TECHNICAL NOTE

DYNAMIC STABILITY OF AN ASYMMETRIC SANDWICH BEAM RESTING ON A PASTERNAK FOUNDATION

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Abstract

The parametric dynamic stability of a pinned-pinned asymmetric sandwich beam resting on a Pasternak foundation with viscoelastic core, subjected to an axial pulsating load is investigated. The effects of thickness ratio of two elastic layers (h_{31}), elastic modulus ratio(E_3/E_1), the ratio of modulus of the shear layer of Pasternak foundation to the Youngs modulus of elastic layer (G_s/E_1), the ratio of length of the beam to the thickness of the elastic layer (l_{h1}), the ratio of in phase shear modulus of the viscoelastic core to the Youngs modulus of the elastic layer (G_2/E_1), the ratio of thickness of Pasternak foundation to the length of beam (δ/l), coreloss factor (η) the ratio of thickness of viscoelastic layer to that of elastic layer (h_{21}) on the non-dimensional static buckling load are considered. In addition to these the effects of the above parameters on the regions of parametric instability have been studied.

Keywords: Parametric dynamic stability, Viscoelastic core, Sandwich beam, Pasternak foundation, Coreloss factor and Modulus ratio

Nomenclature

A_i	= (i=1,2,3) areas of cross section of a 3-layered beam, i = 1 for top layer
B	= width of beam
c	= $h_1 + 2h_2 + h_3$
E_i	= (i=1,2,3)Young's modulus
G_2	= in-phase shear modulus of the viscoelastic core
G_2^*	= G_2 (1+jη), complex shear modulus of core
G_s	= modulus of the shear layer of a Pasternak foundation
g^*	= g (1+jη), complex shear parameter
g	= shear parameter
$2h_i$	= (I=1,2,3) thickness of the i^{th} layer i = 1 for top layer
h_{12}	= h_1/h_2
h_{31}	= h_3/h_1
I_i (i=1,2,3)	= second moments of area of cross section about a relevant axis, i = 1 for top layer
J	= $\sqrt{-1}$
K	= modulus of spring in a Pasternak foundation
l	= beam length
l_{h1}	= l/h_1
m	= mass/unit length of beam
$\bar{P}_1$	= non-dimensional amplitude for the dynamic loading
t	= time
t_o	= $\sqrt{m\,l^4/(E_1 I_1 + E_3 I_3)}$
$\bar{t}$	= t/t_o, non-dimensional time
u (*x*,*t*), U_1 (*x*,*t*)	= axial displacement at the middle of the top layer of the beam
w (*x*,*t*)	= transverse deflection of beam
w'	= $\frac{\partial w}{\partial x}$

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w'' $= \dfrac{\partial^2 w}{\partial x^2}$

Y = geometric parameter

$\bar{w}$ $= \dfrac{w}{l}$

$\ddot{\bar{w}}$ $= \dfrac{\partial^2 \bar{w}}{\partial \bar{t}^2}$

$\bar{w}''$ $= \dfrac{\partial^2 \bar{w}}{\partial \bar{x}^2}$

$U_{i,x}$ $= \dfrac{\partial U_i}{\partial x}$ (here i = 1, 3)

$\bar{x}$ $= \dfrac{x}{l}$

δ = thickness of the shear layer of a Pasternak foundation

η = coreloss factor

ρ_i = density of ith layer

$[\phi]$ = a null matrix

ω = frequency of forcing function

$\bar{\omega}$ $= \omega t_o$, non dimensional frequency ratio

Introduction

Many investigators have studied the vibrations and stability of beams on elastic foundations. The problem of beams on elastic foundations occupies an important place in modern structural and foundation engineering. The static case has been studied extensively and the subject is covered in great depth in Hetenyi's book [1]. For the dynamic case, most works have been done within the scope of elementary Bernoulli-Euler beams on elastic foundation. Usually, the subgrade is replaced either by a winkler foundation [2] or by a homogeneous, isotropic semi-infinite elastic continuum [3]. However, Kerr [4] and Soldini [5] have shown that there is a large class of foundation materials occurring in engineering practice, the behavior of which can not be represented by these two models. In an attempt to find a physically close and mathematically simple representation of an elastic foundation for these materials, Pasternak [6] proposed a foundation model consisting of a winkler foundation with shear interactions. This may be accomplished by connecting the ends of the vertical springs to a beam consisting of incompressible vertical elements, which deforms only by transverse shear. The steady state response and the stability boundaries of a variable cross-section beam on an elastic foundation were obtained by Ahuja and Duffield theoretically and experimentally [7]. The effects of rotary inertia, shear deformation and foundation constants on the natural frequencies of a Timosenko beam with various end conditions were studied by Wang and Stephens [8].

A finite element model was developed by Abbas and Thomas to study the dynamic stability of hinged-hinged and fixed-free Timosenko beams on Winkler foundation [9]. The effect of an elastic foundation on the natural frequencies, static buckling loads and regions of dynamic instability of Timosenko beams were investigated by Yokoyama [10].

The main parametric resonances of a tapered Cantiliver beam lying on a Pasternak foundation and having a thermal gradient was addressed by Kar and Sujata [11]. The same authors studied the parametric instability of Timosenko beams resting on a variable Pasternak foundation [12]. The influence of the elastic foundation stiffness and the shear layer constant on buckling loads of a column on a biparametric foundation was investigated by Pantelides [13]. The effects of viscoelastic supports at the ends on the dynamic stability of an asymmetric sandwich beam were studied by Ghosh [14], which is the research work of the main author of this paper. The same author has also studied the effects of asymmetry on a rotating sandwich beam [15].

Since till now no work has been done for the investigation of the effect of Pasternak foundation on the dynamic stability of an asymmetric sandwich beam, in the present work studies of the parametric instability of a pinned-pinned asymmetric sandwich beam with viscoelastic core and resting on a Pasternak foundation has been done, which is the new contribution of this paper. The effects of the various system Parameters on the static buckling loads as well as on the parametric instability of the system are being studied.

Formulation of the Problem

A viscoelastic sandwich beam of length *l*, resting on a Pasternak foundation is shown in Fig.1. The beam is capable of oscillating in the xz plane on the application of an external pulsating load.

The top layer of the beam is made of an elastic material of thickness $2h_1$ and Young's modulus E_1. The core is made of a linearly viscoelastic material with a shear modulus $G_2^* = G_2(1 + j\eta)$, where G_2 is the in-phase shear modulus, η is the core loss factor and $j = \sqrt{-1}$. The bottom layer is of thickness $2h_3$ and Young's modulus E_3. The foundation is comprised of equal and closely placed vertical springs with a spring constant K($N/m/m^2$), supporting a shear layer of thickness δ, with a shear modulus G_s. The beam is subjected to a pulsating axial load $P(t) = P_0 + P_1 \cos \omega t$ at $x = l$.

The following assumptions are made for obtaining the equations of motion.

- The beam transverse deflection is small, and is the same everywhere in a given cross section.
- The metallic layers obey Euler-Bernoulli assumptions of beam theory.
- The layers are perfectly bonded so that displacements are continuous across the interfaces.
- Rotary inertia effects in the layers are negligible.
- Damping in the viscoelastic core is predominantly due to shear.
- Bending and extensional effects in the core are neglected.

The expressions for potential energy, Kinetic energy and work done are as follows :

$$V = \frac{1}{2} E_1 A_1 \int_0^1 U_{1,x}^2 \, dx + \frac{1}{2} E_3 A_3 \int_0^1 U_{3,x}^2 \, dx$$

$$+ \frac{1}{2}(E_1 I_1 + E_3 I_3) \int_0^1 w_{,x}^2 \, dx + \frac{1}{2} G_2^* A_2 \int_0^1 \gamma_2^2 \, dx$$

$$+ \frac{1}{2} G_s b \delta \int_0^1 w'^2 \, dx + \frac{kB}{2} \int_0^1 w^2 \, dx ,$$

$$T = \frac{1}{2} m \int_0^1 w_{,t}^2 \, dx \quad \text{and}$$

$$W_p = \frac{1}{2} \int_0^1 P(t) \, w_{,x}^2 \, dx$$

where U_1 and U_3 are the axial displacements in the top and bottom layers, $w_t = \frac{\partial w}{\partial t}$, $w_{,x} = \frac{\partial w}{\partial x}$ and γ_2 is the shear strain in the middle layer given by

$$\gamma_2 = \frac{U_1 - U_3}{2h_2} - \frac{c w_{,x}}{2h_2}$$

U_3 is eliminated using the Kerwin's assumption [16]. The application of the extended Hamilton's principle.

$\delta \int_{t_1}^{t_2} (T - V + W_p) \, dt = 0$ leads to the following non-dimensional equations of motion.

$$\bar{w}_{,\bar{t}\bar{t}} + (1+Y)\,\bar{w}_{,\overline{xxxx}} - \left(\frac{3}{2} \frac{G_s}{E_1} \frac{\delta}{l} \frac{l_{h1}^3}{1 + \left(E_3/E_1\right) h_{31}^3} - \bar{P}(\bar{t}) \right) \bar{w}_{,\overline{xx}}$$

$$+ \frac{3}{2} \frac{kl}{E_1} \frac{l_{h1}^3}{1 + \left(E_3/E_1\right) h_{31}^3} \, \bar{w} + Y \frac{2h_2}{c} \gamma_{2,\overline{xxx}} = 0 \qquad (1)$$

$$\frac{2 E_1 A_1 h_2 c}{(1+\alpha)\, l} \frac{2 h_2}{c} \gamma_{2,\overline{xx}} - \frac{G_2^* A_2 \, l \, c}{2 h_2} \frac{2 h_2}{c} \gamma_2 + \frac{2 E_1 A_1 h_2 c}{(1+\alpha)\, l} \bar{w}_{,\overline{xxx}} = 0$$

$$(2)$$

Where

$$\bar{w}_{,\overline{xxxx}} = \frac{\partial^4 \bar{w}}{\partial \bar{x}^4}, \; \bar{w}_{,\overline{xx}} = \frac{\partial^2 \bar{w}}{\partial \bar{x}^2}, \; \gamma_{2,\overline{xxx}} = \frac{\partial^3 \gamma_2}{\partial \bar{x}^3}, \; \gamma_{2,\overline{xx}} = \frac{\partial^2 \gamma_2}{\partial \bar{x}^2}.$$

$$Y = \frac{E_1 A_1 C^2}{D(1+\alpha)} \qquad (3)$$

is the non-dimensional geometric parameter with $\alpha = (E_1 A_1)/(E_3 A_3)$ and $D = E_1 I_1 + E_3 I_3$.

Equation (2) can be simplified as

$$\frac{2 h_2 Y}{c} \gamma_{2,\overline{xx}} - \frac{2 \, g^* \, Y h_2 \gamma_2}{c} + Y \bar{w}_{,\overline{xxx}} = 0 \qquad (4)$$

The following are the associated boundary conditions to be satisfied at $\bar{x} = 0$ and $\bar{x} = 1$.

$$(1+Y)\,\bar{w}_{,\overline{xxx}} + Y \frac{2 h_2}{c} \gamma_{2,\overline{xx}} + \left[\bar{P}(\bar{t}) - \frac{3 G_s \delta \, l_{h1}^3}{2 E_1 \, l \,(1 + E_{31} h_{31}^3)} \right] \bar{w}_{,\bar{x}} = 0$$

$$(5)$$

or $\overline{w} = 0$ (6)

$$(1+Y)\,\overline{w},_{\overline{xx}} + Y\frac{2h_2}{c}\gamma_{2,\overline{x}} = 0 \quad (7)$$

or $\overline{w},_{\overline{x}} = 0$ (8)

$$\text{and } \frac{2h_2}{c}\gamma_{2,\overline{x}} + \overline{w},_{\overline{xx}} = 0 \quad (9)$$

or $\gamma_2 = 0$ (10)

In the above, γ_2 is the shear strain in the core layer, $\overline{x} = x/l$, $\overline{t} = t/t_0$,

$$t_0 = \sqrt{m\,l^4/(E_1 I_1 + E_3 I_3)}\ ,$$

$h_{31} = h_3/h_1$, $h_{21} = h_2/h_1 = 1/h_{12}$

Also $\overline{P} = \overline{P}_0 +$

$\overline{P}_1 \cos\overline{\omega}\ \overline{t}$, $\overline{P}_0 = P_0\,l^2/(E_1 I_1 + E_3 I_3)$,

$\overline{w},_{\overline{x}} = \frac{\partial\overline{w}}{\partial\overline{x}}$ $\overline{\omega} = \omega\,t_0$ and $\overline{w},_{\overline{t}} = \frac{\partial\overline{w}}{\partial\overline{t}}$ etc.

Finally, $g* = \frac{G_2^*\,l_{h1}^2\,(1+E_{31}h_{31})}{4E_3\,h_{21}\,h_{31}}$ is the complex shear parameter, $g* = g\,(1+j\eta)$, g being the shear parameter.

Approximate Solution

Solutions of Equations (1) and (2) are assumed in the form

$$\overline{w}(\overline{x},\overline{t}) = \sum_{i=1}^{i=p} w_i(\overline{x})\ f_i(\overline{t}) \quad (11)$$

$$\overline{\gamma}_2(\overline{x},\overline{t}) = \sum_{k=p+1}^{k=2p} \gamma_k(\overline{x})\ f_k(\overline{t}) \quad (12)$$

where p and q areto be suitably chosen for convergence. Here w_i and γ_k are the shape functions and f_i and f_k are the generalized coordinates. w_i and γ_k are to be so chosen to satisfy as many boundary conditions 5 to 10 as possible [13]. For the present case this is done by choosing $w_i = \sin\,(i\pi\overline{x})$ and $\gamma_{\overline{k}} = \cos\,(\overline{k}\pi\overline{x})$, $\overline{k} = k - p$, which fulfills the requirements.

Substituting above equations in equations (1) and (4) as usee of the general Galerkin method [17] yields the following matrix equations of motion in the generalized coordinates.

$$[M]\{\ddot{Q}_1\} + [k_{11}]\{Q_1\} + [k_{12}]\{Q_2\} = \{0\} \quad (13)$$

$$[k_{21}]\{Q_1\} + [k_{22}]\{Q_2\} = \{0\} \quad (14)$$

where $\{Q_1\} = \{f_1, \ldots\ldots\ldots\ldots, f_p\}^T$ (15)

$$\{Q_2\} = \{f_{p+1}, \ldots\ldots\ldots\ldots, f_{2p}\}^T \quad (16)$$

Also, $M_{ij} = \int_0^1 w_i\,w_i\,d\,\overline{x}$ (17)

$$k_{11ij} = (1+Y)\int_0^1 w_i''\ w_j''\ d\,\overline{x} + \Phi\int_0^1 w_i\,w_j\,d\,\overline{x}$$

$$+ [\psi - \overline{P}(\overline{t})]\int_0^1 w_i'\ w_j'\ d\,\overline{x} \quad (18)$$

$$k_{12jk} = Y\int_0^1 w_i''\ u_k'\ d\,\overline{x} \quad (19)$$

and

$$k_{22kl} = Y\int_0^1 u_k'\ u_l'\ d\,\overline{x} + g^*\,Y\int_0^1 u_k\,u_l\,d\,\overline{x} \quad (20)$$

In the above,

$$w_i' = \frac{\partial w_i}{\partial x} \quad (21)$$

$$\lambda_s = \left(\frac{k\,l}{E_1}\right), \quad \Phi = \frac{3\lambda_s\,l_{h1}^3}{2\,(1+E_{31}h_{31}^3)} \quad (22)$$

$$\psi = \frac{3\,G_s\,\delta\, l_{h1}^3}{2\,E_1\, l\,(1 + E_{31}\, h_{31}^3)} \tag{23}$$

Also, $\left[k_{21}\right] = \left[k_{12}\right]^T$ (24)

The equations (13) and (14) further simplified to

$$[M]\{\ddot{Q}_1\} + \left[[k] - \bar{P}_0 [H]\right]\{Q_1\} - \bar{P}_1 \cos(\bar{\omega}\,\bar{t})\,[H]\{Q_1\} = \{0\} \tag{25}$$

where

$$[k] = [\bar{k}] - [k_{12}][k_{22}]^{-1}[k_{12}]^T \tag{26}$$

$$H_{ij} = \int_0^1 w_i'\, w_j'\, d\bar{x} \tag{27}$$

and

$$[\bar{k}]_{ij} = (1 + Y)\int_0^1 w_i''\, w_j''\, d\bar{x} + \phi \int_0^1 w_i\, w_j\, d\bar{x} + \psi \int_0^1 w_i'\, w_j'\, d\bar{x} \tag{28}$$

Static Buckling Loads

Substitution of $\bar{P}_1 = 0$ and $\{\ddot{Q}_1\} = 0$ in (equation 25) leads to eigenvalue problem $[k]^{-1}[H]\{Q_1\} = \frac{1}{P_0}\{Q_1\}$. The static buckling loads $(P_o)_{crit}$ for the first few modes are obtained as the real parts of the reciprocals of the eigenvalues of $[k]^{-1}[H]$.

Regions of Instability

Referring [8] the following equations can be derived

$$\ddot{U}_N + \omega_N^{*2}\, U_N + 2\,\varepsilon \cos\bar{\omega}\,\bar{t} \sum_{M=1}^{M=p} b_{NM}\, U_M = 0\,,$$

$$N = 1, 2 \ldots\ldots\ldots\ldots, p \tag{29}$$

Where b_{NM} are the elements of [B],

ω_N^* are the distinct eigen values of the system, $\varepsilon = \frac{\bar{P}_1}{2} < 1$ and

$$[B] = -[L]^{-1}\,[M]^{-1}\,[H]\,[L] \tag{30}$$

L is the modal matrix of $[M]^{-1}\left[[k] - \bar{P}_0\,[H]\right]$.

So $\{Q_1\} = [L]\,\{U\}$ (31)

where $\{U\}$ is a new set of generalized coordinates.

For subsequent usages

$$\omega_N^* = \omega_{N,R} + J\,\omega_{N,I} \tag{32}$$

$$b_{NM} = b_{NM,R} + J b_{NM,I} \tag{33}$$

The boundaries of the region of instability of main and combination resonances are obtained using the following conditions by Saito and Otomi [18].

Case (A) : Main Resonance

In this case, the regions of instability are given by

$$\left|\frac{\bar{\omega}}{2} - \omega_{\mu,R}\right| < \frac{1}{4}\sqrt{\left(\frac{\bar{P}_1^2\,(b_{\mu\mu,R}^2 + b_{\mu\mu,I}^2)}{\omega_{\mu,R}^2} - 16\,\omega_{\mu,I}^2\right)} \tag{34}$$

when damping is present and

$$\left|\frac{\bar{\omega}}{2} - \omega_{\mu,R}\right| < \frac{1}{4}\,\frac{\left|\bar{P}_1\, b_{\mu\mu,R}\right|}{\omega_{\mu,R}} \tag{35}$$

For the undamped case,

For μ = 1, 2, ..., N.

Case (B) : Combination Resonance of Sum Type

This type of resonance occurs when $\mu \neq v$, μ, v = 1, 2, ... , N and the regions of instability are given by :

$$\left|\frac{\omega}{2} - \frac{1}{2}(\omega_{\mu,R} + \omega_{v,R})\right|$$

$$< \frac{\omega_{\mu,I} + \omega_{v,I}}{8\sqrt{(\omega_{\mu,I} + \omega_{v,I})}}$$

$$\left(\left[\sqrt{\frac{\bar{P}_1^2}{\omega_{\mu,R}\,\omega_{v,R}}(b_{\mu v,R}\, b_{v\mu,R} + b_{v\mu,I}\, b_{\mu v,I}) - 16\omega_{\mu,I}\,\omega_{v,I}}\right]\right) \quad (36)$$

For the damped case and

$$\left|\frac{\omega}{2} - \frac{1}{2}(\omega_{\mu,R} + \omega_{v,R})\right| < \frac{\bar{P}_1}{4}\sqrt{\frac{b_{\mu v,R}\, b_{v\mu,R}}{\omega_{\mu,R}\,\omega_{v,R}}} \quad (37)$$

For the undamped case

Case (C) : Combination Resonance of the Difference Type

This type of resonance occurs when $\mu < v$, $(\mu, v = 1, 2, \ldots, N)$ and the regions of instability are given by

$$\left|\frac{\bar{\omega}}{2} - \frac{1}{2}(\omega_{v,R} - \omega_{\mu,R})\right|$$

$$< \frac{\omega_{\mu,I} + \omega_{v,I}}{8\sqrt{(\omega_{\mu,I} + \omega_{v,I})}}$$

$$\sqrt{\frac{\bar{P}_1^2}{\omega_{\mu,R}\,\omega_{v,R}}(-b_{\mu v,R}\, b_{v\mu,R} + b_{\mu v,I}\, b_{v\mu,I} - 16\omega_{\mu,i}\,\omega_{v,i})} \quad (38)$$

For the damped case and

$$\left|\frac{\omega}{2} - \frac{1}{2}(\omega_{v,R} - \omega_{\mu,R})\right| < \frac{\bar{P}_1}{4}\sqrt{\frac{-b_{\mu v,R}\, b_{v\mu,R}}{\omega_{\mu,R}\,\omega_{v,R}}} \quad (39)$$

for the undamped case.

Numerical Results and Discussion

Numerical results were obtained for various values of the coreloss factor η, the non-dimensional geometric parameters, h_{31}, l_{h1}, h_{21}, δ_1 and the modulus ratio G_2/E_1, G_s/E_1 and E_3/E_1. For relevant values of the parameters, results of the present study were compared with those in Ray, K and Kar, R.C and good agreement was observed [19].

Figures 1a to 1h shows the dependence of the non-dimension al static buckling load on the various system parameters.

As h_{31} (Fig. 1a) is increased, static buckling load initially increases then decreases slowly for mode 1 and rapidly for modes 2 and 3. This shows there exists an optimum h31 for highest static buckling load under the given values of the other parameters. For large h_{31}, these are seen to remain almost constant. The buckling load for variations in E_3/E_1 (Fig.1b) shows a monotonically decreasing nature with increasing E_3/E_1. Here too, these become nearly constant for large E_3/E_1. These static buckling loads are seen to increase proportionately with G_s/E_1 (Fig.1c). while$(\bar{P}_o)_{crit}$ s increase only marginally with an increase in l_{h1}(Fig.1d), these are almost independent of G_2/E_1 (Fig.1e) except for small value of the parameter. Where as $(\bar{P}_o)_{crit}$ s increase linearly with δ/l (Fig.1f), these are almost independent of the coreloss factor (Fig.1g), which is obvious because loss factor of viscoelastic layer does not have any effect on critical buckling load and increase non-linearly with an increase in h_{21} as Fig.1h shows.

In the following, the values of the various parameters used, unless stated otherwise, are as follows:

$K_I/E_1 = 0.001$, $l_{h1} = 50$, $G_2/E_1 = 0.003$, $G_s/E_1 = 0.001$, $\eta = 0.003$, $\delta/I = 0.05$, $E_3/E_1 = 1.0$, $h_{21} = 0.25$, $h_{31} = 0.75$, $P_0 = 0.05$.

Also, $\omega_{N,R}$ is written as ω_N for brevity.

Figures 2 through 6 show the influence of various parameters on the instability zones. The regions inside the v-boundaries represent the zones of instability. If for the change in the value of a parameter, the width of the instability zones increases or the zone is pulled down or shifted towards the lower excitation frequency region, the stability of the system worsens. Otherwise if with the change in the value of the parameter, the width of the instability zones decreases or pulled up or shifted towards the higher excitation frequency region or if the number of the instability zones reduces, then the stability of the system improves. With these above conditions the effects of various parameters on the dynamic stability of the system have been analyzed as follows.

Figure 2 shows the influence of the coreloss factor upon the regions of parametric instability. It can be seen that increase of η improves the dynamic stability of the system by shifting the zones upward and reducing their areas as well as the number.

Figure 3 depicts the influence of h_{31} on the zones of instability. With an increase in h_{31}, the sizes of the stability zones are seen to increase considerably. These are pulled down as well as shifted towards low frequency region, thereby worsening stability.

The effect, of l_{h1} upon the parametric stability of the system is considered in Fig.4. With an increase in the values of l_{hl}, the instability zones move upward and shift to the right, thereby improving the system stability. E_3/E_1 has a marginal effect on zones of instability of the system. An increase E_3/E_1 shifts the zones to the left and moves them up slightly. Figure is not shown. The influence of the modulus ratio G_2/E_1 upon the instability zones is as follows. It can be observed that, the resonance zones move upward and also shift to the right as G_2/E_1 increases. Hence, this parameter has a stabilizing effect. Figure is not shown.

Figure 5 depicts the effect of the modulus ratio G_s/E_1 on the instability zones. A shift towards higher frequency zones and narrowing of unstable zones are observed with increase of G_s/E_1. Thus, this parameter has a stabilizing effect.

KI/E_1 has similar effects as is evident from Fig.6. Figs. 2 to 6 also show that in all cases up to first four of $\omega_{N,R}$, no of combination resonance zones of the sum type of difference type is occuring. From the above it can be noted that the Pasternak foundation can improve the stability of the system.

Conclusion

In the present work an attempt has been made to show the effect of Pasternak foundation on the non-dimensional static buckling load as well as on the zones of instability.

From Figs.1a to 1h the obtained results are that h_{31} has an optimum value for highest static buckling load. The static buckling loads are seen to increase with the increase of G_s / E_1, δ/l and h_{21}, whereas the static buckling load decreases with the increase of E_3/E_1 but for large values of E_3/E_1, no change in the buckling load occurs. Marginal increasing in buckling load is found with the increase of l_{h1} and the buckling load remains almost independent of G_2/E_1.

From Figs. 2 to 6 it was found that the stability of the system improves with increase of coreloss factor, l_{h1}, G_2/E_1, G_s/E_1 and KI/E_1 and marginal effect on the zones of instability of the system was found with increase of E_3/E_1. Further it was found that the stability of the system worsens as h_{31} increases. The combination resonance zones either of sum type or of difference type do not appear in any of the cases.

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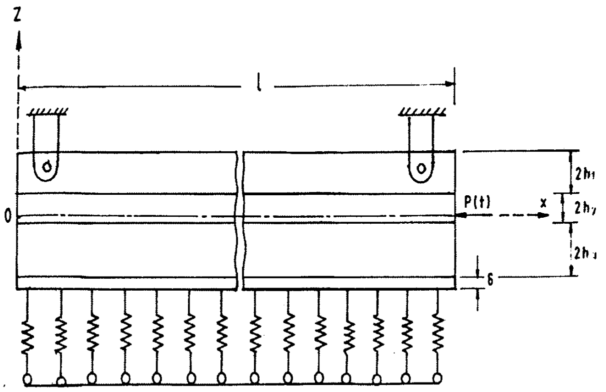

Fig.1 System Configuration

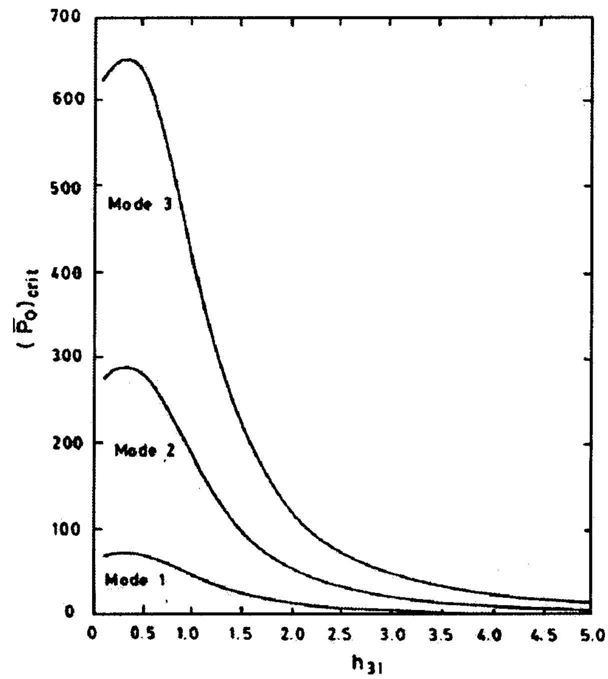

Fig.1a Variation of $(\bar{P}_o)_{crit}$ *with* h_{31}

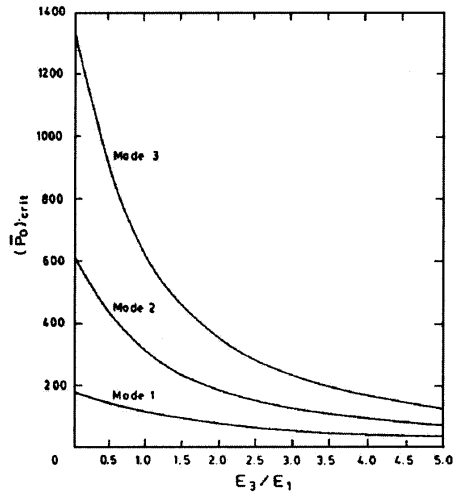

Fig.1b Variation of $(\bar{P}_o)_{crit}$ with E_3/E_1

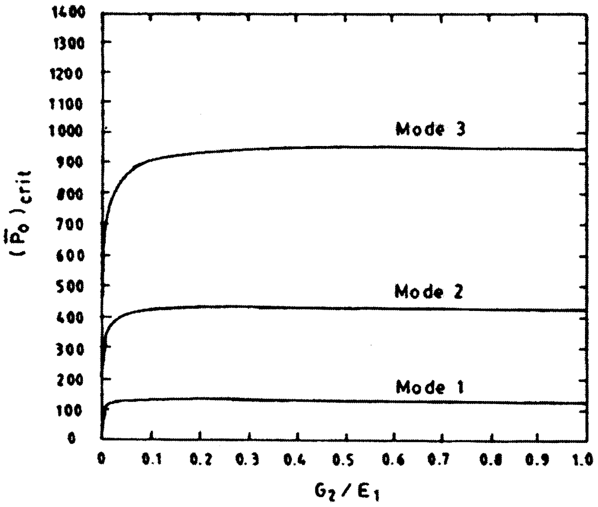

Fig.1e Variation of $(\bar{P}_o)_{crit}$ with G_2/E_1

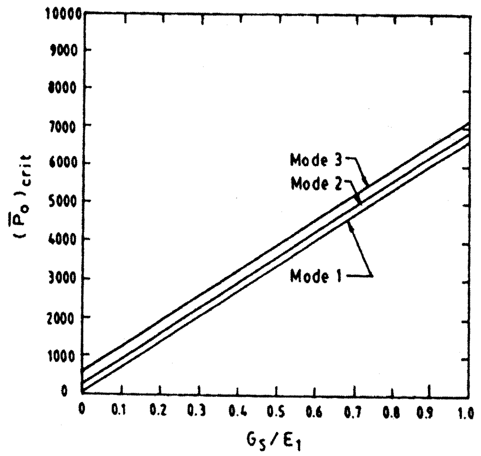

Fig.1c Variation of $(\bar{P}_o)_{crit}$ with G_S/E_1

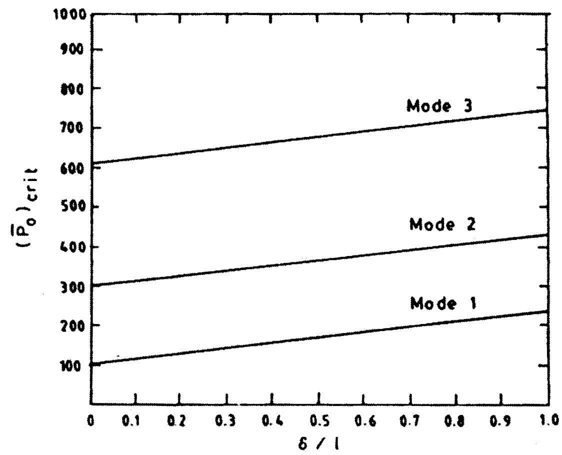

Fig.1f Variation of $(\bar{P}_o)_{crit}$ with δ/l

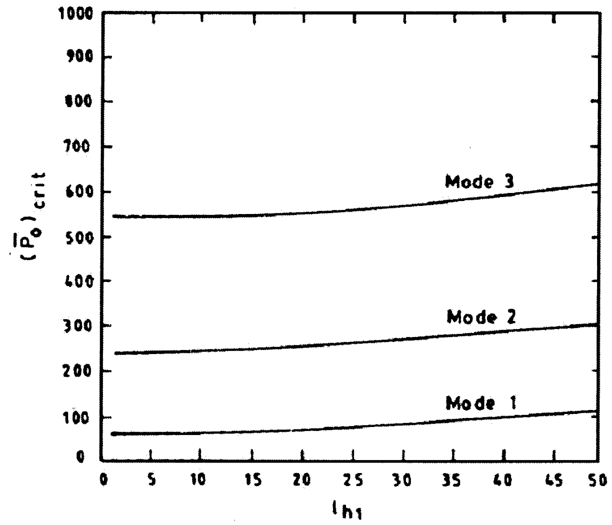

Fig.1d Variation of $(\bar{P}_o)_{crit}$ with l_{h1}

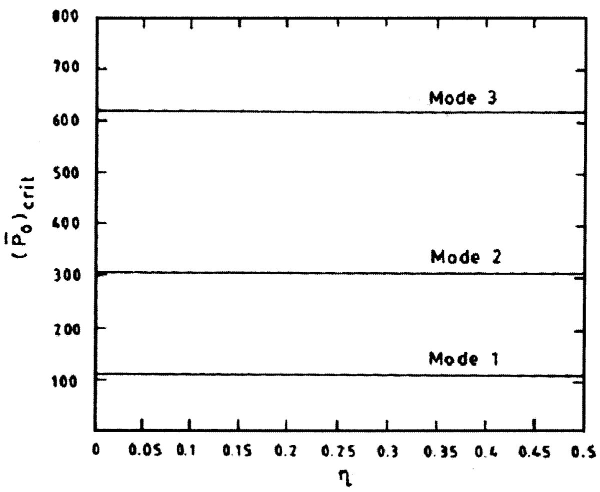

Fig.1g Variation of $(\bar{P}_o)_{crit}$ with η

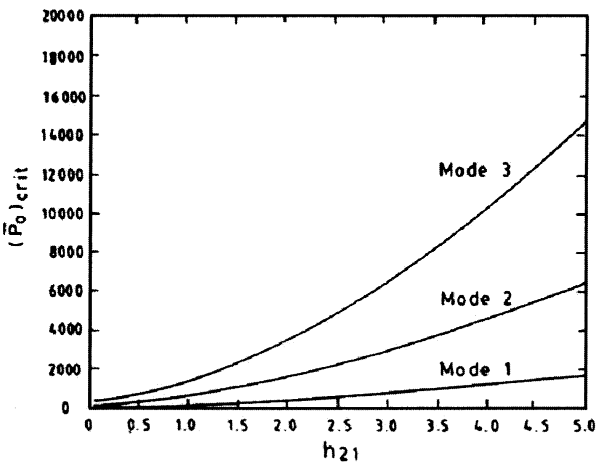

Fig.1h Variation of $(\bar{P}_o)_{crit}$ *with* h_{21}

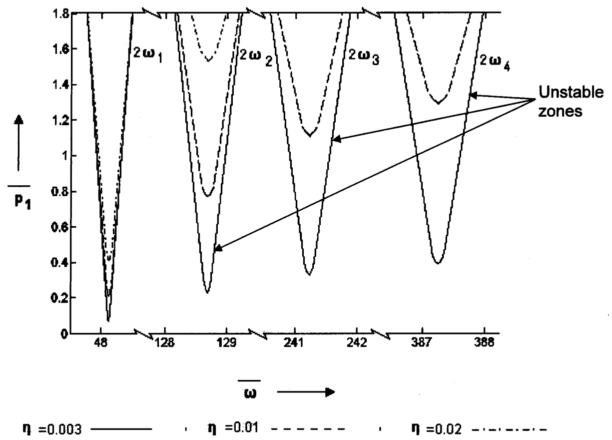

Fig.2 Effect of η *on the Instability Zones*
[Regions of parametric instability for three values of η*,*
$(\eta = 0.003,\ \eta = 0.01$ *and* $\eta = 0.02)]$

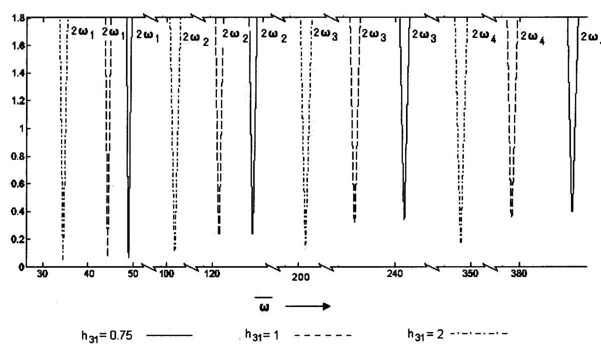

Fig.3 Effect of h_{31} *on the Instability Zones*
[Regions of parametric instability for three values of h_{31}*,*
$(h_{31} = 0.75,\ h_{31} = 1$ *and* $h_{31} = 2)]$

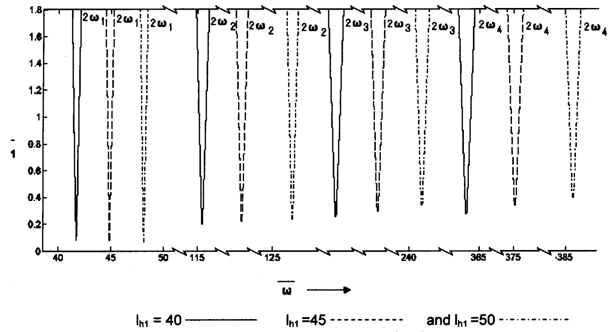

Fig.4 Effect of l_{h1} *on the Instability Zones*
[Regions of parametric instability for three values of l_{h1}*,*
$(l_{h1} = 40,\ l_{h1} = 45$ *and* $l_{h1} = 50)]$

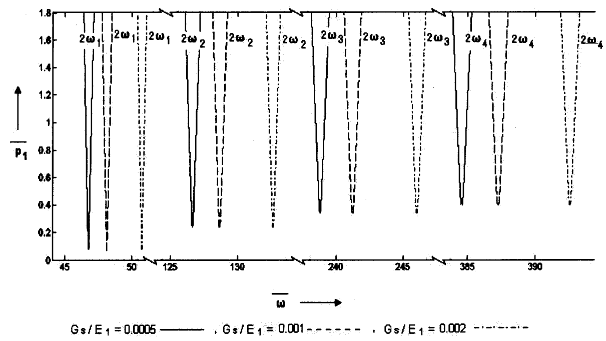

Fig.5 Effect of G_s/E_1 *on the Instability Zones*
[Regions of parametric instability for three values of G_s/E_1*,*
$(G_s/E_1 = 0.0005,\ G_s/E_1 = 0.001$ *and* $G_s/E_1 = 0.002)]$

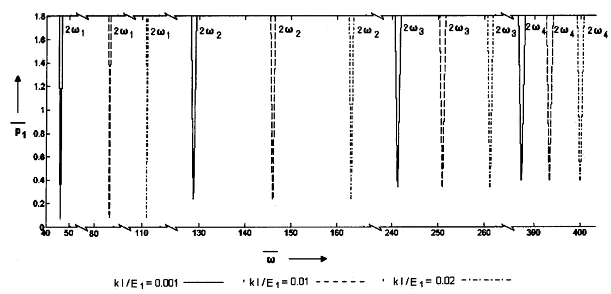

Fig.6 Effect of KI/E$_1$ *on the Instability Zones*
[Regions of parametric instability for three values of KI/E$_1$*,*
(KI/E$_1$ *= 0.001, KI/E*$_1$ *= 0.01 and KIs/E*$_1$ *= 0.02)]*