POSTBUCKLING OF LAMINATED COMPOSITE CIRCULAR CONICAL SHELLS UNDER COMBINED THERMO-MECHANICAL LOADS

S. Singh*, B.P. Patel** and Y. Nath**

Abstract

In this paper, the postbuckling characteristics of cross- and angle-ply laminated composite conical shells subjected to combined thermo-mechanical loading are studied using shear deformable semi-analytical finite element. The nonlinear governing equations, considering geometric nonlinearity based on Sanders type of kinematic approximations, are solved using Newton-Raphson iteration procedure coupled with adaptive displacement control method to trace the postbuckling equilibrium path. It is brought out from the study that the inward displacement in the postbuckling region is significantly higher as compared to the outward one. The ratio of the lowest load in the postbuckling path and the bifurcation load increases with the increase in the pressure loading proportion and decreases with the increase in the axial loading proportions. The relative load carrying capacity of shells under different set of loading combinations changes with the increase in the maximum postbuckling displacement. It is also observed that the nonlinearity in the load interaction relation corresponding to lowest load in the postbuckling path is less compared to that for bifurcation load.

Keywords: Postbuckling, Combined loading, Semi-analytical, Adaptive displacement control, Shell

Introduction

The laminated composite conical shells are increasingly being used as stand alone component or transition elements between cylinders of different diameters in various engineering applications, such as tanks and pressure vessels, missiles and spacecraft, submarines etc. These shells may often be subjected to external pressure, axial compression, torsional and thermal loading individually or in combination. Compressive membrane state of stress introduced due to the above loading conditions may lead to the loss of stability by buckling and be a crucial failure phenomenon especially for thin shells. It may also be necessary to understand the ensuing behavior after buckling. Further, the traditional shell design approach based on the linear bifurcation buckling load reduced by a knock-down factor sometimes may result in overly conservative designs or even potentially unsafe ones. Thus the determination of the critical load and the better understanding of the postbuckling behavior under various loads are some of the important problems for the design of light-weight laminated structures.

The investigation on the buckling analysis of laminated composite conical shells subjected to mechanical loading has received the attention of many researchers e.g. the work of Goldfeld and Arbocz [1], and Goldfeld et al. [2] and the references cited therein. The variation of the stiffness coefficients for the buckling analysis of filament wound angle-ply laminated conical shells is accounted in Refs. [1, 2]. The studies on the thermoelastic buckling/postbuckling characteristics of the laminated conical shells are limited [3-5]. The investigations on the buckling/postbuckling under combined loading conditions are carried out considering laminated cylindrical shells [6-15]. To the best of the authors knowledge, the study on the buckling/postbuckling of laminated conical shells subjected to combined loading is not dealt in the literature. This may be attributed to the inherent complexity of the basic equations in curvilinear circular conical coordinates that are a system of coupled nonlinear partial differential equations with variable coefficients.

The semi-analytical approach based on the trigonometric series for the expansion of the displacement field

* Department of Mechanical Engineering, Kumaon Engineering College, Dwarahat, Almora-263 653, India
** Department of Applied Mechanics, Indian Institute of Technology Delhi, Hauz Khas, New Delhi-110 016, India
Email : badripatel@hotmail.com

in the circumferential direction for the buckling/postbuckling studies of circumferentially closed circular shells under axisymmetric prebuckling stress state is preferred due to its computational efficiency [16-19]. In the above-cited work on semi-analytical approaches, the transverse shear deformation effect is neglected. However, for the laminated composite circumferentially closed shells the transverse shear deformation may be significant even for the thin cases due to smaller wavelength of the buckling/postbuckling deformation to thickness ratio and low transverse shear modulii. The objective of the present work is to study the pre- and post-buckling characteristics of cross- and angle-ply laminated composite conical shells subjected to combined loading using shear deformable semi-analytical finite element.

Formulation

A laminated composite circular conical shell is considered with the coordinates s, θ and z along the meridional, circumferential and radial/thickness directions, respectively, as shown in Fig.1. The displacements, u, v, w at a point (s, θ, z) from the median surface are expressed as functions of mid-surface displacements u_o, v_o and w_o, and independent rotations β_s, and β_θ of the meridional and hoop sections, respectively, as

$$u(s, \theta, z) = u_o(s, \theta) + z\,\beta_s(s, \theta)$$

$$v(s, \theta, z) = v_o(s, \theta) + z\,\beta_\theta(s, \theta)$$

$$w(s, \theta, z) = w_o(s, \theta) \tag{1}$$

Using the semi-analytical approach, u_o, v_o, w_o, β_s and β_θ are represented by a Fourier series in the circumferential angle θ. For the nth harmonic, these can be written as

$$u_s(s, \theta) = u_o^o(s) + \sum_{i=1}^{M_1}\left[u_o^{c_i}(s)\cos(in\,\theta) + u_o^{s_i}(s)\sin(in\,\theta)\right]$$

$$v_o(s, \theta) = v_o^o(s) + \sum_{i=1}^{M_1}\left[v_o^{c_i}(s)\cos(in\,\theta) + v_o^{s_i}(s)\sin(in\,\theta)\right]$$

$$w_o(s, \theta) = w_o^o(s) + \sum_{i=1}^{M_2}\left[w_o^{c_i}(s)\cos(in\,\theta) + w_o^{s_i}(s)\sin(in\,\theta)\right]$$

$$\beta_s(s, \theta) = \beta_s^o(s) + \sum_{i=1}^{M_2}\left[\beta_s^{c_i}(s)\cos(in\,\theta) + \beta_s^{s_i}(s)\sin(in\,\theta)\right]$$

$$\beta_\theta(s, \theta) = \beta_\theta^o(s) + \sum_{i=1}^{M_2}\left[\beta_\theta^{c_i}(s)\cos(in\,\theta) + \beta_s^{s_i}(s)\sin(in\,\theta)\right]$$

$$\tag{2}$$

where superscript o refers to the axisymmetric component of displacement field variables, and c_i and s_i refer to the asymmetric components of the field variables having the circumferential variation proportional to $\cos(in\theta)$ and $\sin(in\theta)$, respectively. The approximation of the field variables in the meridional direction is made using a C^0 continuous three-noded element having $4M_1 + 6M_2 + 5$ nodal degrees of freedom.

The strain-displacement relations are based on Sanders [20] type of kinematic approximations: (i) small strains, (ii) moderately large rotations; and (iii) thin shell ($z/r << 1$) such that $1+z/r \approx 1$, however, transverse shear deformation is important due to smaller wavelength of the postbuckling deformation to thickness ratio and lower G/E ratio. The Green's strains are written in terms of the mid-surface deformations as,

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_p^L \\ 0 \end{Bmatrix} + \begin{Bmatrix} z\varepsilon_b \\ \varepsilon_s \end{Bmatrix} + \begin{Bmatrix} \varepsilon_p^{NL} \\ 0 \end{Bmatrix} \tag{3}$$

where, the linear membrane strains $\left\{\varepsilon_p^L\right\}$, bending strains $\left\{\varepsilon_b\right\}$, shear strains $\left\{\varepsilon_s\right\}$ and nonlinear membrane strains $\left\{\varepsilon_p^{NL}\right\}$ in Eq. (3) are written as [20, 21].

$$\left\{\varepsilon_p^L\right\} = \begin{Bmatrix} \dfrac{\partial u_o}{\partial s} \\[2mm] \dfrac{u_o \sin\phi}{r} + \dfrac{\partial v_o}{r\partial\theta} + \dfrac{w_o \cos\phi}{r} \\[2mm] \dfrac{\partial u_o}{r\partial\theta} - \dfrac{v_o \sin\phi}{r} + \dfrac{\partial v_o}{\partial s} \end{Bmatrix};$$

$$\{\varepsilon_b\} = \begin{Bmatrix} \dfrac{\partial \beta_s}{\partial s} \\[2mm] \dfrac{\beta_s \sin\phi}{r} + \dfrac{\partial \beta_\theta}{r\partial\theta} \\[2mm] \dfrac{\partial \beta_s}{r\,\partial\theta} + \dfrac{\partial \beta_\theta}{\partial s} - \dfrac{\beta_\theta \sin\phi}{r} + \dfrac{\cos\phi}{r}\beta_n \end{Bmatrix};$$

$$\{\varepsilon_s\} = \begin{Bmatrix} \beta_s + \dfrac{\partial w_o}{\partial s} \\[2mm] \beta_\theta + \dfrac{\partial w_o}{r\,\partial\theta} - \dfrac{v_o \cos\phi}{r} \end{Bmatrix};$$

$$\{\varepsilon_p^{NL}\} = \begin{Bmatrix} \dfrac{1}{2}\left(\dfrac{\partial w_o}{\partial s}\right)^2 + \dfrac{1}{2}\beta_n^2 \\[3mm] \dfrac{1}{2}\left(\dfrac{\partial w_o}{r\,\partial\theta} - \dfrac{v_0 \cos\phi}{r}\right)^2 + \dfrac{1}{2}\beta_n^2 \\[3mm] \dfrac{\partial w_o}{\partial s}\left(\dfrac{\partial w_o}{r\,\partial\theta} - \dfrac{v_0 \cos\phi}{r}\right) \end{Bmatrix}; \qquad (4)$$

Here, r and ϕ are the radius of parallel circle and the semi-cone angle, respectively. The rotation about the normal to the middle surface is defined as [20, 21]

$$\beta_n = \frac{1}{2}\left(\frac{v_o \sin\phi}{r} + \frac{\partial v_o}{\partial s} - \frac{\partial u_o}{r\,\partial\theta}\right) \qquad (5)$$

The stress resultant vector $\{\mathbf{N}\} = \{N_{ss}\ N_{\theta\theta}\ N_{s\theta}\}^T$ and the moment resultant vector $\{\mathbf{M}\} = \{M_{ss}\ M_{\theta\theta}\ M_{s\theta}\}^T$ can be expressed in terms of the membrane strains $\{\varepsilon_p\} = \{\varepsilon_p^L\} + \{\varepsilon_p^{NL}\}$ and the bending strains $\{\varepsilon_b\}$ through the constitutive relations as

$$\begin{Bmatrix} \{\mathbf{N}\} \\ \{\mathbf{M}\} \end{Bmatrix} = \begin{bmatrix} [\mathbf{A}] & [\mathbf{B}] \\ [\mathbf{B}] & [\mathbf{D}] \end{bmatrix} \begin{Bmatrix} \{\varepsilon_p\} \\ \{\varepsilon_b\} \end{Bmatrix} - \begin{Bmatrix} \{\bar{\mathbf{N}}\} \\ \{\bar{\mathbf{M}}\} \end{Bmatrix} \qquad (6)$$

where [A], [D] and [B] are the extensional, the bending and bending-extensional coupling stiffness coefficients

matrices. $\{\bar{\mathbf{N}}\}$ and $\{\bar{\mathbf{M}}\}$ are the thermal stress and moment resultants, respectively.

The transverse shear stress resultant vector, $\{\mathbf{Q}\} = \{Q_{sz}\ Q_{\theta z}\}^T$ is related to the transverse shear strain $\{\varepsilon_s\}$ through the constitutive relation as

$$\{\mathbf{Q}\} = [\mathbf{E}]\{\varepsilon_s\} \qquad (7)$$

where $[E]$ is the transverse shear stiffness coefficients matrix.

It may be noted here that the stiffness coefficients [A], [D], [B] and [E] in Eqs. (6) and (7) are functions of the meridional coordinate (s) for the angle-ply laminated filament wound conical shells.

For a laminated shell consisting of N layers with stacking angles $\theta_i\,(i = 1, \ldots, N)$ and layer thicknesses $h_i\,(i = 1, \ldots, N)$, the necessary expressions to compute the stiffness coefficients and thermal stress/moment resultants, available in the literature [22] are used.

The ply-angle θ_i and layer thickness h_i for the filament wound angle-ply conical shells are expressed as [1, 2] :

$$\theta_i = \arcsin\left(\frac{r_i}{r}\sin\theta_i^1\right) \qquad (8a)$$

$$h_i = h_i^1\,\frac{r_1}{r}\,\frac{\cos\theta_i^1}{\cos\theta_i} \qquad (8b)$$

where, r_1, θ_i^1 and h_i^1 are radius of the parallel circle, ply-angle and layer thickness at the small end of the conical shell. For cross-ply laminated shells ($\theta_i^1 = 0^0$ or 90°), the ply-angle is considered to be constant along the meridional direction.

The governing equations, derived using the condition for the extremum of the total potential consisting of strain energy and potential of applied external forces, can be written as [4, 5]

$$\left[[\mathbf{K}] - [\mathbf{K_T}] + (1/2)\,[\mathbf{K_1}(\delta)] + (1/3)\,[\mathbf{K_2}(\delta)]\right]\{\delta\}$$

$$= \{\mathbf{F_M}\} + \{\mathbf{F_T}\} \qquad (9)$$

where $[\mathbf{K}]$ is the linear stiffness matrix, $[\mathbf{K}_1]$ and $[\mathbf{K}_2]$ are the nonlinear stiffness matrices linearly and quadratically dependent on the field variables. $[\mathbf{K_T}]$ is the geometric stiffness matrix due to the thermal stress resultants. $\{\mathbf{F_M}\}$ and $\{\mathbf{F_T}\}$ are the mechanical and the thermal load vectors. $\{\delta\}$ is the vector of degrees of freedom.

The equilibrium path is traced by solving Eq. (9) using Newton-Raphson iteration technique coupled with the adaptive displacement control method. The presence of asymmetric perturbation in the form of a small magnitude load spatially proportional to the linear buckling mode shape [23] is considered such that its effect on the prebuckling deformation of the axisymmetrically loaded shell is very less and when the applied axisymmetric load approaches the bifurcation load, the solution continues to the postbuckling path. The solution starts with the load increments, and whenever during the marching, the tangent stiffness matrix becomes semi-positive or negative definite, the solution approach is switched to the adaptive displacement control. The degree of freedom having the highest rate of change in the previous $(i\text{-}1)^{\text{th}}$ step is selected as the control parameter in the current i^{th} step and is updated in each step. The step size is decided based on the increment in the previous step and the number of equilibrium iterations. The equilibrium is achieved for each load/displacement step until the convergence criteria suggested by Bergan and Clough [24] are satisfied within the specific tolerance limit of less than 0.001%.

The detailed description of the finite element formulation, convergence study and validation for buckling/post-buckling of laminated shells are reported in Refs. [4, 5, 25] and are not presented here for the sake of brevity.

Results and Discussion

The postbuckling characteristics of laminated truncated circular conical shells subjected to combined loading are investigated using the semi-analytical finite element formulation. The radius of the parallel circle of the conical shell is expressed as $r = r_1 + (r_2 - r_1)\, s/L$ where r_1 and r_2 are the radii at the left ($s = 0$) and right end ($s = L$) of the conical shell, and L is the slant length. The study is carried out to investigate the influences of semi-cone angle (ϕ) and interaction of axial/external pressure/torsional/thermal loadings on the postbuckling characteristics of the cross-and angle-ply laminated circular conical shells.

The material properties used are

E_L = 181 GPa, E_T = 10.3 GPa, G_{LT} = 7.17 GPa, G_{TT} = 7.17 GPa, $\nu_{LT} = \nu_{TT} = 0.28$, $\alpha_L = 0.02 \times 10^{-6}/{}^0\text{C}$, $\alpha_T = 1125\,\alpha_L$

where E, G, ν and α are Young's modulus, shear modulus, Poisson's ratio and coefficient of thermal expansion, respectively. The subscripts L and T are the longitudinal and the transverse directions respectively with respect to the fibers. All the layers are of equal thickness at the left end ($s = 0$) and the ply-angle is measured with respect to the meridional axis (s-axis). The first layer is the innermost layer of the shell. The layer ply-angle (θ_i^1) and the thickness (h_i^1) at the left end ($s = 0$) of a shell are specified. The shear correction factor employed is evaluated taking into account of layer properties and stacking sequence [26].

The simply supported boundary conditions of the shells considered are :

Left end (s = 0) :
$$u_o^o = u_o^{c\,i} = u_o^{s\,i} = v_o^o = v_o^{c\,i} = v_o^{s\,i} = w_o^o = w_o^{c\,i} = w_o^{s\,i} = \beta_\theta^o = \beta_\theta^{c\,i} = \beta_\theta^{s\,i} = 0$$

Right end (s = L)

Immovable :
$$u_o^o = u_o^{c\,i} = u_o^{s\,i} = v_o^o = v_o^{c\,i} = v_o^{s\,i} = w_o^o = w_o^{c\,i} = w_o^{s\,i} = \beta_\theta^o = \beta_\theta^{c\,i} = \beta_\theta^{s\,i} = 0$$

Movable 1 :
$$v_o^o = v_o^{c\,i} = v_o^{s\,i} = w_o^o = w_o^{c\,i} = w_o^{s\,i} = \beta_\theta^o = \beta_\theta^{c\,i} = \beta_\theta^{s\,i} = 0$$

Movable 2 :
$$u_o^o = u_o^{c\,i} = u_o^{s\,i} = w_o^o = w_o^{c\,i} = w_o^{s\,i} = \beta_\theta^o = \beta_\theta^{c\,i} = \beta_\theta^{s\,i} = 0$$

Movable 3 :
$$w_o^o = w_o^{c\,i} = w_o^{s\,i} = \beta_\theta^o = \beta_\theta^{c\,i} = \beta_\theta^{s\,i} = 0$$

Based on the progressive mesh refinement, 16 elements idealization in the meridional direction is found to be adequate to model the complete slant length of the shell. The number of terms M_1 in the approximation of the variables (u_o, v_o) and M_2 in (w_o, β_s, β_θ), based on the convergence study, are taken as 4 and 2, respectively, for the parametric study.

The characteristics of the eight-layered cross-ply $(0°/90°)_4$ laminated immovable simply supported conical shells ($r_1/h = 200$, $L/r_1 = 1$, $h = 0.003$ m) subjected to combined uniform temperature rise ($\Delta T = \lambda \eta \Delta T_{cr}$; ΔT_{cr} is the linear bifurcation temperature) and external radial pressure $p = \lambda \kappa p_{cr}$; p_{cr} is the linear bifurcation pressure) are analyzed considering different values of the semi-cone angle (ϕ). The pre- and post-buckling characteristics as maximum transverse displacement parameter (w_{omax}/h) versus load factor (λ) curves are shown in Fig.2 for different combinations of thermal (η) and pressure (κ) loading proportions. The results are presented for the circumferential wave numbers (n) corresponding to the lowest critical load. It can be seen from this figure that the prebuckling displacement is outward for predominant thermal loading cases and inward for predominant external radial pressure cases. In the postbuckling region, inward displacement is significantly higher compared to the outward displacement especially for predominant external pressure loading cases. The degree of hardening nonlinearity observed after the minimum load in the postbuckling path increases with the increase in the semi-cone angle (ϕ). The inward displacement in the postbuckling region varies significantly with the change in the loading constants η and κ. The critical buckling temperature versus critical pressure interaction curves corresponding to the bifurcation point are shown in Fig.3. It can be seen from this figure that the interaction curves are of nonlinear nature.

The postbuckling characteristics for the combined axial compression ($P = \lambda \mu P_{cr}$; P_{cr} is the linear bifurcation axial load) and external radial pressure ($p = \lambda \kappa p_{cr}$) of the eight-layered cross-ply $(0°/90°)_4$ laminated simply supported (*movable1*) conical shells ($r_1/h = 200$, $L/r_1 = 1$, $h = 0.003$ m) are shown in Fig.4 for different combinations of axial (μ) and pressure (κ) loading proportions. It can be seen from these results that the inward displacement is significantly higher compared to the outward displacement for all loading combinations. The ratio of the lowest load in the postbuckling path and the bifurcation load increases with the increase in the pressure loading proportionality constant (κ). The critical axial load versus critical pressure interaction curves corresponding to the bifurcation point and the point corresponding to lowest load in the postbuckling path are shown in Fig.5. It can be seen from this figure that the nonlinearity in the interaction

relation for lowest load is less compared to that for the bifurcation load.

The postbuckling characteristics for the combined uniform temperature rise ($\Delta T = \lambda \eta \Delta T_{cr}$), external radial pressure ($p = \lambda \kappa p_{cr}$) and torsional load ($T = \lambda \zeta T_{cr}$; T_{cr} is the linear bifurcation torsional load) of the eight-layered cross-ply $(0°/90°)_4$ laminated simply supported (*movable2*) conical shells are shown in Fig.6. The effect of the interaction of axial compression ($P = \lambda \mu P_{cr}$), external radial pressure ($p = \lambda \kappa p_{cr}$) and torsional load ($T = \lambda \zeta T_{cr}$) on the postbuckling characteristics of the eight-layered cross-ply $(0°/90°)_4$ laminated simply supported (*movable3*) conical shells is shown in Fig.7. It can be seen from Figs. 6 and 7 that the relative load carrying capacity (load factor λ) of different set of loading combinations changes with the increase in the maximum displacement (w_{omax}/h).

The postbuckling characteristics of the eight-layered angle-ply $(45°/-45°)_4$ laminated simply supported (*movable2*) conical shells for the combined uniform temperature rise ($\Delta T = \lambda \eta \Delta T_{cr}$), external radial pressure ($p = \lambda \kappa p_{cr}$) and torsional load ($T = \lambda \zeta T_{cr}$) are shown in Fig.8. The effect of the interaction of axial compression ($P = \lambda \mu P_{cr}$), external radial pressure ($p = \lambda \kappa p_{cr}$) and torsional load ($T = \lambda \zeta T_{cr}$) on the postbuckling characteristics of the eight-layered angle-ply $(45°/-45°)_4$ laminated simply supported (*movable3*) conical shells is shown in Fig.9. The circumferential wave number corresponding to the lowest bifurcation load for the angle-ply shells differs significantly for axial, torsional, external pressure and temperature loading applied individually. It can be seen from Figs.8 and 9 that the ratio of the minimum load in the postbuckling path to the bifurcation load depends on the loading proportions. Further, the circumferential wave numbers (n) corresponding to the bifurcation point and lowest load point in the postbuckling path can be different.

Conclusions

The postbuckling characteristics of simply supported cross-and angle-ply laminated composite conical shells subjected to combined thermo-mechanical loading changing simultaneously are studied using shear deformable semi-analytical finite element. The following observations can be made from the analysis:

- Inward displacement in the postbuckling region is significantly higher compared to the outward one.

- The degree of hardening nonlinearity observed after the minimum load in the postbuckling path increases with the increase in the semi-cone angle.

- The ratio of the lowest load in the postbuckling path and the bifurcation load increases with the increase in the pressure loading and decreases with the increase in the axial loading proportions.

- The nonlinearity in interaction relation corresponding to lowest load in the postbuckling path is less compared to that for bifurcation load.

- The relative load carrying capacity of shells under different set of loading combinations changes with the increase in the maximum displacement.

- The circumferential wave numbers corresponding to the lowest bifurcation load of the angle-ply shells differ significantly for axial, torsional, external pressure and temperature loading applied individually.

- The shells with immovable boundary conditions reveal greater hardening behaviour compared to those with movable boundary conditions.

References

1. Goldfeld, Y. and Arbocz, J., "Buckling of Laminated Conical Shells given the Variations of the Stiffness Coefficients", AIAA Journal, 2004,Vol.42, pp. 642-649.

2. Goldfeld, Y., Arbocz, J. and Rothwell, A., "Design and Optimization of Laminated Conical Shells for Buckling", Thin-Walled Structures, 2005, Vol.43, pp. 107-133.

3. Wu, C. -P. and Chiu, S. -J., "Thermoelastic Buckling of Laminated Composite Conical Shells", Journal of Thermal Stresses, 2001, Vol. 24, pp. 881-901.

4. Patel, B. P., Shukla, K. K. and Nath, Y., "Thermal Postbuckling Characteristics of Laminated Conical Shells with Temperature Dependent Material Properties", AIAA Journal, 2005, Vol. 43, pp. 1380-1388.

5. Patel, B. P., Shukla, K. K. and Nath, Y., "Thermal Postbuckling Analysis of Laminated Cross-ply Truncated Circular Conical Shells", Composite Structures, 2005, Vol. 71, pp. 101-114.

6. Anastasiadis, J. S., Tabiei, A. and Simitses, G. J., "Instability of Moderately Thick Laminated Cylindrical Shells Under Combined Axial Compression and Pressure, Composite Structures, 1994, Vol. 27, pp. 367-378.

7. Tafreshi, A., "Buckling and Postbuckling Analysis of Composite Cylindrical Shells with Cutouts Subjected to Internal Pressure and Axial Compression Loads", International Journal of Pressure Vessels and Piping, 2002, Vol. 79, pp. 351-359.

8. Shen, H. -S., "Postbuckling of Shear Deformable Cross-ply Laminated Cylindrical Shells Under Combined External Pressure and Axial Compression", International Journal of Mechanical Sciences, 2001, Vol. 43, pp. 2493-2523.

9. Adali, S., Verijenko, V. E. and Richter, A., "Minimum Sensitivity Design of Laminated Shells Under Axial Load and External Pressure", Composite Structures, 2001, Vol. 54, pp. 139-142.

10. Bisagni, C. and Cordisco, P., "An experimental Investigation into the Buckling and Postbuckling of CFRP Shells Under Combined Axial and Torsion Loading", Composite Structures, 2003, Vol. 60, pp. 391-402.

11. Meyer-Piening, H. R., Farshad, M., Geier, B. and Zimmermann, R., "Buckling Loads of CFRP Composite Cylinders Under Combined Axial and Torsion Loading-Experiments and Computations", Composite Structures, 2001, Vol. 53, pp. 427-435.

12. Birman, V., and Bert, C. W., "Buckling and Postbuckling of Composite Plates and Shells Subjected to Elevated Temperature", ASME Journal of Applied Mechanics, 1993, Vol. 60, pp. 514-519.

13. Shen, H. -S., "Thermomechanical Postbuckling Analysis of Stiffned Laminated Cylindrical Shell",

ASCE Journal of Engineering Mechanics, 1997, Vol. 123, pp. 433-443.

14. Shen, H. -S., "Postbuckling Analysis of Imperfect Stiffened Laminated Cylindrical Shells Under Combined External Pressure and Thermal Loading", International Journal of Mechanical Sciences, 1998, Vol. 40, pp. 339-355.

15. Shen, H. -S., "Thermomechanical Postbuckling of Shear Deformable Laminated Cylindrical Shells with Local Geometric Imperfections", International Journal of Solids and Structures, 2002, Vol. 39, pp. 4525-4542.

16. Teng, J. G. and Hong, T., "Postbuckling Analysis of Elastic Shells of Revolution Considering Mode Switching and Interaction", International Journal of Solids and Structures, 2006, Vol. 43, pp. 551-568.

17. Jabareen, M. and Sheinman, I., "Effect of the Nonlinear Pre-buckling State on the Bifurcation Point of Conical Shells", International Journal of Solids and Structures, 2006, Vol. 43, pp. 2146-2159.

18. Sheinman, I. and Goldfeld, Y., "Imperfection Sensitivity of Laminated Cylindrical Shells According to Different Shell Theories", ASCE Journal of Engineering Mechanics, 2003, Vol. 129 (9), pp. 1048-1053.

19. Sheinman, I. and Jabareen, M., "Postbuckling of Laminated Cylindrical Shells in Different Formulations", AIAA Journal, 2005, Vol. 43, pp. 1117-1123.

20. Sanders, J. L., "Nonlinear Theories for Thin Shells", Quarterly of Applied Mathematics, 1963, Vol. 21, pp. 21-36.

21. Kraus, H., "Thin Elastic Shells", John Wiley, New York, 1967.

22. Jones, R. M., "Mechanics of Composite Materials", Taylor and Francis, Philadelphia, 1999.

23. Teng, J. G. and Song, C. Y., "Numerical Models for Nonlinear Analysis of Elastic Shells with Eigenmode-affine Imperfections", International Journal of Solids and Structures, 2001, Vol. 38, pp. 3263-3280.

24. Bergan, P. G. and Clough, R. W., "Convergence Criteria for Iterative Process", AIAA Journal, 1972, Vol. 10, pp. 1107-1108.

25. Patel, B. P., Singh, S. and Nath, Y., "Postbuckling Characteristics of Angle-ply Laminated Joined Conical-cylindrical-conical Circular Shells, Journal of Aerospace Sciences and Technologies, 2008, Vol. 60, pp. 127-138.

26. Vlachoutsis, S., "Shear Correction Factors for Plates and Shells", International Journal for Numerical Methods in Engineering, 1992, Vol. 33, pp. 1537-1552.
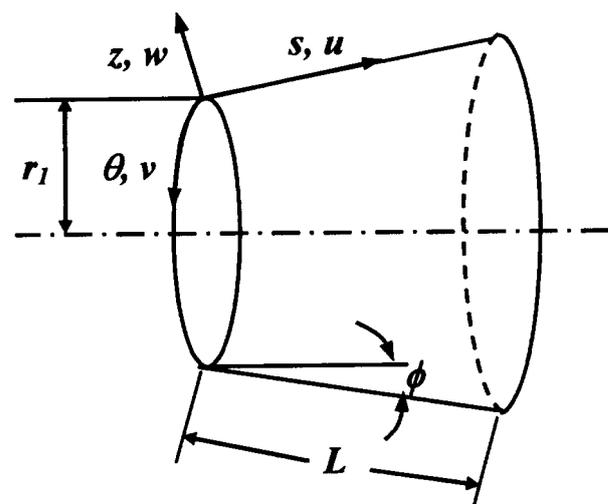
Fig.1 Geometrical Details and Coordinate System of Conical Shell
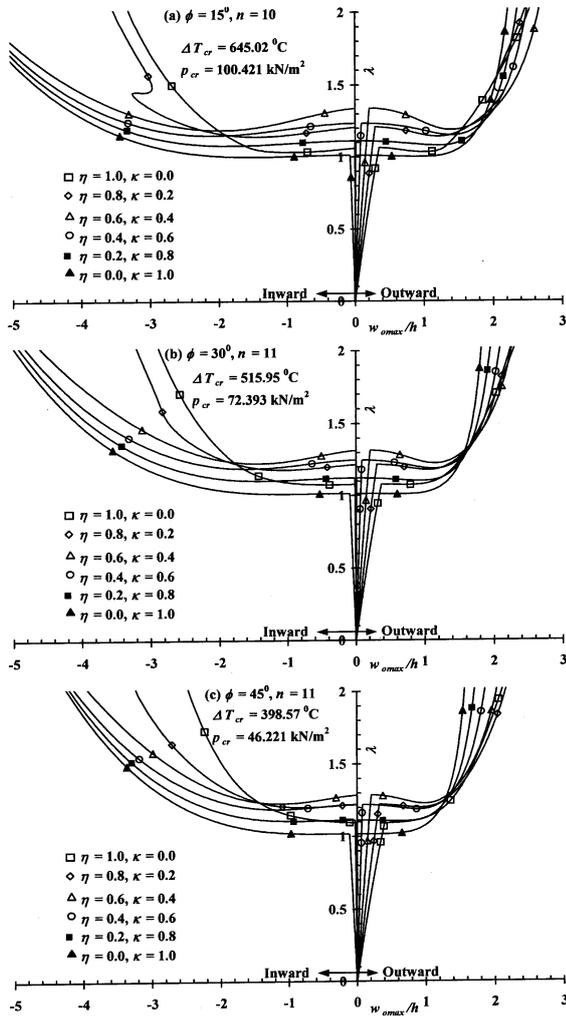
Fig.2 Postbuckling Characteristics of Eight-layered
Cross-ply (0°/90°)₄ Laminated Immovable Simply-supported
Conical Shells (r₁/h = 200, L/r₁ =1, h = 0.003 m) Subjected
to Combined Uniform Temperature Rise and External
Radial Pressure
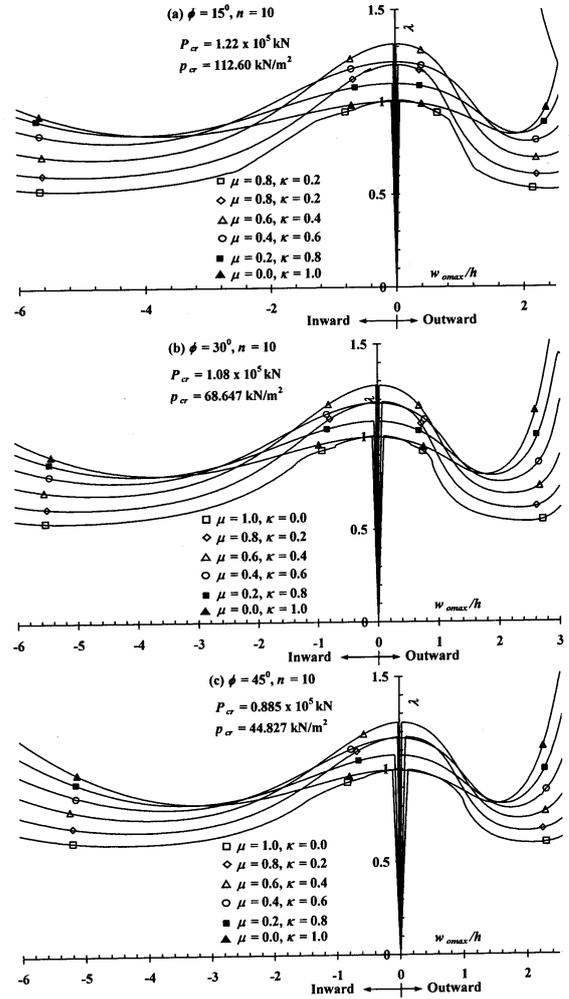
Fig.4 Postbuckling Characteristics of Eight-layered
Cross-ply (0°/90°)₄ Laminated Simply-supported (Movable1)
Conical Shells (r₁/h = 200, L/r₁ =1, h = 0.003 m) Subjected
to Axial Compressive Load and External
Radial Pressure
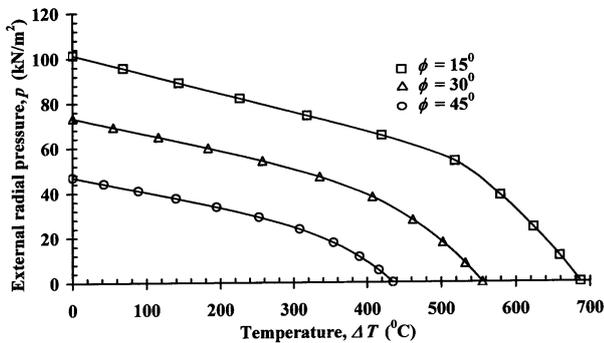
Fig.3 Critical Temperature Versus Critical Pressure Interac-
tion Curves for Eight-layered Cross-ply (0°/90°)₄ Laminated
Immovable Simply-supported Conical Shells
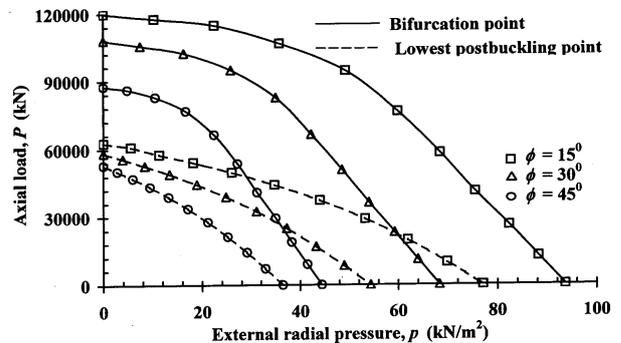
Fig.5 Critical Axial Load Versus Critical Pressure Interac-
tion Curves for Eight-layered Cross-ply (0°/90°)₄ Laminated
Simply-supported (Movable1) Conical Shells
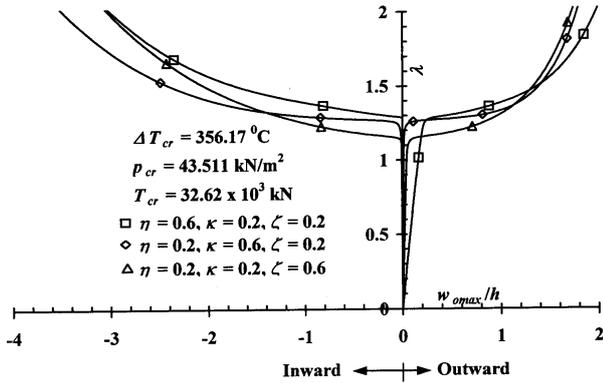
Fig.6 Postbuckling Characteristics of Eight-layered Cross-ply (0°/90°)₄ Laminated Simply-supported (Movable2) Conical Shells ($r_1/h = 200$, $L/r_1 = 1$, $h = 0.003$ m, $\phi = 45°$, $n = 10$) Subjected to Combined Uniform Temperature Rise, External Radial Pressure and Torsional Loading
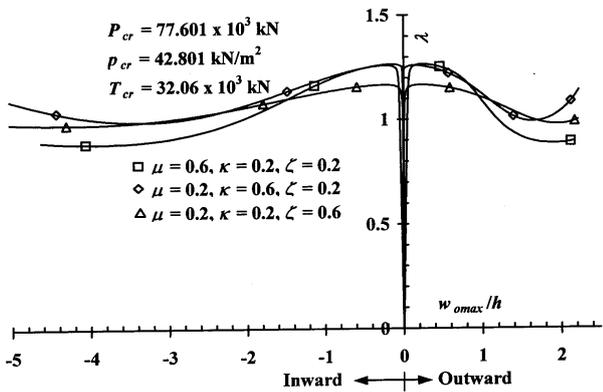
Fig.7 Postbuckling Characteristics of Eight-layered Cross-ply (0°/90°)₄ Laminated Simply-supported (Movable3) Conical Shells ($r_1/h = 200$, $L/r_1 = 1$, $h = 0.003$ m, $\phi = 45°$, $n = 10$) Subjected to Combined Axial Compression, External Radial Pressure and Torsional Loading
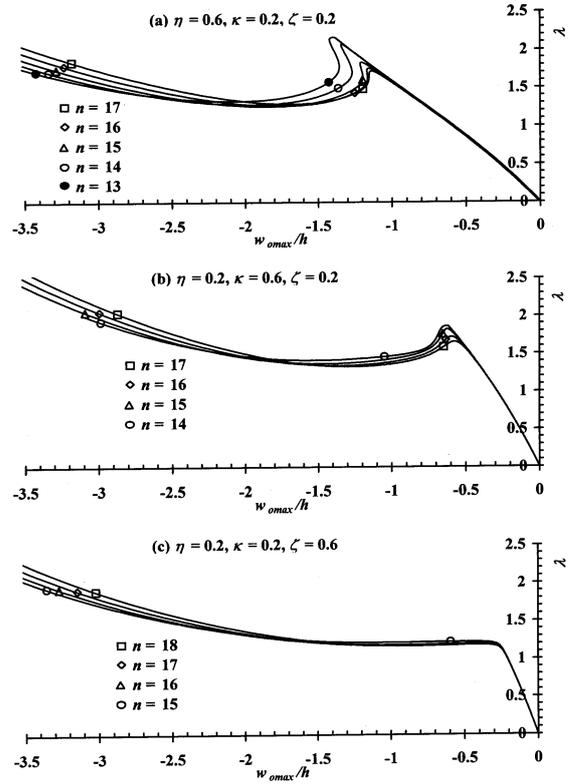
*Fig.8 Postbuckling Characteristics of Eight-layered Angle-ply (45°/-45°)₄ Laminated Simply-supported (Movable2) Conical Shells ($r_1/h = 200$, $L/r_1=1$, $h = 0.003$ m, $\phi = 45°$) Subjected to Combined Uniform Temperature Rise, External Radial Pressure and Torsional Loading :
$\Delta T_{cr} = -136.17\ ^0C$ ($n = 13$), $p_{cr} = 6.652$ kN/m² ($n = 17$), $T_{cr} = 131.639 \times 10^2$ kN ($n = 18$)*
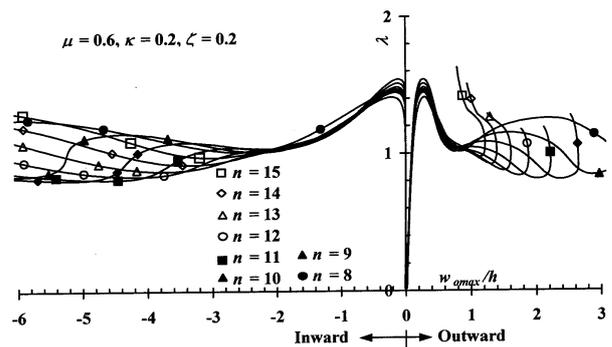
*Fig.9 Postbuckling Characteristics of Eight-layered Angle-ply (45°/-45°)₄ Laminated Simply-supported (Movable3) Conical Shells ($r_1/h = 200$, $L/r_1=1$, $h = 0.003$ m, $\phi = 45°$) Subjected to Combined Axial Compression, External Radial Pressure and Torsional Loading :
$P_{cr} = 145.39 \times 10^2$ kN ($n = 8$), $p_{cr} = 7.359$ kN/m² ($n = 18$), $T_{cr} = 123.435 \times 10^2$ kN ($n = 17$)*