TECHNICAL NOTE

FINITE ELEMENT ANALYSIS OF LAMINATED COMPOSITE SHELLS WITH PIEZOELECTRIC ELEMENTS

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Abstract

The concept of incorporating some degree of smartness or intelligence in a system is gaining wide acceptance for utilisation in a variety of practical applications from aircraft to medical diagnostics because of the capability of smart materials to execute specific functions intelligently in response to changes in environmental stimuli. In this paper, the effectiveness of a simple finite element model in ANSYS is established for analyzing laminated composite shells with piezoelectric elements under static and dynamic loading. The substrate in smart shell is discretised using SOLSH190 element and the piezoelectric layer is discretised using coupled-field element, SOLID5. The basic behaviour of a smart composite shell depends on the behaviour of substrate with the response reduced/controlled by the properties of smart materials used, which is predicted effectively by the present finite element model.

Nomenclature

e_{31}, e_{32}, e_{24}, e_{15}, e_{33}	= piezoelectric stress coefficients in Cm^{-2}
h	= thickness of the shell
E_{11}, E_{22}	= Young's moduli along and transverse direction of fibre
G_{12}, G_{13} G_{23}	= in-plane and transverse shear moduli
L	= length of the shell
u, w	= displacement in hoop and radial direction
ε_{11}, ε_{22}, ε_{33}	= electrical permittivity in Fm^{-1}
ν_{12}, ν_{13}, ν_{23}	= Poisson's ratios
ρ	= density of the material

Introduction

A review of the historical evolution of material science highlights the distinct transition from structural materials like steel, concrete, wood, graphite-epoxy laminates etc. to smart materials as humankind's scientific and technological prowess has matured. Smart materials have the capability to select and execute specific functions intelligently in response to changes in environmental stimuli. In recent years, the novel usages of smart materials in active structures for deflection control, noise attenuation, shape control and vibration suppression have attracted serious attention. Therefore, the integration of smart materials and structural composites has become the subject of focus in the area of smart materials and structures.

A variational formulation of the problem of equilibrium of completely anisotropic piezoelectric shells has been given by Chau [1]. Saravanos [2] has introduced a mixed laminate theory that combines equivalent single layer assumptions for the mechanical displacement with a layerwise representation for the electric potential. A shell finite element and its application on active vibration control of smart piezoelectric composite plate/shell structures has been developed by Balamurugan and Narayanan [3]. Finite element modelling of piezolaminated smart structures for the active vibration control with distributed sensors and actuators has been studied by Narayanan and Balamurugan [4]. Pinto Correia et al. [5] have developed conical panel shaped shell finite element using a mixed laminate theory to analyse adaptive shell structures. Karagülle et al. [6] have studied the active vibration control in cantilever beam and circular disc using ANSYS. An analytical solution for the static deformation and steady-state vibration of simply supported hybrid cylindrical shells consisting of fibre reinforced layers with embedded

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piezoelectric shear sensors and actuators has been given by Vel and Baillargeon [7]. Ray [8] has published a study on smart damping of laminated thin cylindrical panels using piezoelectric fibre reinforced composites by finite element method. Santos et al. [9] have presented the bending and free vibration of multilayered cylindrical shells with piezoelectric elements by a semi-analytical axisymmetric shell finite element model using 3D linear elasticity theory. Study on static and dynamic analyses of smart cylindrical shell using finite element method has been done by Kumar et al. [10]. A piezolaminated composite degenerated shell finite element has been developed to study the active vibration control of structures with distributed piezosensors and actuators by Balamurugan and Narayanan [11]. Malgaca and Karagülle [12] studied experimental analysis of active vibration control of smart beam under harmonic excitation and its simulation using ANSYS.

The studies carried out in the field of smart structures are mainly concentrated in the analysis of smart beams with limited literature available on the analysis of smart shells. Results on the analysis of smart shells using ANSYS, the finite element software, is not available in literature. In this paper, the effectiveness of a simple finite element model in ANSYS is established for analyzing laminated composite shells with piezoelectric elements under static and dynamic loading.

Smart Materials

Smart structures can be characterised by the type of device that is used to do the sensing or the actuation. The most common of these devices are: piezoelectric, fibre-optic, shape memory alloys, electro-rheological fluids and magneto-rheological fluids. In the present study, piezoelectric materials are used as smart material. Piezoelectric materials are solids which generate an electric charge in response to a mechanical deformation (direct piezoelectric effect) and conversely, develop mechanical deformation when subjected to an electric field (converse piezoelectric effect). The coupled mechanical and electrical properties of piezoelectric materials make them well suited for use as sensors and actuators. The two common types of piezoelectric materials are lead zirconate titanate (PZT) ceramics and polyvinylidene fluoride (PVDF) polymers. Piezoelectric materials have been found to be very attractive for smart structure applications. Accordingly, piezoelectric materials are used as smart materials in the present study.

Modelling of Smart Shells

Since the study is concerned with the analysis of shells with piezoelectric elements, it is essential to use a suitable element to model substrate which can be used in combination with that for piezoelectric elements. In ANSYS, following elements are available to model layered composite materials, viz., SHELL99, SHELL91, SHELL181, SOLSH190, SOLID186 Layered Solid, SOLID46 and SOLID191, called layered elements. The elements available for piezoelectric analysis are PLANE13, PLANE223, SOLID5, SOLID98, SOLID226 and SOLID227, called coupled-field elements. The piezoelectric analysis of composite shells with piezoelectric elements necessitates the usage of coupled-field solid element, SOLID5, so as to enable polarisation across the thickness. Accordingly, out of the various layered elements presented above, the element SOLSH190, an 8-noded layered solid shell element suited to model both thin and moderately thick shells and solids is used to model the shells. This element allows upto 250 different material layers and has three degrees of freedom per node, viz., translation in X, Y and Z directions. The element SOLID5 has six degrees of freedom per node (U_X, U_Y, U_Z, TEMP, VOLT and MAG). Piezoelectric layer is assumed to be continuous and surface-bonded to the host structure, which is treated as an additional layer. Modelling of piezoelectric material/layer requires permittivity or dielectric constants, the piezoelectric matrix and the elastic coefficient matrix to be specified as material properties.

Deflection Control

Validation

In order to validate the modelling of piezoelectric layer, a cantilever piezoelectric bimorph beam, used by Balamurugan and Narayanan [11], of span, l = 100 mm and breadth, b = 20 mm as shown in Fig.1a, is analysed. The beam consists of two PZT layers of 1 mm thick each and is assumed to be perfectly bonded. Each layer of the beam is discretised into 25 x 5 x 3 mesh along length,

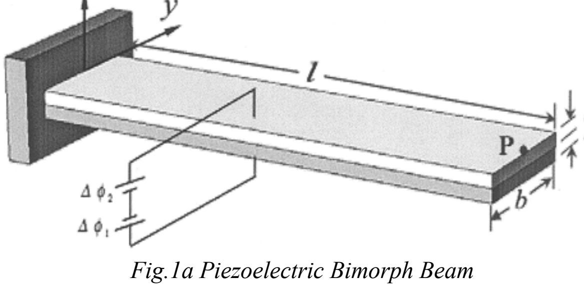

Fig.1a Piezoelectric Bimorph Beam

breadth and thickness directions respectively. The electric potentials, $\Delta\phi_1$ and $\Delta\phi_2$, are prescribed at the lower surface of the bottom layer and the upper surface of the top layer, while the interface between the layers is ground. The elastic properties of the PZT material are given below.

The constitutive matrix is given by,

$$\begin{bmatrix} 107.6 & 63.12 & 63.85 & 0 & 0 & 0 \\ & 107.6 & 63.85 & 0 & 0 & 0 \\ & & 100.4 & 0 & 0 & 0 \\ & & & 19.62 & 0 & 0 \\ & \text{symm}. & & & 22.24 & 0 \\ & & & & & 19.62 \end{bmatrix} \times 10^9\ \text{Nm}^{-2}$$

The piezoelectric coefficients are,

$$e_{31} = e_{32} = -9.6\ \text{Cm}^{-2},\ e_{24} = e_{15} = 12\ \text{Cm}^{-2},\ e_{33} = 15.1\ \text{Cm}^{-2}$$

The dielectric constants are,

$$\varepsilon_{11} = \varepsilon_{22} = 1.714\times 10^{-8}\ \text{Fm}^{-1},\ \varepsilon_{33} = 1.8673\times 10^{-8}\ \text{Fm}^{-1}$$

An electric field of 10 Vmm^{-1} is applied on the bimorph beam and the tip deflection is determined. The deformed pattern of the beam due to applied voltage is shown in Fig.1b.

Table-1 presents a comparison of the result of present study with those of various researchers and is found to be in reasonable agreement.

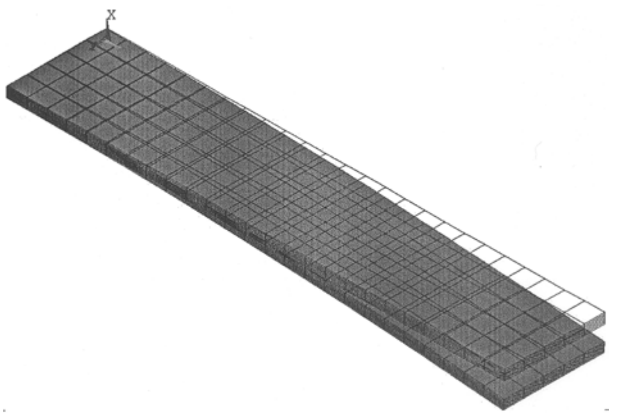

Fig.1b Deformed Pattern due to Applied Voltage

Table-1 : Tip Deflection of Piezoelectric Bimorph Cantilever Beam

Studies	Variation of Electric Field along Thickness	Tip Deflection (μm)
Present study	Linear	14.80
Kögl and Bucalem [13]	Linear	16.81
	Quadratic	15.39
Balamurugan and Narayanan [14]	Linear	16.99
	Quadratic	15.56
Balamurugan and Narayanan [11]	Quadratic	15.29

Performance of Piezoelectric Material as Actuator

A cantilever graphite-epoxy semicircular shell (45/-45/0)$_s$ with continuous piezoelectric layer at top and bottom, used by Saravanos [2], is analysed to study its behaviour. The thickness of each composite ply is 0.12 mm, whereas the thickness of each piezoelectric layer is 0.24 mm. The circumferential length, *l*, is 314 mm, *l/L* ratio is 5 and radius, R, is 100 mm as shown in Fig.2. The shell is discretised into 15 x 4 mesh in hoop and length directions respectively. The piezoelectric layer is discretised into three layers in the thickness direction.

The material properties of graphite-epoxy are given below.

E_{11} = 132.4 GPa., $E_{22} = E_{33}$ = 10.8 GPa., G_{23} = 3.6 GPa., $G_{12} = G_{13}$ = 5.6 GPa.,

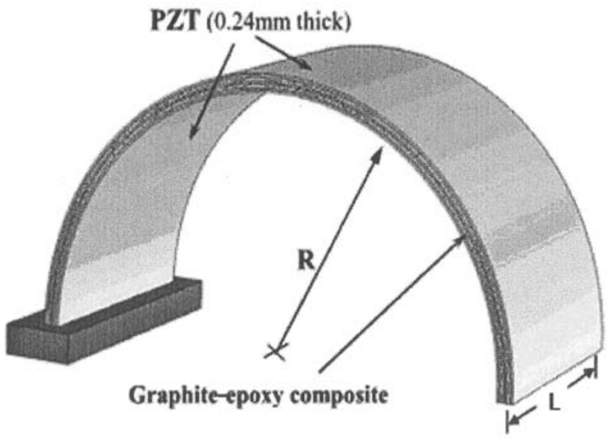

Fig.2 Semicircular Cantilever (45/-45/0)s Shell

$\nu_{12} = \nu_{13} = 0.24$, $\nu_{23} = 0.49$, $\rho = 1578$ kgm^{-3}

The elastic properties of piezoelectric material are given below.

$E_{11} = E_{22} = 81.3$ GPa., $E_{33} = 64.5$ GPa., $G_{23} = G_{13} = 25.6$ GPa.,$G_{12} = 30.6$ GPa.,

$\nu_{12} = 0.33$, $\nu_{13} = \nu_{23} = 0.43$, $\rho = 7600$ kgm^{-3}.

The piezoelectric coefficients are,

$e_{31} = e_{32} = -\ 14.80$ Cm^{-2}, $e_{24} = e_{15} = 12.67$ Cm^{-2}.

The dielectric constants are,

$\varepsilon_{11} = \varepsilon_{22} = 1.3054 \times 10^{-8}$ Fm^{-1}, $\varepsilon_{33} = 1.1505 \times 10^{-8}$ Fm^{-1}

A voltage of +100V is applied on the free surface of each piezoceramic layer. The variation of non-dimensional hoop (u/h) and radial (w/h) displacement is plotted in Fig.3 and is compared and are in good agreement with the results available in literature. Saravanos [2] and Pinto Correia et al. [5] used mixed laminate theory to formulate the finite element in the analysis.

Performance of Piezoelectric Material as Sensor

A cantilever graphite-epoxy $(0/90/45/-45)_s$ semicircular shell, used by Saravanos [2], with extreme continuous PZT layers is considered to study the performance of piezoelectric materials as sensors. The geometric properties and the material properties of the shell are same as in the section Performance of piezoelectric material as actuator. Both the piezoelectric layers are considered as sensors and a radial line load of 10 N is applied at the free end.

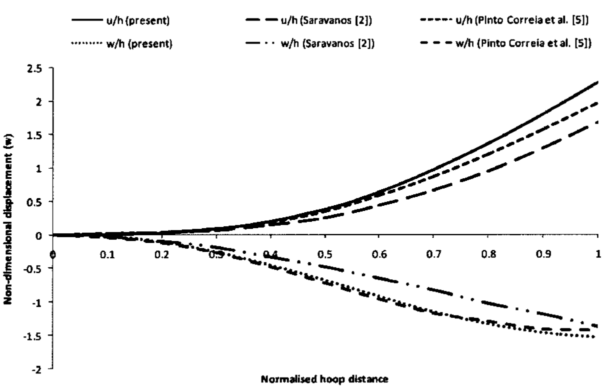

Fig.3 Non-dimensional Radial and Hoop Deflection of the Semicircular Cantilever Shell

Fig.4 shows the normalised sensor potential, $\frac{\phi\, d_{31}}{h} \times 10^6$, sensed by the outer piezoelectric layer along the hoop direction, which agrees reasonably with the available results. Saravanos [2] and Pinto Correia et al. [5] asummed a linear variation and Balamurugan and Narayanan [11] adopted a quadratic variation of electric field through the thickness of the piezoelectric layer. SOLID5 element in ANSYS is formulated with linear variation of electric field through thickness.

Effect of Actuator Voltage

In order to study the effectiveness of piezoelectric material in controlling the deflection of laminated composite shells, symmetric angle-ply $(45/-45/0)_S$ and antisymmetric angle-ply $(45/-45/0)_2$ shells with two straight edges fixed and curved edges free, subjected to a uniformly distributed load of 12 Nm^{-2}, are considered. The geometric and material properties of the shell are same as in the section Performance of piezoelectric material as actuator. Fig.5 gives the non-dimensional central deflection, $\overline{w} = \frac{w}{h} \times 10^{3}$ with different positive voltages applied

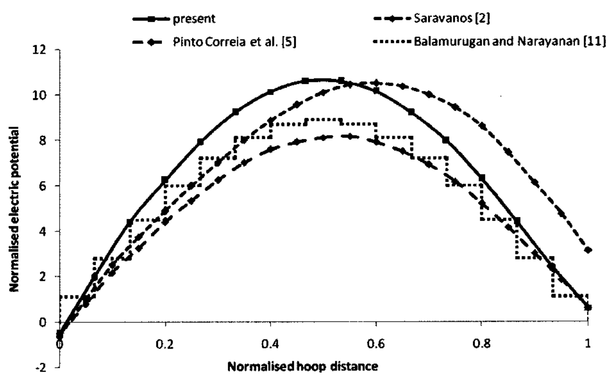

Fig.4 Normalised Sensor Potential along Hoop Direction of Cantilever Shell

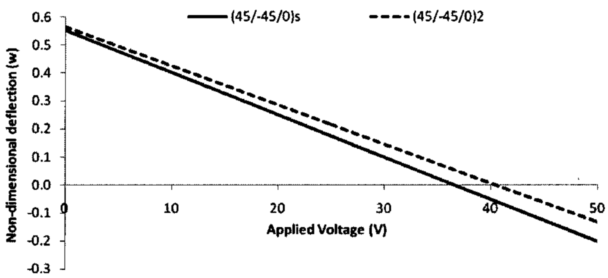

Fig.5 Non-dimensional Central Deflection of Shell due to Different Applied Voltage

at top and bottom piezoelectrodes. From the figure, it is observed that the deflection decreases with increase in voltage, but, with a specific voltage to attain deflection control depending on the geometry, material, loading etc. of the shell, beyond which it deflects in the reverse direction.

Effect of Actuator Position

4-layer symmetric angle-ply shell, with all sides fixed and having length-to-thickness ratio, $L/h = 100$, included angle, $\varphi = 60°$ and thickness, $h = 5$ mm, with piezoelectric layers at different levels are analysed in order to study the effect of actuator position in the deflection control. The material used is graphite-epoxy with the following properties.

E_{11} = 138 GPa., E_{22} = 8.96 GPa., G_{12} = 7.1 GPa., ν_{12} = 0.30.

Properties of the piezoelectric material are as given in the section Performance of piezoelectric material as actuator. Three different cases are considered. In the first case, single piezoelectric layer is bonded at the lower surface, i.e., (p/45/-45/-45/45), in the second case, single piezoelectric layer is bonded at the upper surface, i.e., (45/-45/-45/45/p), and in the third case two piezoelectric layers are bonded as dual actuator at upper and lower surfaces, i.e., (p/45/-45/-45/45/p). The shells are subjected to a transverse uniformly distributed load of 800 Nm^{-2}. In each case, voltage is applied incrementally and the corresponding Root Mean Square Error (RMSE) of the non-dimensional radial deflection along the central line ($L/2$) with reference to the undeformed geometry is calculated as RMSE = $\sqrt{(\text{non - dimensional radial deflection})^2/n}$,where n is the number of nodes along the central line. The variation of RMSE with different applied voltages is presented in Fig.6. When the piezoelectric layer is used at top and bottom, the least RMS Error (0.005) is achieved at a low voltage compared to that of the shell with piezoelectric layer either at top or bottom. Moreover, it is clear that the shell with piezoelectric layer either at top or bottom does not have any significant difference in the behavioural pattern. Fig.7 shows the non-dimensional radial central line deflection in hoop direction when piezoelectric layer is provided both at top and bottom. It is seen that a voltage of 20 V enables to control the deflection to an appreciable level.

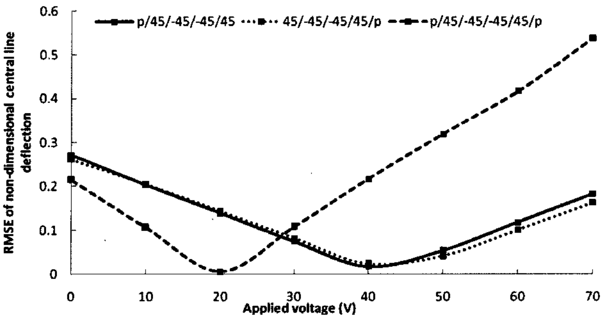

Fig.6 Effect of Actuator Position on Deflection Control

Active Vibration Control

Validation

The modelling procedure for the active vibration control of shells is validated by analysing a smart beam given by Karagülle et al. [6] as shown in Fig. 8.

The dimensional parameters are given by,

Beam : 504 x 25.4 x 0.8 mm
Actuator : 72 x 25.4 x 0.61 mm
Actuator distance (d_a) = 12 mm
Sensor distance (d_s) = 48 mm

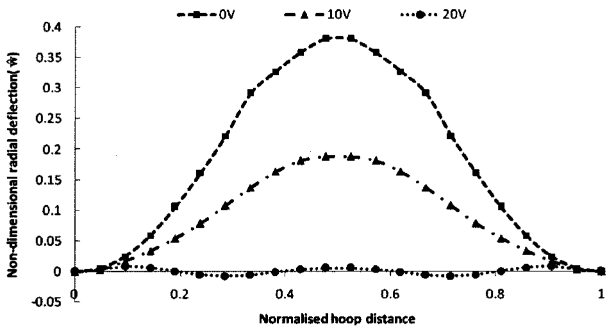

Fig.7 Non-dimensional Central Line Deflection with Different Applied Voltages

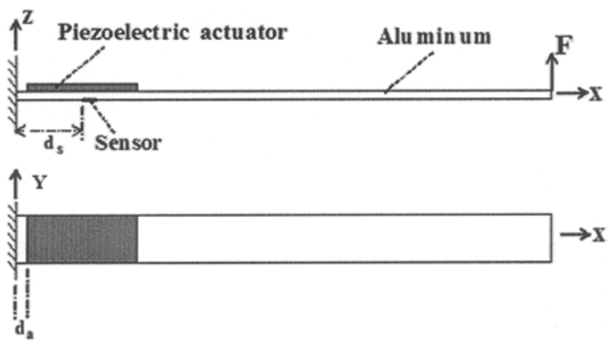

Fig.8 Smart Beam

The material properties of the beam were: E = 68 x 10^9 Nm^{-2}, Poisson's ratio, ν = 0.32 and Density, ρ = 2800 kgm^{-3}. The properties of the piezoelectric material are given as,

Constitutive matrix:

$$\begin{bmatrix} 126 & 79.5 & 84.1 & 0 & 0 & 0 \\ & 126 & 84.1 & 0 & 0 & 0 \\ & & 117 & 0 & 0 & 0 \\ & & & 23 & 0 & 0 \\ & \text{symm .} & & & 23.2 & 0 \\ & & & & & 23 \end{bmatrix} \times 10^{9}\ Nm^{-2}$$

Piezoelectric coefficients:

$e_{31} = e_{32}$ = -6.5 Cm^{-2}, $e_{24} = e_{15}$ = 17 Cm^{-2}
e_{33} = 23.3 Cm^{-2}

Dielectric constants:

$\varepsilon_{11} = \varepsilon_{22}$ = 1.503 x 10^{-8} Fm^{-1}, ε_{33} = 1.3 x 10^{-8} Fm^{-1}

The beam is discretised into 28 x 4 x 1 mesh and the sensor is discretised into 4 x 4 x 3 mesh. A step load of 0.1 N (F) for a duration of 0.0159 second is applied at the tip of the beam as depicted in Fig.8. Rayleigh damping is assumed with coefficients $\alpha = \beta = 0.001$. Newmark's method is used for time integration with a time step (d_t) of 0.0159 second. Constant gain feedback control theory is used for the active vibration control, the block diagram of which is depicted in Fig.9. The input reference value is taken as zero in the vibration control. The instantaneous value of the strain, ε, at the sensor location at a time step is subtracted from zero to find the error signal value. The error value is multiplied by the control gain to calculate the voltage value (V_a) which is used as the input to the actuator nodes. K_s, K_c and K_v are the sensor, control and power amplification factors respectively. K_s, and K_v are taken as 1000 and the value of K_c is changed in the analysis. The process is continued until the steady-state value is reached approximately. A macro in APDL (ANSYS Parametric Design Language) is written to perform the active vibration control.

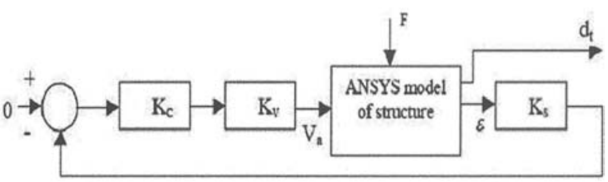

Fig.9 Block Diagram of Constant Gain Feedback Control Theory

The time history of tip deflection of the beam for different gain values is determined and compared with that of Karagülle et al. [6] in Fig.10. The variation of actuator voltage with K_c = 5 is also plotted in Fig.11. The plots show that the present study is in excellent agreement with those in literature.

Smart Shell

A laminated composite cantilever shell of inner radius 100 mm, width 50 mm and thickness 0.96 mm with a sequence $(0/90/45/-45)_s$ is considered to study the active vibration control. The material properties of the shell are same as in section Performance of piezoelectric material as actuator. A piezoelectric patch of size 50 x 25 x 0.5 mm is bonded to the outer surface of the shell, 8° from the fixed end, as shown in Fig.12. The finite element model is given in Fig.13.

A step load of 0.5 N for a duration of 0.004 second is applied at the tip of the shell in the hoop direction. The time step (d_t) for analysis is taken as 0.004 second. The control system is same as before. The instantaneous hoop strain at the point of 14.3 degree from the fixed support in hoop direction is used to find out the actuator voltage. As

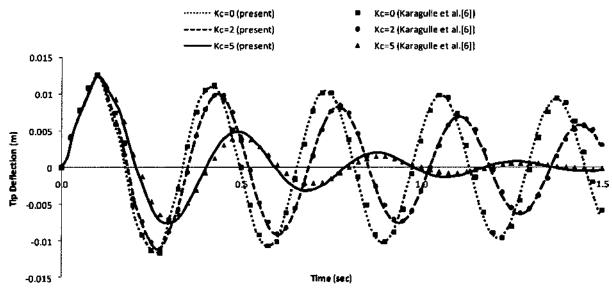

Fig.10 Tip Deflection of Smart Beam for Different Values of Control Gain

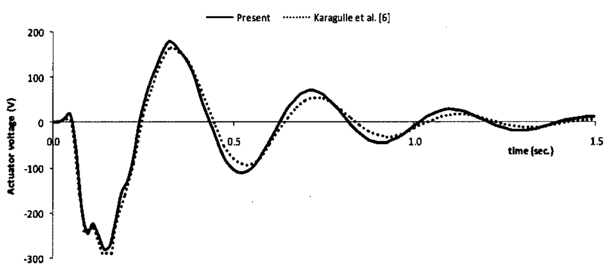

Fig.11 Actuator Voltage for K_c = 5

for the beam, an APDL macro is written to perform the active vibration control as given below.

```
antype, trans                ! defining transient analysis
trnopt, full                 ! full transient option
dt = 0.004                   ! time step
nt = 512                     ! tip node
ns = 174                     ! sensor node
h = 100.96e-3                ! radius of outer surface of shell
h1 = 101.46e-3               ! radius of outer surface of piezo
ks = 1000                    ! sensor amplification factor
kv = 1000                    ! power amplification factor
kc = 5                       ! control amplification factor
kbc, 1                       ! step load option initialisation
tintp,,0.25,0.5,0.5          ! transient integration parameters
time, dt                     ! initial time
deltim, dt                   ! time step
csys, 1                      ! arrest all the nodes one end
nsel, s, loc, y,180
d, all, ux, 0,,,, uy, uz
nsel, all                    ! ground inner piezoelectrode
nsel, s, loc, x, hd, all, volt, o
nsel, s, loc, x, h1          ! short-circuit the outer electrode
d, all, volt, 0
csys, 0
f, nt, fy, -0.5              ! apply impact load
solve
*do, t, 2* dt, ts, dt        ! start do loop
csys, 0                      ! remove impact load
f, nt, fy, 0
*get, eps, node, ns, epel, y ! extract the hoop strain
err=0-ks*eps                 ! calculate error function
va=kc*kv*err                 ! actuator voltage
csys, 1
nsel, s, loc, x, h           ! ground inner piezoelectrode
d, all, volt, 0
nsel, s, loc, x, h1          ! apply voltage at the outer electrode
d, all, volt, va
time, t
solve
*enddo                       ! end do loop
```

The time history of radial and hoop deflection at the free end of the cantilever shell are plotted in Fig.14 for different values of control gain. The actuator voltage for a control gain of 10 is also plotted against time in Fig.15. It

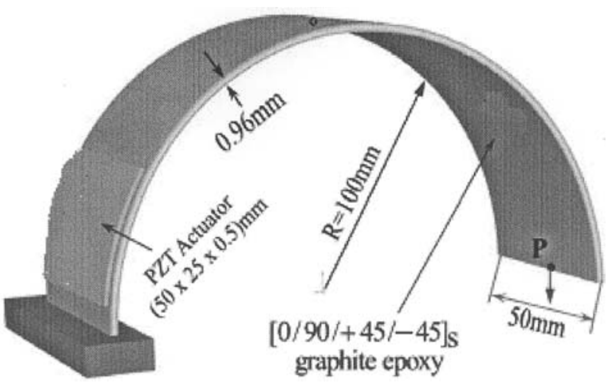

Fig.12 Smart Cantilever Shell

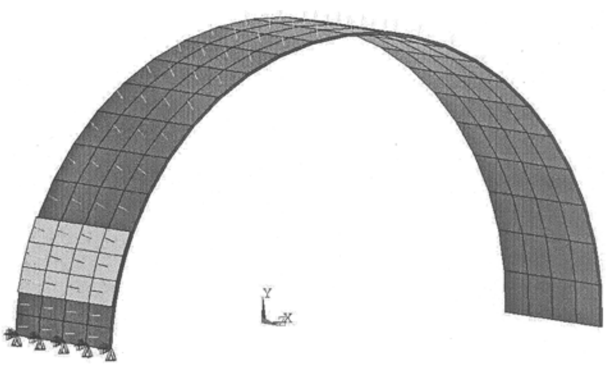

Fig.13 Finite Element Model of Smart Cantilever Shell

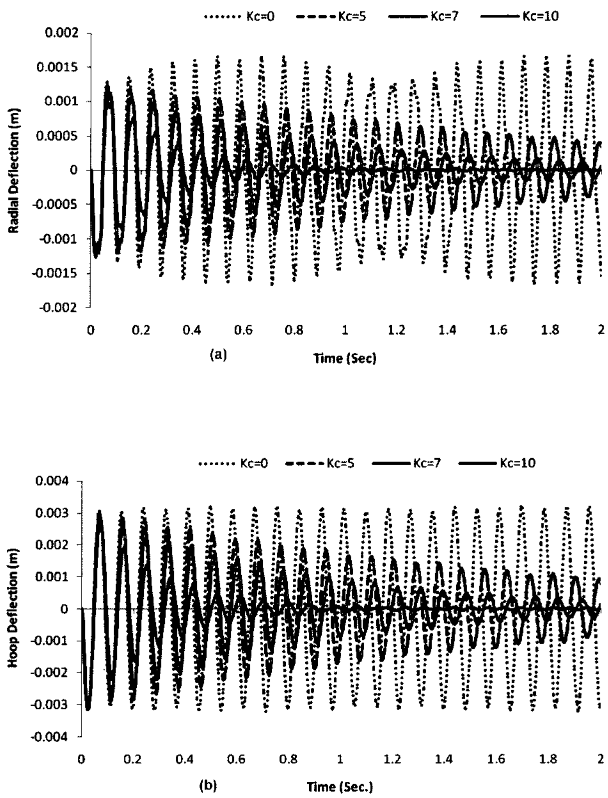

Fig.14 Response of Cantilever Smart Shell for Different Control Gain (a) Radial Deflection (b) Hoop Deflection

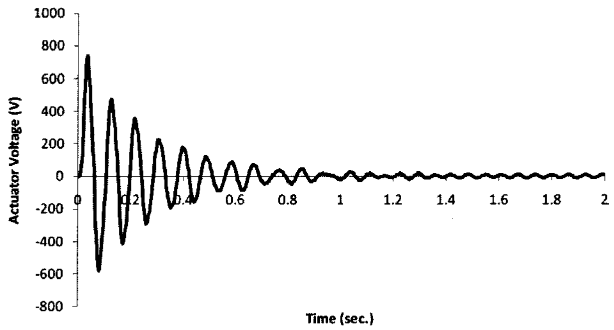

Fig.15 Actuator Voltage for K_c = 10

is observed that as K_c increases, the actuator voltage increases and the vibration settling time decreases. Care should be taken to see that the actuator voltage does not exceed the depoling value of the actuator used, i.e.,1000 V in the present study.

Summary and Conclusions

The behaviour of laminated composite shells with piezoelectric elements is investigated using ANSYS software, in which the substrate is discretised using SOLSH190 element and the piezoelectric layer is discretised using coupled-field element, SOLID5. Following conclusions are drawn from the present study.

- Reduction in deflection is directly proportional to the actuator voltage.
- The shells with piezoelectric layers at top and bottom perform better than that with layers either at top or bottom.
- The amplitude of vibration decreases with increase in voltage without significant change in the frequency.
- An active control of vibration is achieved by suitably amplifying the voltage sensed by the sensor.

In short, the basic behaviour of a smart composite shell depends on the behaviour of substrate with the response reduced/controlled by the properties of smart materials used, which is predicted effectively by the present finite element model.

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