VIBRATION AND SMALL SCALE EFFECTS OF SKEW GRAPHENE SHEETS USING NONLOCAL ELASTICITY THEORY

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Abstract

Nonlocal elasticity theory is a popularly growing technique for the realistic analysis of nano structures. In the present work nonlocal elasticity plate theory has been employed and vibration analyses of skew graphene sheets are carried out. Relevant governing differential equations are reformulated using the nonlocal differential constitutive relations suggested by Eringen. The equations of motion including the nonlocal theory are derived. All edges of the skew graphene sheets are assumed to be simply supported. Naviers approach has been employed to solve the governing differential equations. Bauers skew plate analysis has been extended to include the nonlocal elasticity plate theory. Vibration response of the skew graphene sheets is studied. Effects of the (i) size of the graphene sheets (ii) modes of vibration (iii) nonlocal parameter and (iv) skew angle of graphene sheet on nonlocal vibration frequencies are investigated. It has been observed that the vibration response of the skew graphene sheets are influenced significantly by the nonlocal parameter.

Keywords: *vibration, nano tubes, skew graphene sheet, nonlocal elasticity*

Nomenclature

a, b = Length and breadth of the plate
D = Bending rigidity of the graphene sheet
E = Young's modulus of the graphene sheet material
E_b = Young's modulus of the beam material
h = Thickness of the beam
h_b = Length (or breadth) of a square graphene sheet
L_b = Length of the beam
M_1^{xx}, M_1^{yy},
M_1^{xy} = Moment resultants
N_0^{xx}, N_0^{yy},
N_0^{xy} = In-plane force resultants
q = Transverse distributed load
$S(x)$ = Fourth order elasticity tensor
ξ = Transformed X co-ordinate
η = Transformed Y co-ordinate
σ^l = Macroscopic local stress tensor
u, v = Displacement of the point (x, y, 0) of graphene sheet along X and Y axis, respectively
w^c = Deflections of the single layered graphene sheet at point (x, y) calculated using CLPT
$\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}$,
$\varepsilon_{xy}, \varepsilon_{yz}, \varepsilon_{xz}$ = Strain tensors
μ = Nonlocal parameter
ν = Poisson's ratio of the graphene sheet material
ρ = Density of the graphene sheet material
ρ_b = Density of the beam material

Introduction

Invention of carbon nano tubes (CNTs) by Iijima (1991), started a new field of research for the accurate analysis of nano-size structures. CNTs have attracted attention of scientific community due to their superior mechanical, chemical and electrical properties. Due to these superior properties CNTs are used in the emerging fields of nano electronics and nano devices. The CNTs also hold exciting promise in useful potential applications as electrodes in super capacitors and as cable material for space elevators. But conducting experiments with nanoscale size specimens is found to be difficult and expensive. There-

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fore, development of appropriate mathematical models for the analysis of nanostructures is an important issue. The approaches which are generally used for the analysis of nano structures are (a) atomistic (Ball 2001, Baughman et al. 2002), (b) hybrid atomistic-continuum mechanics (Bodily and sun 2003, Li and Chou 2003, Pradhan and Phadikar 2008) and (c) continuum mechanics. Both atomistic and hybrid atomistic-continuum mechanics are computationally expensive and are not suitable for analyzing large scale systems. Continuum mechanics approach is less computationally expensive than the former two approaches. Thereby size-dependent continuum-based methods (Zhou and Li 2001, Fleck and Hutchinson 1997, Yang et al. 2002) are becoming popular in modelling nano sized structures as it offers much faster solutions than molecular dynamics simulations for various engineering problems. It has been found that continuum mechanics results are in good agreement with atomistic and hybrid approaches (Duan et al. 2007).

There are exploratory studies on the continuum models for vibration of CNTs or similar micro or nanobeam like elements (Wang et al. 2006, Wang and Varadhan 2005, Fu et al. 2006, Wang and Varadhan 2006, Zhou et al. 2006, Lu et al. 2007) . In these work it has been suggested that nonlocal elasticity theory developed by Eringen (1983 and 2002) used in the continuum models for accurate prediction of vibration response. In nonlocal elasticity theory the small-scale effects are captured by assuming that the stress at a point is a function not only of the strain at that point but also a function of the strains at all other points of the domain. This is due to the scale effect of the nanostructures. As the lengths are reduced the influences of long range inter-atomic and inter molecular cohesive forces on the static and dynamic properties tend to be significant and cannot be neglected. The importance of nonlocal elasticity theory motivated the scientific community to explore the behaviour of micro/nano structures more accurately. The feasibility of nonlocal continuum theory in the field of nanotechnology was first reported by Peddieson et. al (2003). A relevant reference concerning nonlocal theories for bending, buckling and vibration analysis of beams is reported by Reddy (2007).

Nanoplates possess superior mechanical properties (Luo and Chang 2000, Zhang and Huang 2006) as that of nano tubes. But when compared to that of one dimensional structure, limited work has been found on vibration analysis of two dimensional nanoplates (Zhang and Huang 2006, He et al. 2006, Kitipornchai et al 2005, Behfar et al. 2005). Only classical plate theory (CLPT) has been considered in modelling the nanoplates (Zhang and Huang 2006, He et al. 2006, Kitipornchai et al. 2005, Behfar et al. 2005). Further these mathematical models did not include scale effects. Thus it is importance to incorporate nonlocal elasticity theories in the vibration analyses of nanoplates due to the scale effect of nano structures. Work related to bending, vibration and buckling analyses of CNTs and graphene sheets using nonlocal elasticity are found in (Murmu and Pradhan 2009a, Murmu and Pradhan 2009b, Murmu and Pradhan 2009c, Murmu and Pradhan 2009d, Murmu and Pradhan 2009e, Murmu and Pradhan 2009f Murmu and Pradhan 2009g, Murmu and Pradhan 2009h, Murmu and Pradhan 2010, Pradhan 2009, Pradhan and Phadikar 2008, Pradhan and Phadikar 2009a, Pradhan and Phadikar 2009b, Pradhan and Phadikar 2009c, Pradhan et al. 2009 and Pradhan and Sarkar 2009). In the present paper attempt is made to study the effect of nonlocal plate theory on the vibration response of skew graphene sheets. Navier's approach has been employed to solve the governing differential equations for all sides simply supported graphene sheets. Vibration of plates with skew angle using local theory is being reported by Bauer (1983). Using the present nonlocal elasticity plate theory vibrations of skew graphene sheets are studied and discussed in this article. Results for (i) nonlocal nanobeams and (ii) nonlocal rectangular plates obtained from the present formulation are compared with the corresponding results available in the literature (Reddy 2007, Pradhan and Phadikar 2009c). Further, effects of (i) size of the graphene sheets (ii) modes of vibration (iii) nonlocal parameter and (iv) skew angle of graphene sheets on nonlocal vibration frequencies are being investigated.

Formulation

The coordinate system used for the graphene sheet is shown in Fig.1. Origin is chosen at one corner of the midplane of the graphene sheet. The X, Y and Z coordinates of the axes are taken along the length, breadth and thickness of the graphene sheet, respectively. Following stress resultants are used in the present formulation

$$N_0^{xx} = \int_{-h/2}^{h/2} \sigma_{xx}^{nl}\, dz, \quad N_0^{yy} = \int_{-h/2}^{h/2} \sigma_{yy}^{nl}\, dz, \quad N_0^{xy} = \int_{-h/2}^{h/2} \sigma_{xy}^{nl}\, dz$$

$$V_0^{xx} = \int_{-h/2}^{h/2} \sigma_{xz}^{nl}\, dz, \quad V_0^{yy} = \int_{-h/2}^{h/2} \sigma_{yz}^{nl}\, dz, \quad M_1^{xx} = \int_{-h/2}^{h/2} z\, \sigma_{xx}^{nl}\, dz$$

$$M_1^{yy} = \int_{-h/2}^{h/2} z\, \sigma_{yy}^{nl}\, dz, \quad M_1^{xy} = \int_{-h/2}^{h/2} z\, \sigma_{xy}^{nl}\, dz, \quad m_0 = \int_{-h/2}^{h/2} \rho\, dz \tag{1}$$

Here h denotes the height of the graphene sheet. σ_{xx}^{nl}, σ_{yy}^{nl}, σ_{zz}^{nl}, σ_{xy}^{nl}, σ_{yz}^{nl} and σ_{xz}^{nl} represent the nonlocal stress tensors. In classical local elasticity theory, stress at a point depends only on the strain at that point. While in the present nonlocal elasticity theory it is assumed that the stress at a point depends on the strains at all the points of the continuum. In other words, according to this nonlocal theory strain at a point depends on both stress and spatial derivatives of the stress at that point. According to Eringen (1983) the nonlocal constitutive behaviour of a Hookean solid is represented by the following differential constitutive relation

$$(1-\mu\nabla^2)\,\sigma^{nl}=\sigma^{l} \tag{2}$$

Here μ is the nonlocal parameter; σ^l is the local stress tensor at a point which is related to strain by generalized Hooke's law

$$\sigma^{l}(x)=S(x):\varepsilon(x) \tag{3}$$

Where S is the fourth order elasticity tensor and ':' denotes the double dot product. The following governing equation has been derived in reference (Pradhan and Sahu, 2010).

$$-D\nabla^4 w^c+\mu\nabla^2\Big[-q-\frac{\partial}{\partial x}\left(N_0^{xx}\frac{\partial w^c}{\partial x}\right)-\frac{\partial}{\partial y}\left(N_0^{yy}\frac{\partial w^c}{\partial x}\right)-\frac{\partial}{\partial x}\left(N_0^{xy}\frac{\partial w^c}{\partial x}\right)$$

$$-\frac{\partial}{\partial y}\left(N_0^{xy}\frac{\partial w^c}{\partial x}\right)+m_0\frac{\partial^2 w^c}{\partial t^2}\Big]+q+\frac{\partial}{\partial x}\left(N_0^{xx}\frac{\partial w^c}{\partial x}\right)+\frac{\partial}{\partial y}\left(N_0^{yy}\frac{\partial w^c}{\partial x}\right)$$

$$+\frac{\partial}{\partial x}\left(N_0^{xy}\frac{\partial w^c}{\partial x}\right)+\frac{\partial}{\partial y}\left(N_0^{xy}\frac{\partial w^c}{\partial x}\right)+m_0\frac{\partial^2 w^c}{\partial t^2} \tag{4}$$

It can be noted that by setting $\mu = 0$ in above Eq. (4) traditional local classical plate theory can be obtained.

Navier's Approach

The developed governing differential equation of section (Formulation) have been solved by Navier's approach for all sides of the graphene sheet simply supported. The simply supported boundary conditions are written as

At x = 0 and x = a u = 0 v = 0 $N_0^{xx}=0$, $M_1^{xx}=0$

At y = 0 and y = b u = 0 v = 0 $N_0^{yy}=0$, $M_1^{yy}=0$

The generalized displacement has been expressed as

$$w^c(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{i\omega t} \tag{5}$$

It is assumed that the graphene sheet is free from any in plane or transverse loadings. So we have

$$N_0^{xx}=N_0^{yy}=N_0^{xy}=q=0$$

Therefore equation (4) is reduced to

$$-D\nabla^4 W-\mu\, m_0\,\omega^2\,\nabla^2 W=-m_0\,\omega^2 W \tag{6}$$

Solution to the equation (6) can be written as mentioned in (Leissa 1969)

From which we get

$$\nabla^2 W=-\left\{k\pm\sqrt{k^2+\frac{k}{\mu}}\right\}W \tag{7}$$

Here

$$k=\frac{\mu\, m_0\,\omega^2}{2D} \tag{8}$$

For nontrivial solution of equation (7) we obtain

$$p=k+\sqrt{k^2+\frac{k}{\mu}} \tag{9}$$

Which implies

$$(\nabla^2+p)\,W=0 \tag{10}$$

Solving the above equation we get

$$k=\frac{p^2}{2p+\dfrac{2}{\mu}} \tag{11.1}$$

On simplification

$$\omega=\left(\sqrt{\frac{D}{m_0\,(1+p\,\mu)}}\right)p \tag{11.2}$$

Tramsforming the axes to the sides of the parallelogram (Fig.1)

$$\xi = x - y \tan\alpha \tag{12.1}$$

$$\eta = \frac{y}{\cos\alpha} \tag{12.2}$$

After transforming the generalised equation (5) and simplifying one get

$$W(\xi, \eta) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin\left(\frac{m\pi\xi}{a}\right) \sin\left(\frac{n\pi\eta}{a}\right) \tag{13}$$

After transforming of equation (10) for skew graphene sheet one can write

$$\frac{\partial^2 W}{\partial \xi^2} + \frac{\partial^2 W}{\partial \eta^2} + 2\sin\alpha \frac{\partial^2 W}{\partial\xi\,\partial\eta} + p\cos^2\alpha\, W = 0 \tag{14}$$

Substituting the boundary condition one gets

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\Bigg[\left\{p\cos^2\alpha - \pi^2\left[\frac{m^2}{a^2}+\frac{n^2}{b^2}\right]\right\} W_{mn}\sin\left(\frac{n\pi\xi}{a}\right)\sin\left(\frac{n\pi\eta}{b}\right)$$

$$-\,2\sin\alpha\frac{nm\pi^2}{ab}W_{mn}\cos\left(\frac{m\pi\xi}{a}\right)\cos\left(\frac{n\pi\eta}{b}\right)\Bigg] = 0 \tag{15}$$

Expanding the Cosine functions of above equation into Fourier Sine series

$$\cos\left(\frac{k\pi\xi}{a}\right)\cos\left(\frac{l\pi\eta}{b}\right) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}^{kl}\sin\left(\frac{m\pi\xi}{a}\right)\sin\left(\frac{n\pi\eta}{b}\right) \tag{16}$$

Where for m = k and n = 1 and $a_{mn}^{kl} = 0$

And for m ≠ k and n ≠1

$$a_{mn}^{kl} = \frac{4}{\pi^2}\,\frac{mn\,[1-(-1)^{m+k}]\,[1-(-1)^{n+l}]}{(m^2-k^2)\,(n^2-l^2)} \tag{17}$$

It may be noted that Fourier coefficient is nonzero only when m+1 and n+1 are odd numbers. Equating the coefficients of

$\sin\left(\frac{m\pi\xi}{a}\right)\sin\left(\frac{n\pi\eta}{b}\right)$ in equation (15) yields

$$\left\{p\cos^2\alpha - \pi^2\left[\frac{m^2}{a^2}+\frac{n^2}{b^2}\right]\right\} W_{mn} - 2\sin\alpha\frac{\pi^2}{ab}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty} k\,l\,a_{mn}^{kl}\,W_{kl} = 0 \tag{18}$$

Equation (18) has a non trivial solution only if the determinant of the coefficients vanishes. The equation for the determinant of the skew graphene sheet is written as

$$\begin{vmatrix}
\begin{bmatrix}\alpha_{11}^{11} & 0 & 0\\ 0 & \alpha_{12}^{12} & 0\\ 0 & 0 & \alpha_{13}^{13}\end{bmatrix}\cdot & \cdot\begin{bmatrix}0 & \beta_{11}^{22} & 0\\ \beta_{12}^{21} & 0 & 0\\ 0 & \beta_{13}^{22} & 0\end{bmatrix} & \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\\
\begin{bmatrix}0 & \beta_{21}^{12} & 0\\ \beta_{22}^{11} & 0 & 0\\ 0 & \beta_{23}^{12} & 0\end{bmatrix}\cdot & \cdot\cdot\begin{bmatrix}\alpha_{21}^{21} & 0 & 0\\ 0 & \alpha_{22}^{22} & 0\\ 0 & 0 & \alpha_{23}^{23}\end{bmatrix}\cdot & \begin{bmatrix}0 & \beta_{21}^{32} & 0\\ \beta_{22}^{31} & 0 & 0\\ 0 & \beta_{23}^{32} & 0\end{bmatrix}\cdot\\
\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} & \begin{bmatrix}0 & \beta_{31}^{22} & 0\\ \beta_{32}^{21} & 0 & 0\\ 0 & \beta_{33}^{22} & 0\end{bmatrix} & \cdot\cdot\begin{bmatrix}\alpha_{31}^{31} & 0 & 0\\ 0 & \alpha_{32}^{32} & 0\\ 0 & 0 & \alpha_{33}^{33}\end{bmatrix}\cdot
\end{vmatrix} = 0 \tag{19}$$

Where

$$\alpha_{mn}^{mn} = p\cos^2\alpha - \pi^2\left[\frac{m^2}{a^2}+\frac{n^2}{b^2}\right]$$

$$\beta_{mn}^{kl} = -\,32\sin\alpha\,\frac{m\,n\,k\,l}{ab\left[m^2-k^2\right]\left[n^2-l^2\right]} \tag{20}$$

For α = 0 i.e., the rectangular membrane, all diagonal elements of the determinant vanish so that the determinant becomes

$$\prod_{m=1}^{\infty}\prod_{n=1}^{\infty}\left[p - \pi^2\left[\frac{m^2}{a^2}+\frac{n^2}{b^2}\right]\right] \tag{21}$$

Solution of equation (21)

$$\omega_{mn} = \sqrt{\frac{D}{m_0}}\left[\frac{m^2}{a^2}+\frac{n^2}{b^2}\right]\pi^2 \tag{22}$$

The solution for local rectangular nanoplate is identically same as mentioned in the reference (Pradhan and Phadikar 2009c).

Equation (19) is solved for nontrivial solution (Bauer 1983) and vibration frequencies and vibration modes are computed.

Results and Discussions

The governing differential equation for the vibration of nonlocal skew graphene sheets is written in Eq. (10). It can be seen that putting $\mu = 0$ in the equation traditional local elastic graphene sheet vibration equation is obtained. It is interesting to note that by putting $\mu = 0$, one can obtain the corresponding local elasticity equation for graphene sheets. This derived local elasticity equations of graphene sheets matches with those reported by Kitipornchais et al. (2005). Further, putting $b = \infty$ in Eq.(21) nonlocal solutions for free vibration of nanobeam are obtained. These derived equations do match with the nonlocal equations for free vibration of beam reported by Reddy (2007). Euler-Bernoulli theory (EBT) is considered in the analysis. A beam with elastic modulus E_b = 30GPa, length L_b =10m, height h_b = varied, density ρ_b=1 kg/m^3 are considered. Non-dimensional natural frequencies are expressed as $\overline{\omega}_b = \omega_b \times L_b^2 \sqrt{\frac{\rho_b h_b}{E_b I_b}}$. Frequency ratio is defined as the ratio of frequency obtained using nonlocal theory to the frequency obtained using the local theory. Young's modulus E, Poisson's ratio v and density ρ are assumed as 1.02TPa, 0.3 and 1 kg/m^3, respectively. Present frequency ratio results of rectangular graphene sheets have been verified with those reported in Pradhan and Phadikar (2009c). Comparisons of these nanobeams and nanoplates results with those available in the literature (Reddy 2007, Pradhan and Phadikar 2009c) are presented in Tables-1 and 2, respectively. Frequency values for $\mu = 0$ cases are available in Pradhan and Sahu 2010 and Pradhan and Phadikar 2009c where one can infer about the actual frequencies of different cases from frequency ratios.

All edges of the skew graphene sheet (Fig.1) are assumed to be simply supported. Navier's approach has been used to solve the governing differential equations. Bauer's skew plate analysis (1983) has been extended to include nonlocal plate theory. Vibrations of these nonlocal skew graphene sheets are studied. One can found from the Figs.2-5 that the frequency ratios for free vibration of the skew graphene sheets are significantly influenced by (i) size (length or breadth) of the graphene sheets (ii) modes of vibrations (iii) nonlocal parameter and (iv) skew angle of the graphene sheets. In Fig.2 the frequency ratios have been plotted for various nonlocal parameters and various sizes of the graphene sheets. The skew angle is considered to be 5 degree. From this figure one can observe that the nonlocal effect is more significant for the graphene sheets

Table-1 : Nondimensional Natural Frequencies of Beams Using EBT

L/h	μ	Nondimensional Natural Frequency from EBT (Reddy 2007)	Nondimensional Natural Frequency from EBT (Present)
100	0.0	9.8696	9.8696
	0.5	9.6347	9.6347
	1.0	9.4159	9.4159
	1.5	9.2113	9.2113
	2.0	9.0195	9.0195
20	0.0	9.8696	9.8696
	0.5	9.6347	9.6347
	1.0	9.4159	9.4158
	1.5	9.2113	9.2112
	2.0	9.0195	9.0194

Table-2 : Comparison of Frequency Ratio Results of Rectangular Nanoplate

	μ = 1 nm^2	μ = 1 nm^2	μ = 2 nm^2	μ = 2 nm^2
Length (nm)	Frequency Ratio (Pradhan and Phadikar 2009c)	Frequency Ratio (Present)	Frequency Ratio (Pradhan and Phadikar 2009c)	Frequency Ratio from Present
5	0.7473	0.7475	0.6173	0.6226
10	0.9140	0.9138	0.8438	0.8467
15	0.9614	0.9588	0.9192	0.9223
20	0.9807	0.9762	0.9508	0.9540
25	0.9859	0.9845	0.9719	0.9698
30	0.9928	0.9892	0.9772	0.9787

smaller than 15nm x 15nm. As the size of the graphene sheet increases the nonlocal effect is diminished exponentially because the effect of inter-atomic and inter molecular cohesive forces decrease with size of the sheet. The maximum jump in the frequency ratio is observed for nonlocal parameter going from 0nm^2 to 1nm^2. This jump in the frequency ratio is gradually reduced as one employs nonlocal parameter 2-4 nm^2.

Similarly in Figs.3-4 frequency ratios are plotted for various vibration modes. In Fig.3 modes with $m \neq n$ are plotted and in Fig.4 modes with $m = n$ are plotted. The nonlocal parameter is assumed to be 1nm^2. The results shown in Figs.3-4 are in line with the rectangular graphene sheet results reported by Pradhan and Phadikar (2009c). In Fig.5 frequency ratios have been plotted for various nonlocal parameters and skew angles of the graphene sheets. The skew graphene sheet is considered to be of 10nm length. With the increase in skew angle the frequency ratios are found to decrease. Further, the nonlocal effects become significant with the increase in the magnitude of the graphene sheet skew angle.

Conclusions

Nonlocal elasticity equations of Eringen are employed and vibration analysis of skew graphene sheets is carried out. Effects of (i) size of the graphene sheets (ii) modes of vibrations (iii) nonlocal parameter and (iv) skew angle of graphene sheets on the vibration frequency ratios are investigated. Nonlocal effects significantly increase with decrease in the graphene sheet size. This effect is more significant for graphene sheets which are smaller than 15nm x 15nm. It has been observed that natural frequency ratios decrease with increase in mode numbers. Further, frequency ratio decreases with increase in magnitude of the nonlocal parameter. Furthermore, the nonlocal effects become significant with the increase in the magnitude of the graphene sheet skew angle.

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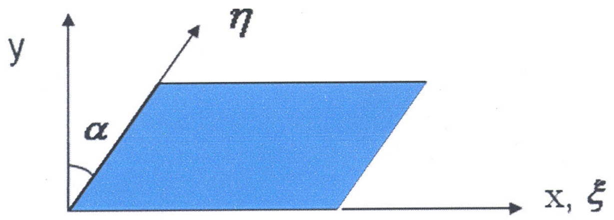

Fig.1 Skew Graphene Sheet with Co-ordinate Axis

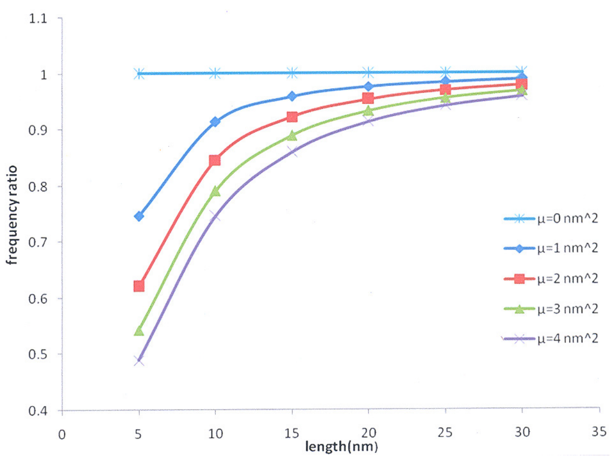

Fig.2 Frequency Ratio Vs Length for α = 5 deg, m=n=1 and Various Nonlocal Parameters

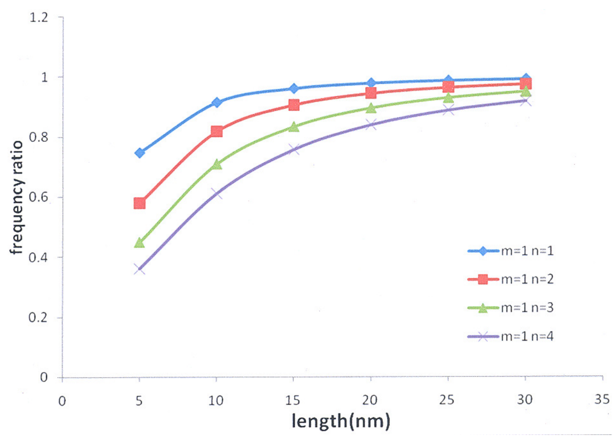

Fig.3 Frequency Ratio Vs Length for α = 5 deg, μ = 1 nm^2 and $m \neq n$

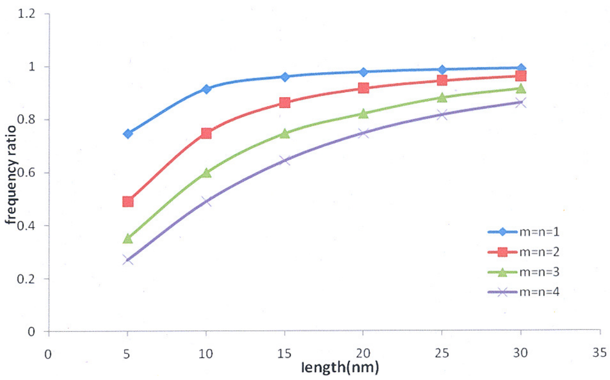

Fig.4 Frequency Ratio Vs Length for α *= 5 deg,* μ *= 1* nm^2 *and m = n*

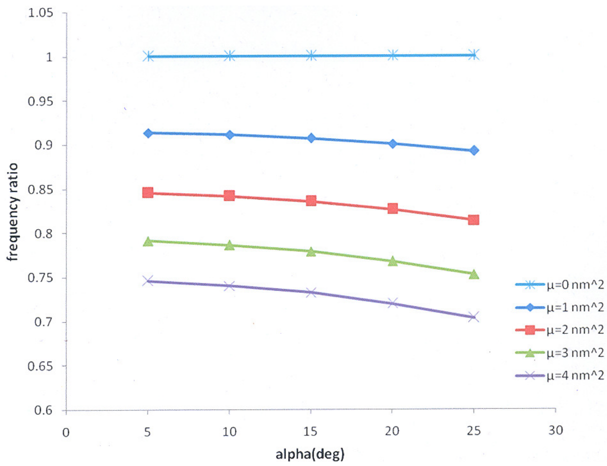

Fig.5 Frequency Ratio Vs α *(deg) for m=n=1, L = 10 nm and Various Nonlocal Parameters*