TECHNICAL NOTE

STUDY ON EXTENDING RANGE OF ARTILLERY ROCKET USING CONTROL SURFACES

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Abstract

Extension of range of artillery rocket using gliding technique is very effective way. Extension in the range is achieved by control surfaces without adding any extra charge. Controls are applied after completion of launch phase and trajectory is controlled by canards or flat tail control fins. Contribution in this work is to find the optimum time to initiate the controls during the flight so as to get maximum range. Analysis has been carried out using six degree of freedom equations of motion.

Nomenclature

ϕ	= Roll angles in degree
θ	= Pitch Angle in degree
φ	= Yaw angle in degree
α	= Angle of attack in degree
β	= Side slip angle in degree
δ	= Flight path angle in degree
γ	= Flight Yaw angle in degree
Cf_x	= Control Force coefficient in x direction
Cf_y	= Control Force coefficient in y direction
Cf_z	= Control Force coefficient in z direction
D	= Drag Force
G	= Gravitational force
g	= Acceleration due to gravity
I	= Moment of Inertia
I_{xx}	= Axial moment of Inertia in x direction
I_{yy}	= Axial moment of Inertia in y direction
I_{zz}	= Axial moment of Inertia in z direction
L	= Lift Force
m	= Mass of the Artillery Rocket
p	= Roll rate
q	= Pitch rate
r	= Yaw rate
T	= Thrust Force
u	= Velocity component in body frame in x direction
v	= Velocity component in body frame in y direction
w	= Velocity component in body frame in z direction
$O\text{-}XYZ$	= Inertial coordinate system
$C\text{-}X_bY_bZ_b$	= Body coordinate system
$C\text{-}tnb$	= Velocity Coordinate system

Introduction

Artillery weapon system is used by ground, air, or naval forces such as guns, mortars, Howitzers, Rocket launchers, or overall cannons. Conventional artillery rocket is effective against stationary and area-type targets. Performance of conventional artillery rocket is limited in terms of range and accuracy. Nowadays, there is requirement of long range artillery rockets so as to achieve indirect fire. Extension in range of artillery rocket can be obtained in different ways like increasing muzzle velocity with additional charge, boosting by rocket motor, reduction in drag using base bleed or increasing range of a rocket with gliding flight [2].

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Gliding move the object smoothly and effortlessly. The same concept can be applied to artillery rocket for the extension of the range. Each artillery rocket has its minimum and maximum range depending upon its elevation and line of fire. Conventional artillery rocket can not achieve more range than its fixed maximum range. However by adding gliding effect as additional feature to the conventional artillery rocket, range can be increased [3]. Normally in artillery rocket, gliding phase starts after reaching Vertex height. Once the rocket reaches to vertex, the control surfaces are unfolded & control the lift so that the rocket can move in same attitude. The trajectory can be divided into three phases as follows (Fig.1).

- **Launch Phase**: The first phase corresponds to launch phase. Once the rocket is launched it moves towards its desired path due to thrust force. The launch phase continues till the controls are not activated.
- **Controlled Phase**: The second phase is controlled phase. In this phase, control surfaces are deflected so as to increase the lift force to balance the gravity force. This will make rocket to glide.
- **Ballistic Phase**: In third phase controls are stopped and rocket follows a ballistic trajectory.

Thrust Profile: Thrust profile has been defined in trapezoidal shape (Fig.2). From t_1 to t_2 linear thrust, from t_2 to t_3 constant thrust and t_3 to t_4 linear thrust.

where t_1 is start of propellant burning and t_4 is All Burnt Point (ABP) time. The values of t_2 and t_3 define the interval of constant Thrust. The value of Constant Thrust can be defined as per Rocket design parameters.

In this paper, study has been carried out for the effect of initiation of control at various points during the flight such as within power phase, after power phase and before vertex height.

Mathematical Model

A vector equation obtained from Newton's second law forms a mathematical model to study dynamical motion of the artillery rocket. A general model to study the dynamical motion of the artillery rocket is obtained by resolving the equations of motion in coordinate system.

Coordinate Systems: Coordinate systems used in the study are inertial coordinate system, body coordinate system and velocity coordinate system (Fig.3) which are defined as follows:

Inertial coordinate system (O-XYZ): Launch point is defined as origin O. XZ plane is the horizontal plane. XY is vertical plane of fire. X component defines range, Y defines altitude and Z defines drift of the rocket from required path.

Body Coordinate System ($C\text{-}X_bY_bZ_b$): Body coordinate system is fixed with respect to the geometry of rocket. The origin for this coordinate system is defined at center of gravity of the body C. CX_b defines the axis of rotation symmetry and CY_b is in the plane of reflection symmetry. CZ_b completes right handed system.

Velocity Coordinate System (C-tnb): The origin for this coordinate system is defined at center of gravity of the body C. Ct is along the instantaneous direction of motion, Cn is normal to motion in vertical plane and Cb completes the right handed system.

The relations between these systems have been developed using transformation matrices and obtained by applying Eulerian rotations sequentially. To develop the transformation matrices convention is taken as anticlockwise sense to be positive [4].

Transformation matrix from Inertial Coordinate System to Body Coordinate System consists of three rotations and can be obtained as product of three rotation matrices. Ax (Φ) denotes the rotation matrix obtained by applying rotation through the angle Φ about x-axis.

$$A_{IB} = A_x(\phi)\, A_y(\varphi)\, A_z(\theta)$$

$$\left[A_{IB}\right] = \begin{bmatrix} \cos\theta\cos\varphi & \cos\varphi\sin\theta & -\sin\varphi \\ \sin\phi\sin\varphi\cos\theta - \cos\phi\sin\theta & \cos\phi\cos\theta + \sin\phi\sin\theta\sin\varphi & \sin\phi\cos\varphi \\ \sin\phi\sin\theta + \cos\phi\sin\varphi\cos\theta & \cos\phi\sin\theta\sin\varphi - \sin\phi\cos\theta & \cos\phi\cos\varphi \end{bmatrix} \quad (1)$$

Transformation matrix from Inertial Coordinate System to Velocity Coordinate System is

$$A_{IV} = A_y(\gamma)\, A_z(\delta)$$

$$\left[A_{IV}\right]=\begin{bmatrix}\cos\gamma\cos\delta & \cos\gamma\sin\delta & -\sin\gamma\\ -\sin\delta & \cos\delta & 0\\ \sin\gamma\cos\delta & \sin\gamma\sin\delta & \cos\gamma\end{bmatrix} \quad (2)$$

Transofrmation matrix from Body Coordinate System to Velocity Coordinate System is

$$A_{BV} = A_y(\beta)\, A_z(\alpha)$$

$$\left[A_{BV}\right]=\begin{bmatrix}\cos\beta\cos\alpha & \cos\beta\sin\alpha & -\sin\beta\\ -\sin\alpha & \cos\alpha & 0\\ \sin\beta\cos\alpha & \sin\beta\sin\alpha & \cos\beta\end{bmatrix} \quad (3)$$

Equations of Motion: The trajectory followed by the rocket is defined by its initial position and initial velocity. The motion of the rocket is studied with the help of Newton's second law which defines the equations in vector form as

$$m\,\frac{d\bar{V}}{dt} = \bar{G} + \bar{T} + \bar{D} + \bar{L}$$

$$\frac{d\bar{H}}{dt} = \bar{M} \quad (4)$$

where

$\bar{H}$ = Angular momentum vector
$\bar{M}$ = Sum of all moments

Equations of motion can be generated by resolving the above vector equations (4) in three directions. The forces taken into consideration are gravity force, thrust force, drag force and lift force. Using transformation matrix (1) gravity force is transformed to corresponding vector in body frame. Transpose of transformation matrix (3) is used to change drag force and lift force to express in body frame. General six degrees of freedom equations [1] of motion in body frame are given below.

$$m\,u' = m\,(rv - qw) - mg\cos\varphi\sin\theta - D\cos\beta\cos\alpha - L\sin\alpha + T + Cf_x$$

$$m\,v' = m\,(pw - ru) - mg\,(\cos\phi\cos\theta + \sin\phi\sin\theta\sin\varphi) - D\cos\beta\sin\alpha + L\cos\alpha + Cf_y$$

$$m_3\,w' = m\,(qu - pv) - mg\,(\cos\phi\sin\theta\sin\varphi - \sin\phi\cos\theta) + D\sin\beta + Cf_z$$

$$I_{xx}\,p' = qr\,(I_{yy} - I_{zz})$$

$$I_{yy}\,q' = pr\,(I_{zz} - I_{xx})$$

$$I_{zz}\,r' = pq\,(I_{xx} - I_{yy})$$

A simulation study has been carried out in MATLAB. The equations of motion are integrated using forth order Runge-Kutta method.

Simulation and Analysis

The rocket is fired from the launcher with initial elevation angle and it follows its desired path. In this paper, the effect of initiation of control on rocket range has been studied and analyzed. Analysis for increased lift in terms of percentage of 'g' value also forms part of this work to obtain the maximum range. Control phase is defined either for desired time of flight or for desired velocity gained. Initiation of control has been applied for seven different elapse time related to ABP position. Lift has been increased by deflecting elevator for five different controlled values in terms of 'g' starting from 30% of 'g' value to 70% of 'g' value with the step of 10%. Controlled values less than 30% of 'g' are not considered in this work, as for this controlled values angle of attack increases upto 300 which is very large compared to other controlled values and disturbs stability. Controlled values for greater than 70% of 'g' are not taken into consideration as they are not giving any gain in range. Simulation has been carried out and results are generated in comparison with no control phase trajectory i. e. standard trajectory.

Figure 4 shows the trajectories when control is started at ABP. Trajectories in Fig.4 are distinguished with the symbols like no symbol for standard trajectory, dots for increment in lift by 30% of 'g' value, circles for increment in lift by 40% of 'g' value, stars for increment in lift by 50% of 'g' value, diamonds for increment in lift by 60% of 'g' value and squares for increment in lift by 70% of 'g' value. Range for standard trajectory or 0% increment in the lift force is 22450m. It is observed that the maximum range achieved is 43343 m for 30% of 'g' value and the increment is 93.06% as compared to standard trajectory range.

Comparative Study

Table-1 shows that the maximum range achieved by the rocket for various starting points of the control during the flight and corresponding increments in the range with respect to standard trajectory range at particular % of 'g'

Table-1 : Maximum Range for Various Starting Point of Control (30 % of 'g' Value)

Sl. No.	Maximum Range (m)	% in Range increase w.r.t. Uncontrolled Rocket Range 22450 m	Maximum Range Achieved for Control Apply at % of ABP
1	24777	10.36	50
2	28252	25.84	60
3	31841	41.83	70
4	35743	59.21	80
5	39793	77.25	90
6	**43343**	**93.06**	**100**
7	42526	89.42	110

Table-2 : Detailed Analysis for ABP at 4.05 Second

Sl. No.	Maximum Range (m)	% in Range increase w.r.t. Standard Trajectory Range	Maximum Range Achieved for Control apply at % of ABP
1	43116	92.05	97.53
2	43213	92.49	97.78
3	43296	92.86	98.02
4	43366	93.16	98.27
5	**43413**	**93.38**	**98.52**
6	43397	93.30	98.77
7	43389	93.27	99.01
8	43386	93.26	99.26
9	43369	93.18	99.51
10	43358	93.13	99.75
11	43343	93.06	100.00

value due to increase in lift [2], [3]. It is observed that the maximum range is achieved at 30% of 'g' value for all the starting point of the control during the flight. It is seen that applying the control at 50% of ABP from launching the rocket gives maximum range as 24777 m with increment in lift by 30% of 'g' value where as the standard trajectory range is 22450m and the gain in range is 10.36% of standard trajectory range. Applying control at 100% of ABP gives maximum gain in range. It is observed that applying control after ABP will give gain in range but it is less as compared to applying control at ABP (Fig.5).

The detailed analysis has been carried out for ABP at 4.05sec. Control has been applied from 97.5% to 100% of ABP and maximum range is recorded.

From Table-2, it is observed that the maximum range achieved for control applied at 98.52% of ABP is 43413. The gain in range is 93.38% of standard trajectory range. This is due to thrust profile which is decreasing.

Conclusion

- Initiation of control near All Burnt Point gives maximum gain in the range.
- Due to thrust profile, the maximum range is achieved just before the all burnt point.
- Applying control very early during the flight will increase the range as compared to standard trajectory Range, but the gain in range is relatively less.

References

1. Garnell, P., "Guided Weapon Control Systems", 2nd Edition, Pergamon Press, Oxford, England, 1980.

2. McCoy, R. L., "Modern Exterior Ballistics", Schiffer Publication, 1999.

3. Wenjun, Yi, Bisheng Zhang and Zhongyan Wang., "Analysis of Ballistic Performance of a Glide Extended-Range Projectile", 21st International Symposium on Ballistics, Adelaide, Australia, 2004, pp. 458-465.

4. Grignon, C., Zilliani, A., Fleek, V., David, S. and Arriguci, A., "Steering of a Spin Stabilized Gliding Projectile: First Approach", 20th International Symposium on Ballistics, Orlando, 2003, pp.90-98.

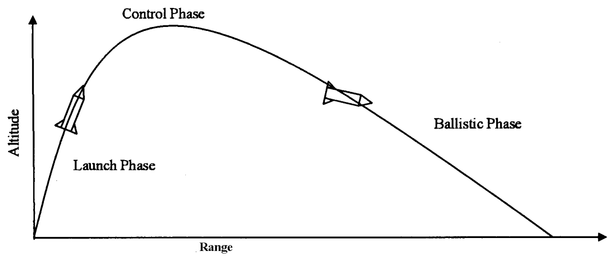

Fig.1 Different Phases of Trajectory

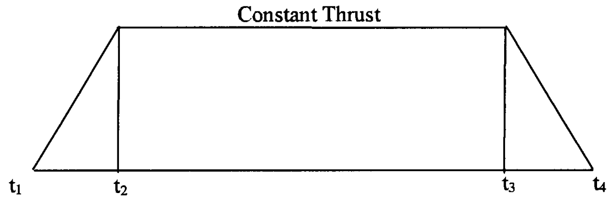

Fig.2 Thrust Profile

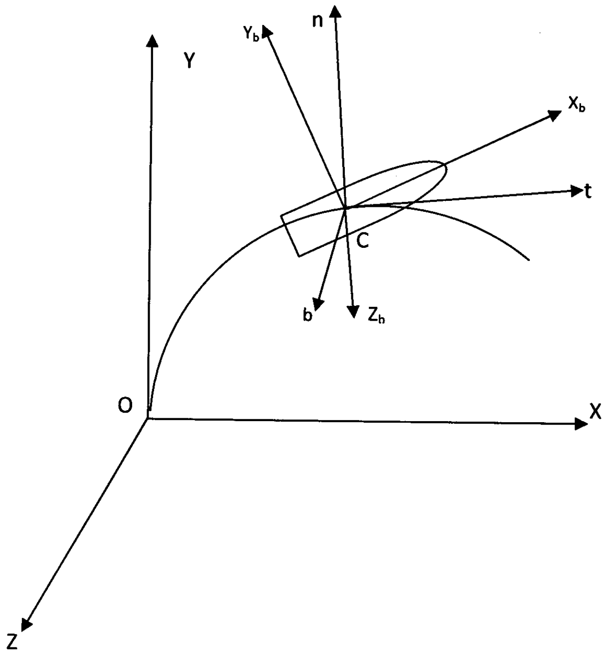

Fig.3 Coordinate Systems

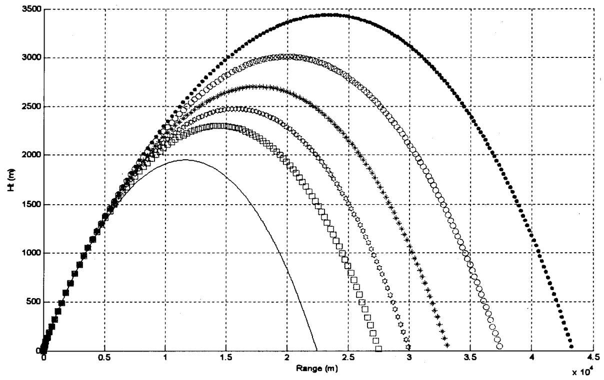

Fig.4 Control Starts at ABP

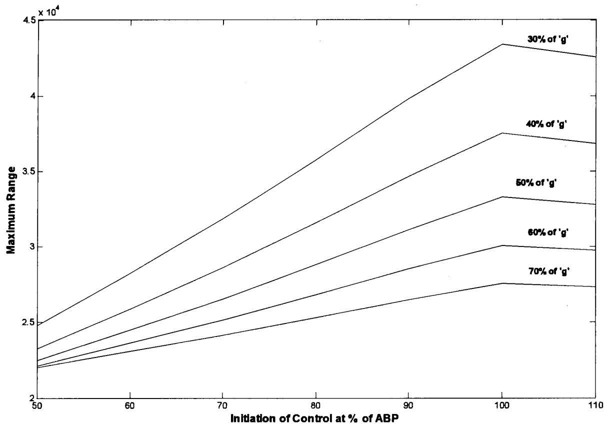

Fig.5 Comparative Study of Maximum Ranges Achieved