NONLINEAR VIBRATION CHARACTERISTICS OF POINT SUPPORTED ISOTROPIC AND SYMMETRICALLY LAMINATED PLATES

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Abstract

Nonlinear free and forced vibration characteristics of composite square plates either supported at four corner points or point supported at the middle of its edges as well as corners are investigated here using different high precision plate bending finite elements. It is observed that, higher order interpolation polynomial for the transverse displacement helps the plate bending finite elements to converge for the linear vibration frequencies. However, convergence for large amplitude vibration problems depends on the interpolation polynomials for in-plane displacements as well as transverse displacement. In general, the hardening nonlinearity (i.e., the increase of nonlinear frequency with vibration amplitude) of point supported plates is less than those of immovable simply supported plates.

Key Words*: point supported composite plate, finite element, nonlinear frequency*

Introduction

The increased utilization composite plates and shells in different engineering structures have attracted the attention of many researchers to investigate their vibration characteristics. It is observed from the existing literature [1-4] that the nonlinear dynamics of simply supported and clamped plates have received considerable attention of the researchers. However, flexural vibration characteristics of point supported composite plates have been sparsely treated in the literature even though such plates find wide application in civil, aerospace, automotive and marine applications.

Few analytical and numerical studies on the linear vibration frequencies of point supported isotropic [5-11] and composite [12-16] plates are available in the literature. Fan and Cheung [5] used spline finite strip method, Narita [6], Kim and Dickinson [7] and Kitipornchai et al. [8] employed Ritz method in combination with Lagrange multiplier approach, Gorman [9] applied superposition method, while, Raju and Amba-Rao [10] and Utjes et al. [11] employed finite element method to study the linear vibration frequencies of corner supported isotropic plates. Recently, Cheung and Zhou [12] used static beam function approach, Zhou et al. [13] used finite layer method, Zhou and Ji [14] followed Levys solution procedure, Narita and Hodgkinson [15] applied Ritz method, while Setoodeh and Karamani [16] used finite element method to study the vibration frequencies of composite plates with corner support or internal point support. However, all the above works deals with the vibration behavior of point supported rectangular plates using linear theory.

Nonlinear dynamics of plate like structures is a complex phenomenon involving both flexural vibration and in-plane strain. This stiffness interaction between the in-plane and bending degrees of freedoms are generally expressed through von Karman's strain-displacement assumptions. The nonlinear analysis of point supported plates involving high stress regions near the localized supports due to the in-plane deformation of the waiving edges is further complicated. However, the nonlinear flexural vibration analysis of point supported rectangular plate is not yet commonly available in the literature.

In the present paper, nonlinear vibration characteristics of point supported rectangular isotropic and composite plates are studied using different high-precision plate-bending finite elements developed by the authors [17-19] over the years. Two different types of support conditions are considered, i.e., (a) the plate are supported at four corner points and (b) the plate is point supported at four corners as well as at the middle of four edges. The flexural motion of the plate is assumed to be harmonic and the

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in-plane movement is assumed to be periodic (with a constant period of vibration). The nonlinear matrix amplitude equation is obtained by employing Galerkin's method [20] to study the free and forced vibration characteristics of point supported isotropic and composite plates.

Finite Element Formulations

The displacement components at a generic point (*x*, *y*, *z*) of a shear deformable rectangular plate can be expressed as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\,\phi_x \\ v(x,y,z) &= v_0(x,y) + z\,\phi_y \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

Here, u_0, v_0, w are the mid-surface displacements; ϕ_x and ϕ_y are the nodal rotations. Following von Kármán strain-displacement relation, the in-plane and shear strains can be written as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ v_{0,x} + u_{0,y} \end{Bmatrix} + \begin{Bmatrix} w_{,x}^2/2 \\ w_{,y}^2/2 \\ w_{,x}\, w_{,y} \end{Bmatrix}$$

$$+ z \begin{Bmatrix} \phi_{x,x} \\ \phi_{y,y} \\ \phi_{x,y} + \phi_{y,x} \end{Bmatrix} = \{\varepsilon_m\} + z\{\kappa\} \tag{2a}$$

and

$$\begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} w_{,x} + \phi_x \\ w_{,y} + \phi_y \end{Bmatrix} \tag{2b}$$

Here, $()_{,x}$ and $()_{,y}$ represent the partial differentiation with respect to *x* and *y*. The membrane $\{N\}$, bending $\{M\}$ and shear $\{Q\}$ stress resultants may be expressed as

$$\begin{aligned} \{N\} &= \{N_{xx}\ N_{yy}\ N_{xy}\}^T = [A_{ij}]\{\varepsilon_m\} + [B_{ij}]\{\kappa\} \\ \{M\} &= \{M_{xx}\ M_{yy}\ M_{xy}\}^T = [B_{ij}]\{\varepsilon_m\} + [D_{ij}]\{\kappa\} \\ \{Q\} &= [S]\{\gamma\} \end{aligned} \tag{3}$$

where $[A]$, $[B]$, $[D]$, and $[S]$ are extensional, extension-bending, bending, and shear stiffness coefficients respectively. Following standard procedure, the governing equation for the nonlinear flexural vibration of a composite plate under transverse harmonic pressure $q_0 \sin\theta t$ may be written as

$$\begin{bmatrix} M_{mm} & 0 \\ 0 & M_{bb} \end{bmatrix} \begin{Bmatrix} \ddot{\delta}_m \\ \ddot{\delta}_b \end{Bmatrix} + \begin{bmatrix} K_{mm} & K_{mb} \\ K_{bm} & K_{bb} \end{bmatrix} \begin{Bmatrix} \delta_m \\ \delta_b \end{Bmatrix}$$

$$+ \begin{bmatrix} 0 & N_1(w) \\ N_2(w) & N_3(w) \end{bmatrix} \begin{Bmatrix} \delta_m \\ \delta_b \end{Bmatrix} = \begin{Bmatrix} 0 \\ q_0 \sin\theta t \end{Bmatrix} \tag{4}$$

Here, $\boldsymbol{M}$ and $\boldsymbol{K}$, are the mass and linear stiffness matrix respectively, $\boldsymbol{N}_1$ and $\boldsymbol{N}_2$ are nonlinear stiffness matrices linearly depends on transverse displacement w; $\boldsymbol{N}_3$ is quadratic nonlinear matrix. Subscript '*m*' and '*b*' corresponds to membrane (u_0, v_0) and bending (w, ϕ_x, ϕ_y) components of the degrees of freedom and the corresponding mass and stiffness matrices.

Solution Procedure

The displacement components for the nonlinear vibration of composite plates under transverse harmonic pressure $q_0 \sin\theta t$ are assumed to be of the form

$$\delta_m(t) = \delta_m \sin^2\theta t = \begin{Bmatrix} u_0 \sin^2\theta t \\ v_0 \sin^2\theta t \end{Bmatrix} \text{ and}$$

$$\delta_b(t) = \delta_b \sin\theta t = \begin{Bmatrix} w \sin\theta t \\ \phi_x \sin\theta t \\ \phi_y \sin\theta t \end{Bmatrix} \tag{5}$$

Now, substituting the assumed solution into the governing equation (4) and taking the weighted residual [20] along the path $\int_0^{T/4} \{R_m\} \sin^2\omega t = \{0\}$ and $\int_0^{T/4} \{R_b\} \sin\omega t = \{0\}$ the following matrix-amplitude equation is obtained

$$\begin{bmatrix} \frac{3}{4}K_{mm} & \frac{8}{3\pi}K_{mb} + \frac{3}{4}N_1 \\ \frac{8}{3\pi}K_{bm} + \frac{3}{4}N_2 & K_{bb} + \frac{3}{4}N_3 \end{bmatrix} \begin{Bmatrix} \delta_m \\ \delta_b \end{Bmatrix}$$

$$- \theta^2 \begin{bmatrix} -M_{mm} & 0 \\ 0 & M_{bb} \end{bmatrix} \begin{Bmatrix} \delta_m \\ \delta_b \end{Bmatrix} = \begin{Bmatrix} 0 \\ q_0 \end{Bmatrix} \qquad (6)$$

Free Flexural Vibration

In the case of free flexural vibration $(q_0 = 0)$, the matrix amplitude equation (6) is solved iteratively using the following high precision plate bending elements to study the nonlinear vibration frequencies of point supported isotropic and composite plates.

Nonlinear Forced Vibration Under Transverse Harmonic Pressure

For the case of forced vibration under transverse harmonic load $q_0 \sin \theta\, t$ (θ is in the vicinity of linear vibration frequency ω_L), the above matrix amplitude equation is solved to obtain the steady-state flexural vibration amplitude $\{\delta_m, \delta_b\}^T$ (the maximum nodal displacements) of isotropic and composite plates corresponding to non-dimensional excitation frequency θ/ω_L and load parameter q_0

Element 1: This is a 16-node plate bending element with five degrees of freedom per node namely u_0, v_0, w, ϕ_x, and ϕ_y. The cubic polynomial shape functions employed to describe the field variables are expressed as follows:

$$u_0 = \left[1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^3y, x^2y^2, xy^3, x^3y^2, x^2y^3, x^3y^3\right]$$

$$\{c_i\},\ i = 1, 16$$

$$v_0 = \left[1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^3y, x^2y^2, xy^3, x^3y^2, x^2y^3, x^3y^3\right]$$

$$\{c_i\},\ i = 17, 32$$

$$w = \left[1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^3y, x^2y^2, xy^3, x^3y^2, x^2y^3, x^3y^3\right]$$

$$\{c_i\},\ i = 33, 48$$

$$\phi_x = \left[1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^3y, x^2y^2, xy^3, x^3y^2, x^2y^3, x^3y^3\right]$$

$$\{c_i\},\ i = 49, 64$$

$$\phi_y = \left[1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^3y, x^2y^2, xy^3, x^3y^2, x^2y^3, x^3y^3\right]$$

$$\{c_i\},\ i = 65, 80$$

Here, c_i are constants and are expressed in terms of nodal displacements in the finite element discretization. The shear correction factor is taken as 5/6. Assumed strain fields are employed to overcome shear locking [17].

Element 2: Here, shear strains are taken as independent degrees of freedom and the nodal rotations are expressed as $\phi_x = -w_{,x} + \gamma_{xz}(x, y)$ and $\phi_y = -w_{,y} + \gamma_{yz}(x, y)$. A four nodded plate bending element [18] with ten degrees of freedom namely u_0, v_0, w, $w_{,x}$, $w_{,y}$, $w_{,xx}$, $w_{,xy}$, $w_{,yy}$, γ_{xz} and γ_{yz} are employed here. The displacement components u_0, v_0, w, γ_{xz} and γ_{yz} are expressed as

$$u_0 = [1, x, y, xy]\ \{c_i\}, \qquad i = 1, 4$$

$$v_0 = [1, x, y, xy]\ \{c_i\}, \qquad i = 5, 8$$

$$w = \big[1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^4, x^3y, x^2y^2, xy^3, y^4, x^5,$$

$$x^4y, x^3y^2, x^2y^3, xy^4, y^5, x^5y, x^3y^3, xy^5\big]\ \{c_i\}, \qquad i = 9, 32$$

$$\gamma_{xz} = [1, x, y, xy]\ \{c_i\}, \qquad i = 33, 36$$

$$\gamma_{yz} = [1, x, y, xy]\ \{c_i\}, \qquad i = 37, 40$$

This element is free from shear locking phenomenon and all the stiffness and mass matrices are evaluated using full integration scheme.

Element 3: This is a similar four node plate bending element with 14 degrees of freedom per node with the following modified interpolation polynomials for u_0, v_0, w, γ_{xz} and γ_{yz} [19] :

$$u_0 = \left[1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^3y, x^2y^2, xy^3, x^3y^2, x^2y^3, x^3y^3\right]$$

$$\{c_i\},\ i = 1, 16$$

$$v_0 = \left[1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^3y, x^2y^2, xy^3, x^3y^2, x^2y^3, x^3y^3\right] \{c_i\}, \; i = 17, 32$$

$$w = \left[1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^3y, x^2y^2, xy^3, x^3y^2, x^2y^3, x^3y^3\right] \{c_i\}, \; i = 33, 48$$

$$\gamma_{xz} = [1, x, y, xy] \{c_i\}, \qquad i = 49, 52$$

$$\gamma_{yz} = [1, x, y, xy] \{c_i\}, \qquad i = 53, 56$$

This element is also free from shear locking phenomenon and all the stiffness and mass matrices are evaluated using full integration scheme. This element has been successfully employed in Ref. [19] to study the nonlinear stability analysis of composite skew plates involving stress concentration at the corners.

Results and Discussion

Large amplitude free flexural vibration characteristics of point supported (Fig.1) square composite plates of length "a" and thickness "h" are studied here. The material properties, unless specified otherwise, used in the present analysis are

$E_L/E_T = 40.0$, $G_{LT}/E_T = 0.6$, $G_{TT}/E_T = 0.5$,
$v_{LT} = 0.25$, $E_T = 1.00000.0$ and $\rho = 1.0$.

Here, E, G, v and ρ are Young's modulus, shear modulus, Poisson's ratio and density. Subscripts L and T represent the longitudinal and transverse directions respectively with respect to the fibers. All the layers are of equal thickness. The boundary conditions considered here are:

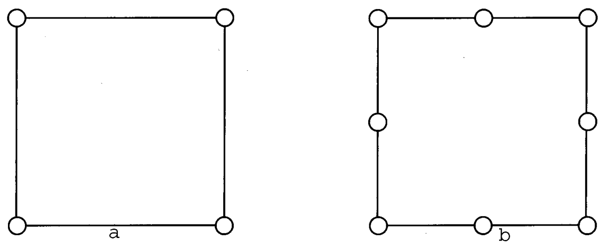

Fig.1 The Point Supported Square (a x a) Plate of Thickness h. Point Supports are Encircled
(a) Point Supported at the Corners
(b) Point Supported at the Corners as well as the Mid-edges

Simply supported plate :

$u_0 = v_0 = w = 0$ along the edges

4-point supported plate :

$u_0 = v_0 = w = 0$ at four corner nodes

8-point supported plate :

$u_0 = v_0 = w = 0$ at middle of edges as well as corner nodes

Before studying the nonlinear behavior of point supported plates, the efficiency of different plate bending elements for the non-dimensional linear vibration frequencies ($\overline{\omega} = \omega\, a^2/\sqrt{\rho\, h/D_0}$,) $D_0 = E_1 h^3 / 12^* (1 - \upsilon_{12}\upsilon_{21})$ of a corner supported thin square angle-ply [45^0/-45^0/45^0/-45^0/45^0] composite plate is studied in Table-1 along with the available analytical solutions of Cheung and Zhou [12] and they match very well. It is observed from Table-1 that all the three elements employed here have good convergence property in calculating linear frequencies and thus a 6x6 mesh of element 1 and 10x10 mesh of element 2 and element 3 are adequate to model the full plate for the linear analysis of corner supported plates. Further, the non-dimensional linear frequencies ($\overline{\omega} = \omega\, a^2/\pi^2 h \sqrt{\rho/E_T}$) of a thin square symmetric cross-ply [0^0/90^0/0^0/90^0/0^0] plate point supported at corners or point supported at the mid-node of the edges as well as corners are studied with different plate bending elements and the converged frequencies are presented in Table-2.

Now, the convergence of different plate bending elements for the nonlinear frequency ratios (ω_{NL}/ω_L) of simply supported and corner supported thin (a/h = 100) isotropic square plates are examined in Table-3. The nonlinear frequencies of simply supported plates are studied by different investigators, while the nonlinear frequencies of corner supported plates are not available in the literature. Eigenvalue equation (6) is solved iteratively ($q_0 = 0$) for different amplitude (w/h) of vibration and a detailed convergence study is carried out for the nonlinear frequencies. It can be observed from Table-3 that, similar to linear frequencies, the nonlinear frequencies also converge well for simply supported plates and a 6x6 mesh of element 1 and 10x10 mesh of elements 2-3 are adequate to model the full plate. However, even if, the convergence is monotonic for the corner supported plate, the rate of convergence for nonlinear frequencies is very slow. This is attributed to the localized in-plane stresses at the corners due to waiving of

Table-1 : Convergence Study of Non-dimensional Linear Frequency ($\overline{\omega} = \omega a^2 / \sqrt{\rho h / D_0}$) of a Corner Supported Square Angle-ply [45^0/-45^0/45^0/-45^0/45^0] Composite Plate (E_1 = 138, E_2 = 8.96, G_{12} = 7.1, υ_{12} = 0.3) (a/b = 1, a/h = 1000, $D_0 = E_1 h^3 / 12 (1 - \upsilon_{12} \upsilon_{21})$)

	Mess Size (dof)	Modes 1	2	3	4	5	6
Element 1	2 x 2 (245)	4.47334	7.93027	8.88789	13.11079	21.43231	25.94171
	4 x 4 (845)	4.47226	7.92203	8.88071	13.10618	21.27514	25.30575
	6 x 6 (1805)	4.47207	7.92187	8.87995	13.10508	21.27119	25.29461
Element 2	4 x 4 (250)	4.47175	7.92141	8.88044	13.10484	21.26651	25.28185
	6 x 6 (490)	4.47175	7.92141	8.88044	13.10484	21.26651	25.28185
	8 x 8 (810)	4.47175	7.92141	8.88044	13.10484	21.26651	25.28185
	10 x 10 (1210)	4.47175	7.92141	8.88044	13.10484	21.26651	25.28185
Element 3	4 x 4 (350)	4.47649	7.92847	8.89208	13.12356	21.39785	25.52658
	6 x 6 (686)	4.47332	7.92365	8.88381	13.11098	21.30273	25.35382
	8 x 8 (1134)	4.47271	7.92256	8.88212	13.10849	21.28377	25.31583
	10 x 10 (1694)	4.47251	7.92221	8.88152	13.10791	21.27788	25.30411
Cheng and Zhou [12]		4.5076	8.0919	8.9916	13.217	21.434	25.665

Table-2 : Non-dimensional Linear Frequency of Square Cross-ply [0^0/90^0/0^0/90^0/0^0] Composite Plates (a/b = 1, a/h = 1000, $\overline{\omega} = \omega a^2 / \pi^2 h \sqrt{\rho / E_T}$)

	Mess Size (dof)	Modes 1	2	3	4	5	6
Supported at Four Corners							
Element 1	6 x 6 (1805)	0.64594	0.97768	1.42595	2.52801	3.1821	3.96606
Element 2	10 x 10 (1210)	0.64599	0.97772	1.42613	2.52804	3.18262	3.96563
Element 3	10 x 10 (1694)	0.6460	0.97773	1.42616	2.52822	3.18295	3.96623
Supported at the Corners and Mid-node of the Edges							
Element 1	6 x 6 (1805)	1.72827	3.18617	3.27993	3.99993	5.44687	5.88163
Element 2	10 x 10 (1210)	1.72702	3.18262	3.19069	3.92774	5.45099	5.80377
Element 3	10 x 10 (1694)	1.72688	3.18295	3.19068	3.92756	5.45231	5.80507

the edges in the in-plane directions as observed in Fig.2, where the nonlinear mode shapes for a simply supported and point supported isotropic plate are plotted. It is observed here that the complete cubic polynomial shape functions for the in-plane displacements (u_0, v_0) helps the elements 1 and 3 to converge faster than element 2 with lower order interpolation polynomial for in-plane displacements (u_0, v_0) and fifth order polynomial for transverse displacement (w). It may be noted further that for the case of thin plates (a/h = 100) the nonlinear frequencies obtained from a 20x20 mesh of element 3 with a total degree of freedom of 6174 is almost near to the corresponding values obtained from a 16x16 mesh of element 1 with a total degree of freedom of 12005. Hence element 3 performs better for thin plates (a/h = 100). It is further observed from Table-3 that the hardening non-linearity in

Table-3 : Convergence Study of Nonlinear Frequency Ratios (ω_{NL} / ω_L) of isotropic Square Plate with Different Elements (a/b = 1, a/h = 100)

	Mess Size (dof)	w_{max} / h					
		0.2	0.4	0.6	0.8	1.0	1.2
Simply Supported Plate							
Element 1	2 x 2 (245)	1.01907	1.07425	1.16074	1.27281	1.40515	1.55339
	4 x 4 (845)	1.01908	1.07425	1.16073	1.27275	1.40498	1.55302
	6 x 6 (1805)	1.01909	1.07431	1.16086	1.27296	1.40527	1.5534
	8x 8 (3125)	1.01911	1.07437	1.16098	1.27316	1.40556	1.55378
	10 x 10 (4805)	1.01912	1.07443	1.16110	1.27335	1.40583	1.55412
Element 2	6 x 6 (490)	1.01973	1.07677	1.16589	1.28081	1.41575	1.56596
	8 x 8 (810)	1.01940	1.07554	1.16340	1.27695	1.4106	1.55978
	10 x 10 (1210)	1.01925	1.07499	1.16228	1.2752	1.40827	1.55696
Element 3	6 x 6 (686)	1.01847	1.07216	1.15669	1.26662	1.39677	1.54282
	8 x 8 (1134)	1.01862	1.07271	1.15784	1.26849	1.39942	1.54627
	10 x 10 (1694)	1.01870	1.07303	1.15850	1.26957	1.40095	1.54826
Corner Supported Plate							
Element 1	4 x 4 (845)	1.01134	1.04453	1.09728	1.16642	1.24863	1.3409
	6 x 6 (1805)	1.01020	1.04030	1.0883	1.1515	1.2271	1.3125
	8x 8 (3215)	1.00960	1.3770	1.0828	1.1423	1.2139	1.2948
	12 x 12 (6845)	1.00877	1.03459	1.07603	1.13105	1.19737	1.2728
	14x 14 (9245)	1.00850	1.03352	1.07372	1.12717	1.1917	1.2652
	16 x 16 (12005)	1.00827	1.03263	1.07181	1.12397	1.18701	1.25891
Element 2	8x 8 (810)	1.0153	1.0597	1.1291	1.2184	1.3225	1.4371
	12 x 12 (1690)	1.0135	1.0529	1.1149	1.1954	1.2899	1.3950
	16 x 16 (2890)	1.0124	1.0488	1.1063	1.1813	1.2699	1.3687
	20 x 20 (4410)	1.0117	1.0459	1.1003	1.1715	1.2558	1.3503
Element 3	8 x 8 (1134)	1.0099	1.0388	1.0850	1.1459	1.2189	1.3013
	12 x 12 (2366)	1.0090	1.0355	1.0778	1.1339	1.2014	1.2781
	16 x 16 (4046)	1.0085	1.0334	1.0773	1.1264	1.1904	1.2634
	20 x 20 (6174)	1.0081	1.0319	1.0701	1.1210	1.1826	1.2529
	24 x 24 (8750)	1.0078	1.0308	1.0679	1.1173	1.1774	1.2460
	28 x 28 (11774)	1.0076	1.0299	1.0659	1.1140	1.1725	1.2395

corner supported plate is less compared to simply supported plate.

Next, nonlinear frequency ratios of isotropic, cross-ply $[0^0/90^0/0^0/90^0/0^0]$ and angle-ply $[45^0/-45^0/45^0/-45^0/45^0]$ point supported (4 point and 8 point) thin (a/h =100) and thick (a/h =10) square plates are presented in Table-4 using 20x20 mesh of element 3 (total degree of freedom 6174) and 16x16 mesh of element 1 (total degree of freedom of 12005) respectively. It may be observed that,

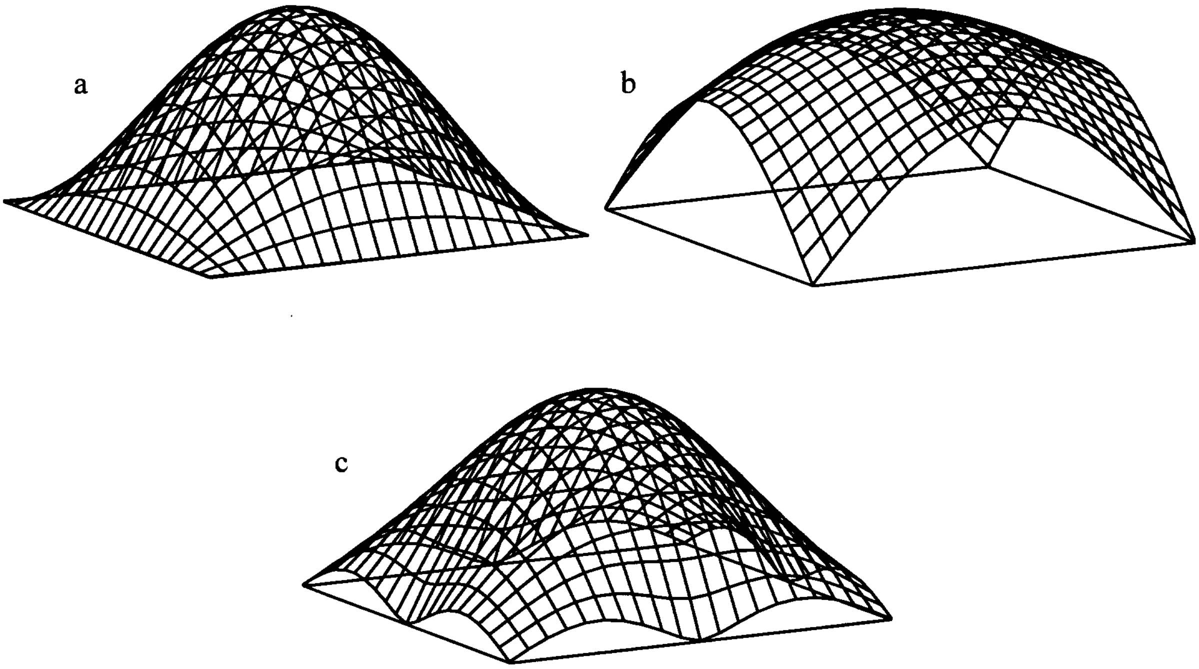

Fig.2 Non-linear First Mode Shape (w/h = 1.0) of Free Flexural Vibration of Square Plates. (a) Simply Supported Plate (b) Point Support at Four Corners (c) Point Support at Four Corners and also at the Mid Edges

Table-4 : The Non-linear Frequency Ratios (ω_{NL} / ω_L) of Point Supported Square Plates

w/h	*a/h* = 100 (Result of Element 3 with 20 x 20 mesh)			*a/h* = 10 (Result of Element 1 with 16 x 16 mesh)		
	Isotropic	[$0^0/90^0/0^0/90^0/0^0$]	[$45^0/-45^0/45^0/-45^0/45^0$]	Isotropic	[$0^0/90^0/0^0/90^0/0^0$]	[$45^0/-45^0/45^0/-45^0/45^0$]
Supported at Four Corners						
0.2	1.00808	1.00744	1.00589	1.01274	1.04388	1.05566
0.4	1.03186	1.02906	1.02335	1.04888	1.13920	1.16514
0.6	1.07010	1.06281	1.05176	1.10355	1.23897	1.15947
0.8	1.12099	1.10552	-	1.17160	1.26714	1.13579
1.0	1.18256	1.14932	-	1.24897	**1.13794**	**1.00717**
1.2	1.25287	1.18816	-	1.33260	1.13049	-
Supported at the Corners and Mid-node of the Edges						
0.2	1.00765	1.00619	1.00455	1.01105	1.04277	1.06993
0.4	1.03030	1.02451	1.01801	1.04329	1.13654	1.10184
0.6	1.06705	1.05418	1.03990	1.09430	1.23028	1.29262
0.8	1.11665	1.09407	1.06949	1.16093	1.32816	1.41061
1.0	1.17760	1.14279	1.10590	1.23995	1.42289	1.50347
1.2	1.24840	1.19877	1.14822	1.32853	1.51984	1.57471

the nonlinear frequencies are in general higher for thick isotropic plates compared to thin ones. However, for the case of composite plates, vibration mode changes for some cases at higher vibration amplitude (w/h) leading to drop in nonlinear frequencies [20] or non-convergence in periodic solution.

Now, the forced vibration characteristics of a corner supported isotropic and cross-ply $[0^0/90^0/0^0/90^0/0^0]$ square plate (a/h = 100) under transverse harmonic pressure $q_0 \sin \theta t$ ($\theta = 0.8\omega_L$; ω_L is the linear vibration frequency) are taken-up for investigation. The forced vibration amplitudes $\{\delta_m, \delta_b\}^T$ are obtained from the matrix amplitude equation (6). Time history analysis with Newmark's time integration technique is carried out starting from the initial condition ($\delta = \{\delta_m, \delta_b\}^T$ at time $t = T/4$) and the dynamic response of transverse displacement (w_c/h) at the centre are presented in Fig.3 for excitation frequencies $\theta = 0.8\omega_L$. The dynamic response is observed to be steady-state and hence the validity of matrix-amplitude equation (6) is established.

Next, the steady-state forced vibration amplitudes (w/h) of a corner supported isotropic and cross-ply $[0^0/90^0/0^0/90^0/0^0]$ square plates under transverse harmonic pressure $q_0 \sin \theta t$ are studied Fig.4. The backbone curves, i.e., the frequency-amplitude relationships, for the case of free flexural vibration ($q_0 = 0$), are presented in the Fig.4 as solid line. The nonlinear forced vibration amplitudes (w_{max}/h) under non-dimensional excitation frequency (θ/ω_L) are presented as scattered symbols for various values of the non-dimensional load parameters $q_N = q_0 a^3 / D$ or $q_N = q_0 a^3 / E_T t^3$ for the case of isotropic and composite plates respectively. It is observed from the figure that, the flexural vibration amplitude (w/h) increases as the excitation frequency (θ) either increases from zero or decreases from a higher value (say, $\theta = 2\omega_L$). As the excitation frequency approaches the linear flexural vibration frequency (ω_L) of the plate from either side, the nonlinear flexural vibration amplitude increases rapidly

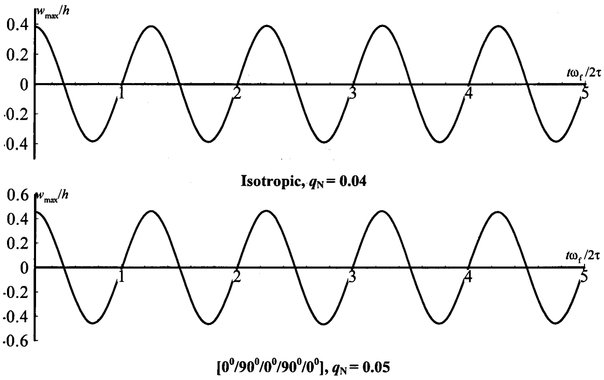

Fig.3 Steady-state Dynamic Response of a Corner Supported Square Plate Under Uniformly distributed Transverse Harmonic Pressure $q_0 \sin \theta t$ ($\theta = 0.8\ \omega_L$; $a/b = 1$; $a/h = 100$; $q_N = q_0 a^3 / D$ for Istropic, $q_N = q_0 a^3 / E_T t^3$ for Composites Plate)

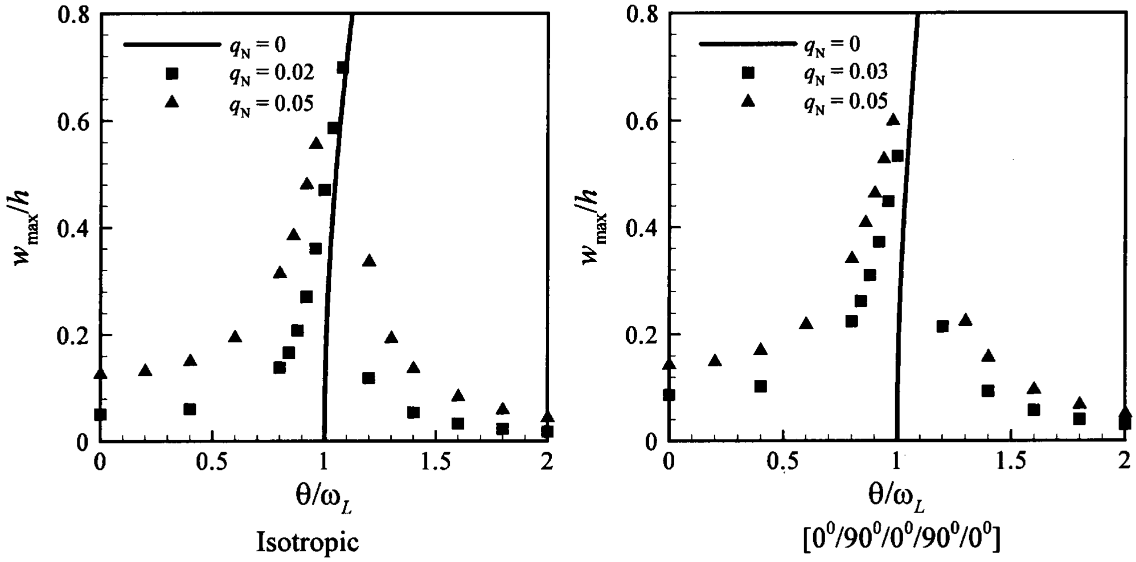

Fig.4 Non-linear Flexural Vibration of a Corner Supported Square Plate Under Uniformly Distributed Transverse Harmonic Pressure $q_0 \sin \theta t$ ($a/b = 1$; $a/h = 100$; $q_N = q_0 a^3 / D$ for Istropic, $q_N = q_0 a^3 / E_T t^3$ for Composites Plate)

(tangential to the backbone curves) as structural damping is not considered in the present study. Further, the vibration at a higher excitation frequency (points on the right side of the backbone curve) is observed to be out-of-phase with the applied load.

Conclusions

Large amplitude free and forced vibration characteristics of point supported isotropic and laminated composite square plates are studied using different high precision plate bending elements. Extensive convergence study is carried out to evaluate the capabilities of plate bending elements (with different interpolation polynomials for the in-plane and transverse displacements) in predicting nonlinear frequencies of point supported isotropic and symmetric laminates. It is seen that although rate of convergence of all the elements is excellent for linear frequencies, convergence of non-linear frequencies is slow due to high in-plane stresses at the point supports. Plate bending elements with higher order interpolation polynomial for both in-plane and transverse displacements converge comparatively faster for the nonlinear analysis of point supported thin plates. However, for the thick plates, interpolation polynomial for the shear rotation (or nodal rotation) plays an important role. It is observed that the hardening non-linearity in point supported plates is less compared to the simply supported plates. For example, the nonlinear frequency ratio (ω_{NL}/ω_L) of a thin isotropic square plate at a vibration amplitude equal to the plate thickness (w_{max}/h = 1) is 1.40583, 1.18256 and 1.17760 for simply supported, 4-point supported and 8-point supported boundary conditions respectively. These results for the large amplitude vibration of point supported composite plates are believed to be new and will serve as benchmark for comparison in future research work.

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