prediction in composite laminates has been published in three special issues of the Journal Composites Science and Technology [1-4]. For the design of laminates, such methods include netting analysis, (assumes that the stresses induced in the structure are carried entirely by the filaments, and the strength of the resin is neglected. It is also assumed that the filaments possess no bending or shearing stiffness, and carry only the axial tensile loads). The proposed theory by Hart-Smith is referred to as the '10% rule' is among the popular failure theories [5].

In this article, both analytical and experimental results for a pristine composite laminate have been discussed. The composite laminate is subjected to uni-axial compression loading. To predict the failure strength, various failure

theories that were considered include Tsai-Wu, Hoffman, Hill, Maximum stress and Quadratic or Yamada Sun Failure Theories (YSFT). In Section-Problem Definition, defines the present problem, and in Section-Theoretical Computation, theoretical / analytical solutions for a composite laminate strength computation. Section-Failure Theories, deals with a discussion on five failure theories that are considered in this article. In Section-Finite Element Analysis is FEA of linear static and buckling analysis. Lastly in Section-Experimental Results discusses the experimental work carried out for pristine composite laminate under uni-axial compression loading.

Problem Definition

Consider a composite square laminate, made of layers of continuous fibers embedded in an organic matrix. The ply orientation of the laminate selected is symmetric with respect to the middle plane of the laminate. The main objective of the study is to evaluate the behavior of a panel consisting of a number of layers / laminae (NL), without any defect (Pristine). Since the composites are critical under compression loading, only uni-axial compression of a laminate is investigated here. The dimensions of the laminate are chosen such that the laminate will not buckle globally due to geometry. Studies were performed to predict the strength of a laminate using theoretical / analytical and Finite Element Analysis (FEA). Table-1 shows the material properties used in the theoretical and FEA.

Boundary Conditions and Loading

The boundary conditions considered on the four edges of the laminate are: (i) 90kN uniform compressive load is applied on the first edge. (ii) The second edge (opposite to the first one) is the fixed boundary condition (displacements) for a three dimensional model are u (T1) = v = w =0; and for the two dimensional model all the six degrees of freedom (u, v, w, R_x, R_y, and R_z) are constrained to zero. (iii) On the remaining two free edges that are restrained by V (knife) edges in test setup, were simulated as v (T2) and w (T3) are constrained to zero for the three-dimensional

model; and for the two-dimensional model except u all other v, w, R_x, R_y, and R_z are constrained to zero.

Material Properties

Computations were carried out by considering the material properties given in Table-1. The laminate layup sequence considered is (45/0/-45/90)$_{3s}$ with thickness of each ply = 0.17 mm making the total laminate thickness to be 4.08 mm. The overall dimensions of the laminate are 150 mm x 150 mm.

Theoretical Computation

Static Strength

It is possible to use classical laminated plate theory (CLPT) [6-7] to arrive at the strength of the laminate (theoretical / analytically). The problem chosen is of pristine laminate. A symmetric laminate subjected to pure compression loading as described in the problem definition and material properties in the earlier section. For geometric and material properties symmetry with respect to the middle surface leads to the condition that all B_{ij} = 0.

Using the CLPT equations, a MATLAB [8] program has been written. The effective Q matrix, obtained from commercial software [9-10] for a laminate will be the same as that of the summed individual Q matrices for each ply. Based on maximum stress criterion, for the pristine laminate, a load of 147.172kN is computed as sustainable using the program.

Buckling Load

From the theory of elastic stability for a plate [11-12] the critical load per unit length (N_x) is given by equation,

$$(N_x)_{cr} = \frac{4\pi^2 D}{b^2} \tag{1}$$

where, $D = \frac{E t^3}{12(1-v^2)}$.

The above equation renders a buckling load of 80.79kN for simply supported and uniformly compressed in one direction.

Table-1 : Mechanical Properties of the Material Used in the Analysis								
Material Properties (MPa)				Strength Properties (MPa)				
E_x	E_y	v_{xy}	G_{xy}	X_T	X_c	Y_T	Y_c	S_{xy}
130000	8000	0.32	3000	585	494	22	28	46

Failure Theories

The various failure theories discussed in this article are: Hill [13], Tsai-Wu [14], Hoffman[15], Quadratic or Yamada Sun Failure Theories (YSFT) [16], and Maximum stress [17]. These are used in the prediction of strength by computing failure indices.

Hill Failure Criterion

According to the Hill Failure Criterion, there is no distinction between tensile and compressive behavior. The failure index is determined based on

$$\frac{\sigma_x^2}{X^2} + \frac{\sigma_y^2}{Y^2} + \frac{\sigma_z^2}{Z^2} - \left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}\right)\sigma_x\sigma_y - \left(\frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}\right)\sigma_y\sigma_z$$

$$- \left(\frac{1}{Z^2} + \frac{1}{X^2} - \frac{1}{Y^2}\right)\sigma_z\sigma_x + \frac{\tau_{xy}^2}{S_{xy}^2} + \frac{\tau_{yz}^2}{S_{yz}^2} + \frac{\tau_{zx}^2}{S_{zx}^2} \geq FI \qquad (2)$$

in which X, Y, Z, S_{xy}, S_{yz}, S_{zx} are maximum allowable stresses and FI is the failure index prescribed by the user.

Hoffman Failure Criterion

The Hoffman Failure Criterion introduces distinction between tensile and compressive stresses to generalize the Hill Failure Criterion, i.e.,

$$C_x\left(\sigma_x - \sigma_y\right)^2 + C_y\left(\sigma_y - \sigma_z\right)^2 + C_z\left(\sigma_z - \sigma_x\right)^2 + \left(\frac{1}{X_t} - \frac{1}{X_c}\right)\sigma_x$$

$$+ \left(\frac{1}{Y_t} - \frac{1}{Y_c}\right)\sigma_y + \left(\frac{1}{Z_t} - \frac{1}{Z_c}\right)\sigma_z + \frac{\tau_{xy}^2}{S_{xy}^2} + \frac{\tau_{yz}^2}{S_{yz}^2} + \frac{\tau_{zx}^2}{S_{zx}^2} \geq FI \qquad (3)$$

where,

$$C_x = \frac{1}{2}\left(\frac{1}{X_tX_c} + \frac{1}{Y_tY_c} - \frac{1}{Z_tZ_c}\right); \quad C_y = \frac{1}{2}\left(\frac{1}{Y_tY_c} + \frac{1}{Z_tZ_c} - \frac{1}{X_tX_c}\right);$$

$$C_z = \frac{1}{2}\left(\frac{1}{Z_tZ_c} + \frac{1}{X_tX_c} + \frac{1}{Y_tY_c}\right)$$

and X_t, X_c, Y_t, Y_c, Z_t, Z_c, S_{xy}, S_{yz}, S_{zx} are maximum allowable stresses and FI is the failure index prescribed by the user.

Tsai-Wu Failure Criterion

The Tsai-Wu failure criterion is another generalization of the Hill failure criterion:

$$\left(\frac{1}{X_t} - \frac{1}{X_c}\right)\sigma_x + \left(\frac{1}{Y_t} - \frac{1}{Y_c}\right)\sigma_y + \left(\frac{1}{Z_t} - \frac{1}{Z_c}\right)\sigma_z + \frac{\sigma_x^2}{X_tX_c} + \frac{\sigma_y^2}{Y_tY_c} + \frac{\sigma_z^2}{Z_tZ_c}$$

$$+ \frac{\tau_{xy}^2}{S_{xy}^2} + \frac{\tau_{yz}^2}{S_{yz}^2} + \frac{\tau_{zx}^2}{S_{zx}^2} + 2F_{xy}\sigma_x\sigma_y + 2F_{yz}\sigma_y\sigma_z + 2F_{zx}\sigma_x\sigma_z \geq FI \qquad (4a)$$

In which X_t, X_c, Y_t, Y_c, Z_t, Z_c, S_{xy}, S_{yz}, S_{zx} are maximum allowable stresses, F_{xy}, F_{yz}, F_{zx} are interactive strength constants and FI is the failure index prescribed by the user. In order for the Tsai-Wu failure surface to be closed, the interactive constants should be bounded by,

$$\left(\frac{1}{x_tx_c}\right)\left(\frac{1}{y_ty_c}\right)\left(\frac{1}{z_tz_c}\right) - F_{12}^2 > 0. \qquad (4b)$$

YSFT or Quadratic Failure Theory

YSFT (Yamada Sun Failure Theory) or Quadratic failure theory [16] is not available, as standard failure theory in the commercial package of the PATRAN. So in PATRAN software package using Patran Command Language (PCL)[18] functions has been implemented and used for computation. X_T and S_{xy} correspond to the ply longitudinal strength and the ply shear strength measured from a symmetric cross-ply laminate. In this the first assumption is $\sigma_y = 0$; the following criterion for laminate failure is proposed

$$\sqrt{\left(\frac{\sigma_x}{X_T}\right)^2 + \left(\frac{\tau_{xy}}{S_{xy}}\right)^2} \geq FI \qquad (5)$$

$FI \geq 1$ failure; $FI < 1$ non-failure.

Maximum Stress Theory of Ply Failure

In the maximum stress failure theory, fiber failure (FF) of the lamina is assumed to occur whenever any normal or shear stress component equals or exceeds the corresponding strength.

This theory is written mathematically as follows, [17]:

$$FI = \max \left(|\frac{\sigma_x}{X_Y}| \, , \, |\frac{\sigma_x}{Y_T}| \, , \, |\frac{\tau_{xy}}{S_{xy}}| \right) \qquad (6)$$

X_T : Tensile strength in longitudinal direction.

X_c : Compressive strength in longitudinal direction.

Y_T : Tensile strength in Transverse direction.

Y_c : Compressive strength in Transverse direction.

S_{xy} : Shear strength in Longitudinal-Transverse (x-y) plane.

Appropriate values of X_t or X_c and Y_t or Y_c are used depending on the sign of σ_x and σ_y respectively for tension or compression. The above equations of FF criterion which are based on experimental experience, means that any additional stresses σ_z, τ_{yz}, τ_{zx} do not influence FF.

Finite Element Analysis

Finite Element Method (FEM) is a numerical method for solving partial differential equations [19]. The concept of FEM is to discretize the field to k discrete elements with simple and normal shapes. The field consists of elements and nodes; each node has three degrees of freedom (DOF) in the case of hexahedral element

Static Analysis

The linear static analysis has been carried out incorporating the failure theories as discussed in Section-Failure Theories. The considered laminate, as presented in Section-Problem Definition, and was checked for 'h' convergence. It is seen that for this configuration, 400 elements of 2646 DOF mesh render best results. Load applied is 90 kN for all the failure indices analysis. Fig.1 shows the magnitude of displacement obtained for the laminate considered. The maximum magnitude of displacement obtained is 0.407 mm as seen in Fig.1. Fig.2 shows the deformation magnitude of 0.404 mm displacement for three dimensional model. It is clear from these two figures that the displacement magnitude differs only in the third decimal, which is less than 1% deviation. Fig.3 shows the variation of strain in the three dimensional model which also shows similar trend as in two dimensional model.

Table-2 lists the stress (σ_x, σ_y and σ_{xy}) values for the four layers only, as the laminate is made of $(0/45/-45/90)_{3s}$ angles, generally known as $\pi/4$ laminate. For all the other plies, the results repeat same value corresponding to the respective ply angle. Table-3 indicates the Failure Indices (FI) obtained from, Tsai-Wu, Hill, Hoffman, Maximum

Table-2 : Maximum Stress Components for the Composite Laminate

Layer No.	Angle Degree	Maximum Stress Components (MPa)		
		σ_x	σ_y	τ_{xy}
L1	45	-181	-14.4	-8.14
L2	0	-355	-6.99	-3.35E-14
L3	-45	-181	-14.4	-8.14
L4	90	-6.99	-21.8	-1.19E-17

Table-3 : Comparison of Failure Indices Obtained from Various Failure Theories

Layer No.	Angle Degree	Failure Indices				
		Tsai-Wu	Hill	Hoffman	Max. Stress	YSFT
L1	45	0.631	0.587	0.622	0.493	0.407
L2	0	0.648	0.610	0.640	0.714	0.718
L3	-45	0.631	0.587	0.622	0.493	0.407
L4	90	0.987	0.985	0.987	0.986	0.014

stress and Yamada Sun Failure Theories. It is observed from the Table-3 the FI values from Tsai-Wu and Hoffman results match closely with each other. The failure indices in the ascending order are Tsai-Wu, Hoffman, Maximum stress, followed by Hill criterion and YSFT. In case of YSFT, it may be noted that an assumption of $\sigma_y = 0$ is considered. Due to the fact that the component σ_y is not considered, it renders the FI as 0.014.

Buckling Analysis

Buckling analysis can be used to find critical loads at which a structure becomes elastically unstable. The governing matrix equation for buckling analysis is a general Eigen value problem of the form:

$$(K + \lambda K_G) u = 0 \qquad (7)$$

Where K is a linear stiffness matrix, K_G is the geometric stiffness (or stress stiffening) matrix, λ is the Eigen value (load factor), and u is the buckling mode shape [20-21], [23].

Figure 4 shows the fringe pattern of buckling mode-I for two-dimensional laminate model. The two-dimensional model of the problem renders a value of 33.0336kN as the load for first buckling mode. Fig.5 shows the fringe pattern of buckling mode-I for three-dimensional laminate model. The three dimensional model of the laminate renders a value of 70.046kN, which is about two times the two-dimensional value.

Experimental Results

Carbon epoxy laminates were fabricated using Vacuum Enhanced Resin Infusion Technology (VERITy) process [24]. All the laminates were cured in an autoclave following a standard cure cycle and then sliced into specimens using a diamond-coated saw. The test laminates are pasted with two to eight strain gauges. Each gauge on the front has corresponding gauge on the back face of the each laminate to aid in ensuring application of pure compressive loading and to detect bending or buckling, or both, if any. This also ensures the alignment of the specimen along the loading direction. All the specimens were tested on a servo-hydraulic computer controlled testing machine of 500kN capacity. The test performed is similar to ASTM D 7137M-07 standard test method for compressive residual strength properties of damaged polymer matrix composite plate [25]. However the dimension chosen for the plate in this article is a square plate of dimension 150 mm without any damage. Fig.6 shows the test setup while carrying out the test.

In Fig.7 there are three curves only, two from experiment and one from FE analysis. Fig.7 shows two experimental curves obtained from a typical test for the front and back mounted strain gauges. These two curves show a very good alignment with the load applied up to 3000 micro strain ($\mu\varepsilon$), where the first instability point occurs. This is considered as an excellent alignment for further analysis and results. The maximum strain withstood by the laminate is around 8000 $\mu\varepsilon$ after which the load immediately falls to zero stress (the return curve) captured by both the front and back strain gauges as shown in Fig.7. The front and back strain gauges suddenly falls to zero stress from about 3000 $\mu\varepsilon$ value and 7500 $\mu\varepsilon$ respectively without any post buckling phenomenon. A drop to zero strain / stress level indicates a complete failure of the laminate. Fig.8 shows the plot of Fig.7 without the failure data. The laminate cannot withstand any further load. The linear static stress analysis has been presented for the pristine laminate. The finite element analysis (FEA) results with square data points compares very well with the experimental stress-strain curve within the elastic regime.

Concluding Remarks

A composite laminate of dimension 150 x 150 x 4.08 mm^3 has been analyzed and experimental results are presented. Theoretical solutions were computed for the laminate using both the classical laminate theory and buckling expressions. Theoretical computations using CLPT technique to determine the strength of a laminate were found to be very attractive, especially when there is no damage. The strength computation from maximum stress theory renders very good approximation, within 5 percent, of the result compared with the experimental result (~130kN). In this article, the five different failure theories are summarized and are compared with reference to $\pi/4$ composite pristine laminate subjected to in-plane compressive load.

It is observed that both the two and three dimensional linear static analysis render a maximum deviation of about 1% in results of stress and strain. However the buckling analysis for the two-and three-dimensional results vary by a factor of 2. The three-dimensional buckling result is closer to theoretical expression and experimental result. It is confirmed from both the two and three-dimensional buckling analysis that the failure, load is less than the ultimate load obtained from a test of the pristine laminate. The experimental result co-relate excellently with the static finite element analysis within the elastic regime.

Acknowledgement

References

1. Hinton, M.J, Soden, P.D. and Kaddour, A.S., "Strength of Composite Laminates Under Biaxial Loads", Applied Composite Materials, 3, 1996, pp.151-162.

2. Hinton, M.J, Kaddour, A.S. and Soden, P.D., "Evaluation of Failure Prediction in Composite Laminates: Background to Part B of the Exercise", Composites Science and Technology, 62, 2002, pp.1481-1488.

3. Hinton, M.J., Kaddour A.S. and Soden, P.D., "Evaluation of Failure Prediction in Composite Laminates: Background to Part C of the Exercise", Composites Science and Technology, 64, 2004, pp.321-327.

4. Soden, P.D., Kaddour, A.S. and Hinton, M.J., "Recommendations for Designers and Researchers Resulting from the World-wide Failure Exercise", Composites Science and Technology, 64, 2004, pp.589-604.

5. Hart-Smith, L.J., "Expanding the Capabilities of the Ten-Percent Rule for Predicting the Strength of Fibrepolymer Composites", Composites Science and Technology, 62, 2002, pp.1515-1544.

6. George Z. Voyiadjis and Peter I. Kattan., "Mechanics of Composite Materials with MATLAB", Springer 2005.

7. Reddy, J.N., "Mechanics of Laminated Composite Plates: Theory and Analysis", CRC press, 1997.

8. http://www.mathworks.in/matlab.htm

9. Ronald F. Gibson., "Principles of Composite Materials Mechanics", McGraw-Hill, Inc., 1994.

10. Robert M. Jones., "Mechanics of Composite Materials, McGraw-Hill, Kogakusha Ltd., 1975.

11. Iyengar, N.G.R., "Buckling and Optimization of Layered Composites", Journal of the Aeronautical Society of India, Vol.41, No.2, 1989.

12. Timoshenko, S.P. and Gere, J.M., "Theory of Elastic stability", International Edition, McGraw-Hill book Company, 1963.

13. Hill, R., "The Mathematical Theory of Plasticity", Oxford University Press, 1950.

14. Tsai, S.W. and Wu, E.M., "A general theory of Strength for Anisotropic Materials", Journal of Composite Materials, Vol.5, No.1, 1971, pp.58-80, doi:10.1177/002199837100500106.

15. Hoffman, O., "The Brittle Strength of Orthotropic Materials", Journal of Composite Materials, Vol.1, No.2, 1967, pp.200-206.

16. Yamada, S.E. and Sun, C.T., "Analysis of Laminate Strength and Its Distribution", Journal of Composite Materials, Vol.12, No.3, 1978, pp.275-284.

17. Puck, A. and Schneider, W., "On Failure Mechanisms and Failure Criteria of Filament-wound Glass-Fibre/resin Composites", Plastics and Polymers, 1969, pp.33-44.

18. http://www.mscsoftware.com/Products/CAE-Tools/

19. Zienckiwicz, O.C. and Taylor, R.L. "The Finite Element Method", 4th Edition, McGraw Hill International editions, 1991.

20. Anil, V., Upadhyay, C.S. and Iyengar, N.G.R., Stability Analysis of Composite Laminate With and Without Rectangular Cutout Under Biaxial Loading", Composite Structures, 80, 2007, pp.92-104.

21. Singh, S., Patel, B. P., Sharma, A., Shukla, K. K. and Nath, Y., "Nonlinear Stability and Dynamics of Laminated Composite Plates and Shells", Springer Proceedings in Physics, Vol.111, No.1,Vibration Problems ICOVP, 2005, pp.415-427.

22. Tekin, M.D. and Altan, M.F., "The Finite Element Analysis of Buckling of Laminated Rectangular Reinforced Concrete Plates with Circular Holes", Mathematical and Computational Applications, Vol.1, No.2, 1996, pp.164-168.

23. Van Der Maas., "The Stability of Elastic Equilibrium", Doctoral Thesis, Delft University, January 10, 2006.

24. Nitesh Gupta and Ramesh Sundaram., "Fiber Optic Sensors for Monitoring Flow in Vacuum Enhanced Resin Infusion Technology (VERITy) Process", Composites: Part A, 40, 2009, pp.1065-1070.

25. ASTM D7137M-07, "Standard Test Method for Compressive Residual Strength Properties of Damaged Polymer Matrix Composite Plate", ASTM International, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959, United States.
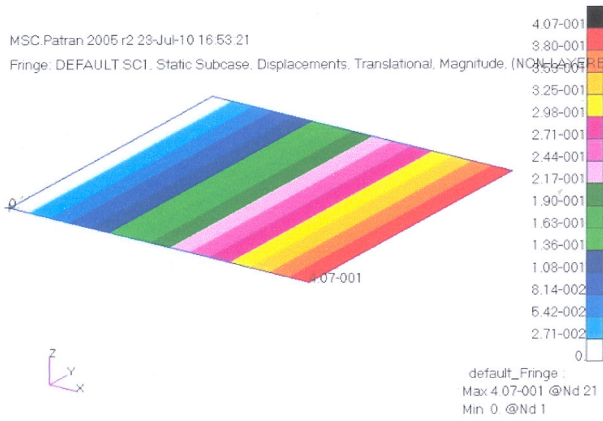
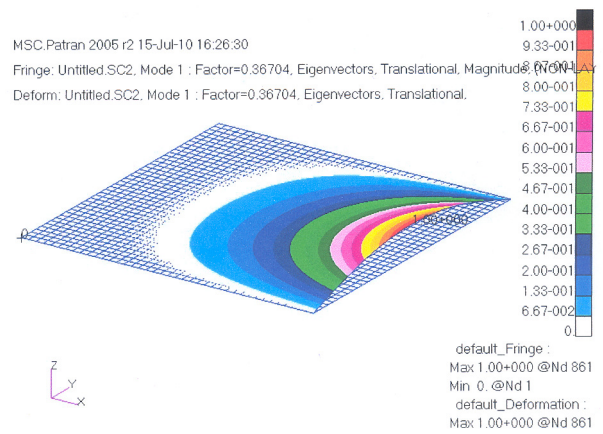*Fig.1 Magnitude of Displacement for 2-dimensional Model*

Fig.2 Magnitude of Displacement for 3-dimensional Model

Fig.3 Axial Strain Tensor for 3-dimensional Laminate in X-direction

Fig.4 Fringe Pattern of Buckling Mode-I for 2-dimensional Laminate

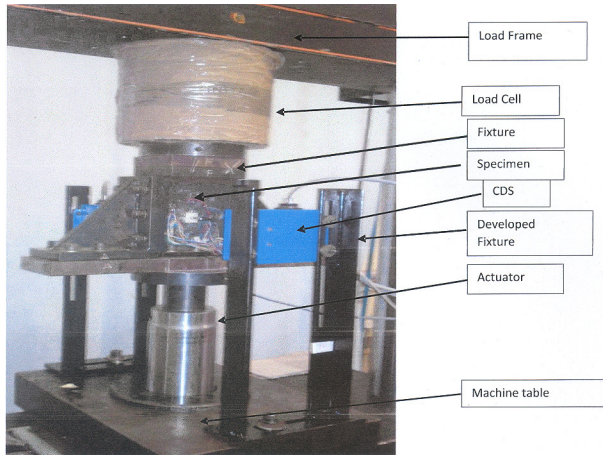*Fig.5 Fringe Pattern of Buckling Mode-I for 3-dimensional Laminate*

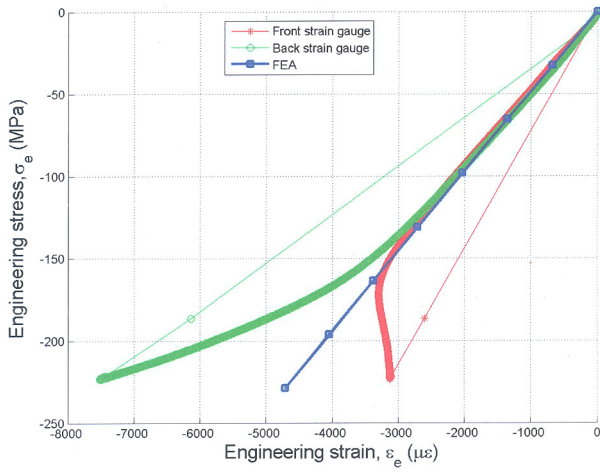*Fig.6 Photograph of the Test Setup During Testing*

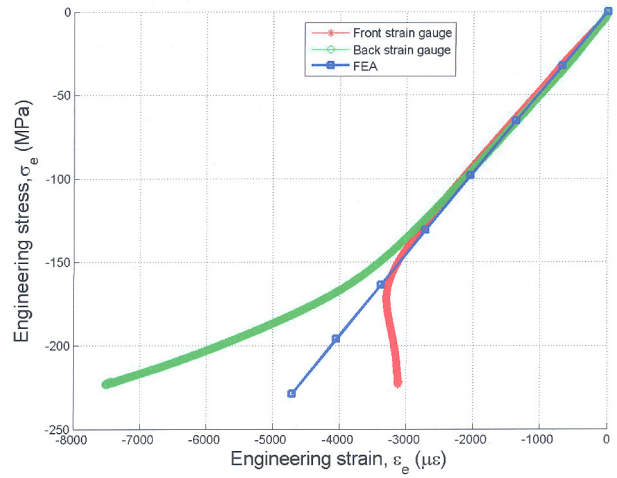*Fig.7 Comparison of Numerical Analysis and Experimental Strain Data*

Fig.8 Plot of FE and Experimental Strain Versus Stress Data Without Failure