# LONGITUDINAL AERODYNAMIC PARAMETER ESTIMATION USING NEURAL NETWORK AND GAUSS-NEWTON METHOD

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## Abstract

A new parameter estimation method based upon Feed Forward Neural Network is proposed. The proposed method utilizes the universal mapping capability of Feed Forward Neural Network to develop flight dynamic model of aircraft. Gauss-Newton method is used to obtain values of aerodynamic parameters by minimizing a chosen error cost function. The method has then been validated using flight data pertaining to longitudinal dynamics of aircraft. Proof of match approach has been followed to verify the estimated model by the proposed method. The results obtained using the proposed method have also been compared with those obtained using wind tunnel tests, and Filter Error method. Unlike, most of the conventional methods, the proposed method does not require a priori description of the model. It also bypasses the requirement of solving the equations of motion. This feature may have special significance in handling flight data of an unstable aircraft.

Keywords: Parameter estimation, Longitudinal aerodynamic, Neural Network, Gauss-Newton method

Nomenclature			= neural network output vector at $k^{\text{th}}$ instant		
$a_x, a_z$	= accelerations component along $x, z$ body axes	Z(k)	= measured output vector at $k^{tn}$ instant		
$\overline{c}$ $C_m$	<pre>= mean aerodynamic cord length = rolling moment coefficient</pre>	Greek Notation			
$C_D, C_L$	= drag and lift coefficient	α	= angle-of-attack		
$C_X, C_Z$ $e^j$	= force coefficient along $x, z$ body axes = column vector with one in the $j^{\text{th}}$ row and	$\Theta$ $\Theta$ $\delta_e$	<ul><li>= pitch angle</li><li>= system parameters</li><li>= elevator deflection</li></ul>		
F <sub>eng</sub>	zeros elsewhere = thrust force by aircraft engine	$\sigma_{eng}$	= engine inclination angle with aircraft body axes		
$I_x, I_y, I_z$	= moments of inertia about $x, y$ and $z$ axes	Abbrevation			
$I_{xz}$	= moment of inertia about <i>xz</i> plane	CG	= Center of Gravity		
m p, q, r	= mass of the aircraft = roll, pitch and yaw rates	ENCG	= Distance between Center of Gravity and Engine		
à	= pitch acceleration	Introduction			
$\frac{q}{q}$	= dynamic pressure	Aircraft parameter estimation is probably the most outstanding and illustrated example of the system identi-			
S U(k) V W	<ul> <li>= reference wing area</li> <li>= neural network input vector at k<sup>th</sup> instant</li> <li>= true airspeed</li> <li>= weight matrix in neural network</li> </ul>	fication methodology. The highly successful application of system identification to flight vehicle has been possible partly due to better measurement techniques and data processing capabilities provided by the digital computers,			

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partly due to the ingenuity of engineers in advantageously using the developments in other fields such as estimation and control theory, and partly due to fairly well-understood basic physical principles leading to adequate aerodynamic modeling and design of appropriate flight test. In the past the most widely used parameter estimation methods have been maximum likelihood method, equation error method and output error method [1-4]. Application of these methods requires a priori postulation of the model [3, 4]. There are several approaches to model building. The so called "white box" approach starts from the first principle and a model is derived from basic physical laws that govern the behavior of the system [5]. This approach works for relatively simple systems, but its complexity increases manifold for recently introduced highly augmented, high performance aircraft. On the other hand, the "black box" approach generates a model based entirely on the input/output measurements of the system without trying to model the internal physical mechanism of the system [5]. Since we wish to estimate and validate such models from measured input-output data, the Artificial Neural Networks (ANNs) provide an alternative approach to model building.

A new thrust area is emerging in the area of aircraft aerodynamic modeling and parameter estimation: development of techniques using (ANNs) for flight vehicle identification. More recently, many scientists and engineers have explored the potential of ANNs in diverse fields such as signal processing, pattern recognition, aircraft aerodynamic modeling, parameter estimation and control [6]. Artificial neural networks have been used in variety of applications because they are adaptive, they learn through examples and they can provide excellent functional approximation [6, 7]. Recently, ANNs modeling has been attempted for aircraft dynamics where aircraft motion variables and control inputs are mapped to predict the total aerodynamic coefficients [8-11]. In all these papers, the emphasis has been on aerodynamic modeling and estimation of aerodynamic coefficients using Feed Forward Neural Networks (FFNNs). Raol and Jategaonkar [12] have used the Recurrent Neural Networks (RNNs) to model aircraft aerodynamics in a way that allows aircraft parameters to be estimated from flight data. However, as the authors [10] pointed out, the RNNs have only a limited scope of aircraft identification application and it is the FFNNs which may prove to be more flexible and thereby have a higher potential for future application for aircraft identification and parameter estimation.

Raisinghani, Ghosh et al. [13] using FFNN proposed two new methods namely the Delta and the Zero method for explicitly estimating aircraft parameters from flight data. Both these methods do not require an a priori postulation of the model and bypass the requirement of solving equations of motion. It is suggested in Ref. [13], that both these methods may be viewed as complimentary to the existing methods for parameter estimation and they present themselves as straight forward methods in which the FFNN is trained for the given flight data, and the parameters are estimated at one go [13]. Both the methods use motion and control variables as the input file, while aerodynamic coefficients are presented as the output file for training a Neural Network (NN). For the purpose of parameter estimation, the trained NN is presented with suitably modified input file, and the corresponding predicted output file of aerodynamic coefficients is obtained. Suitable interpretation and manipulation of such input-output files yield the estimated values of the parameters. The application of the Delta and the Zero method on the real flight data has been demonstrated in Refs. [13-15]. Further, the advantage of FFNN based methods in estimating parameters from flight data of unstable aircraft was also highlighted in Ref. [16].

Application of the Delta and the Zero method to complete data yields N values, or in other words time histories, for the derivative (parameter) to be extracted. These extracted values are plotted as histograms, which usually show a near-normal distribution, from which the mean representing the aerodynamic derivatives can be determined [4, 13]. Standard deviations are used as a measure of confidence in each parameter estimate.

A natural effort to improve the confidence level in the parameter estimate gets directed towards improving FFNN training for modeling. There are several possible techniques including the more advanced Levenberg-Marquardt algorithm [17, 18]. There exist many types of feed forward neural networks in the literature, for example, Multilayer Perceptron (MLP), Radial Basis Function (RBF) network [19, 20] etc. Selectively these could be used to improve modeling using FFNN. However, due to lack of systematic principle/guideline to select the tuning parameters of FFNN, one may have to use his own judgment in freezing the tuning parameters. The application of various FFNN training algorithm generally shows very fast decrease in cost function in the first few iterations, but the stringent tolerances for terminations are reached even after a few hundred iterations. This is attributed to the fact that data is corrupted with noise, and after capturing the main system characteristics, the network tries to account for random noise, leading to small oscillation in the cost function [4].

To avoid such phenomena specifically for parameter estimation Modified Delta method was proposed in Ref. 21. Accordingly the Delta method was improved (Modified Delta) following a different strategy in selecting input-output vector rather than too much working on its training aspects. The Modified Delta method is based on interpreting the stability and control derivatives as follows: if we could obtain variation in the value of an aerodynamic coefficient due to variation in only one of the motion/control variables while the variation in other motion/control variables are zero, then the ratio of the variation of the aerodynamic coefficient to variation of the non-zero motion/control variable will yield the corresponding stability/control derivative [21]. The Modified Delta method was applied on real flight data pertaining to longitudinal and lateral-directional dynamics [21]. It was further shown that the Modified Delta method yielded parameter estimates with lesser standard deviation as compared to parameter estimates obtained through the Delta method. For all these methods standard deviations were used as a measure of confidence in the parameter estimates. Most of the conventional methods extract parameter estimates by minimizing error cost function [1] for N data points. The cost function is minimized by a numerical optimization procedure. The Cramer-Rao bounds [1, 4] are also obtained which are measures of confidence in each parameter estimate. Strictly speaking, there is a lower bound on the variance of each model parameter, but they are commonly used as a measure of the relative quality of identification rather than an absolute measure [5]. The Delta and Modified Delta methods did not have the provision to estimate Cramer-Rao bounds along with the estimates. Further, there was no explicit postulation of cost function using motion variables to be minimized in an optimal way to estimate parameters. This motivated us to develop the proposed method.

In the present work, a new method using FFNN is proposed for estimating aircraft parameters from flight data. This method is christened the Neural Gauss Newton (NGN) method. The reasons for the nomenclature will become apparent once the method is described. A blackbox approach using neural networks is used for building aircraft dynamic model. Parameter estimation technique based on the Gauss-Newton optimization is then used to determine the unknown aerodynamic parameters. The proposed method (NGN method) is validated using real flight data. Longitudinal flight data generated using test aircraft [22], HANSA-3 have been used for validation of the proposed method. The results show that the NGN method has good potential as alternative tool for parameter estimation from flight data. One may straightway obtain the parameter estimates along with Cramer-Rao bound via this method.

The verification of the identified model is a key step in the identification process to assess the predictive capabilities of the extracted model. One approach is to compare the flight determined parameter estimates against the values obtained from wind tunnel tests or analytical predictions. The other approach is known as the proof-of-match [4]. It is a widely used approach which is based on the comparison of the model predictions with the flight measurements. To this end, the flight data omitted from the identification studies is selected to ensure that the model is not tuned to specific data record or input form. In this study initially the verification of estimated model for longitudinal case has been carried out by comparing the flight extracted parameter estimates against the values obtained from wind tunnel tests [22]. Finally, proof-ofmatch exercise has also been carried out to verify the extracted model. The capability of NGN method to carry out proof-of-match exercise without solving equations of motion is also an additional feature of this method.

Filter Error Method [4] (FEM) was also applied on these flight data to estimate aerodynamic parameters. The effect of the magnitude and the sign of the initial guess values of the parameters on the estimates obtained using NGN and FEM methods have also been studied. Various runs were carried out with different sets of initial guess values of the parameters. Runs were also carried out by changing the signs and magnitudes of the initial guess values of the parameters. It has been observed that the NGN method has the capability of extracting the aerodynamic parameters accurately even if the initial guess values are substantially off (both in magnitude and sign) from the nominal values of aerodynamic parameters of the chosen aircraft.

## **Parameter Estimation Algorithm**

The block diagram of the proposed neural network based parameter estimation is presented in Fig.1. The proposed estimation method utilizes the universal function mapping capability of FFNN. The neural model representing the dynamics of the aircraft in maneuver is used for the purpose of parameter estimation. For estimating the



Fig.1 Schematic of NGN Method

numerical values of the parameters, Gauss-Newton method [4] is used for updating the parameter by minimizing the error cost function. The error cost function represents the summation of error between measured and predicted motion variables over the length of flight data used in the estimation process. This fundamental understanding is exploited in the proposed NGN method for estimating aircraft stability and control parameters from flight data.

The estimation process starts with conducting of experiment. Pre-decided maneuvers are attempted to excite selected dynamics of the aircraft. On board data acquisition system is activated to acquire the flight data. This set of flight data is referred to as measured flight data. Referring, Fig.1, it can be seen that the measured flight data undergoes data compatibility check (for flight path reconstruction) and all the variables are transferred to the center of gravity (CG) of the aircraft for further analysis. It is assumed here that during the maneuver, the variation in CG is negligible. The approach followed to formulate the NGN method using the measured flight data is described next.

Let the measured flight data contain the time histories at  $k^{th}$  instant of  $\alpha(k)$ ,  $\theta(k)$ , q(k), V(k),  $a_x(k)$ ,  $a_z(k)$ . Next step is to form input U(k) and output Z(k+1) vectors to be used for building the aircraft dynamic model using neural network architecture as given in Fig.2. The input



Fig.2 Neural-Architecture for Flight Dynamic Modeling

vector U(k) and the output vector Z(k+1) required to build flight dynamic model are defined next.

$$U(k) = \left[ \alpha(k) \ \theta(k) \ q(k) \ V(k) \ C_D(k) \ C_L(k) \ C_m(k) \right]^T$$
(1)

where the values of  $C_D(k)$ ,  $C_L(k)$  and  $C_m(k)$  at the  $k^{th}$  instant are obtained by plugging required values of flight variables into Eqs.(2) - (6).

$$C_D(k) = -C_X(k) \cos \alpha(k) - C_Z(k) \sin \alpha(k)$$
(2)

where

$$C_X(k) = \left( \frac{ma_x^{CG}(k) - F_{eng} \cos \sigma_{eng}}{q(k)} \right) / \overline{q}(k) S$$
(3)

$$C_L(k) = C_X(k) \sin \alpha (k) - C_Z(k) \cos \alpha (k)$$
(4)

$$C_{Z}(k) = \left(ma_{z}^{CG}(k) - F_{eng}\sin\sigma_{eng}\right) / \overline{q}(k) S$$
(5)

$$C_{m}(k) = \left[I_{y}\dot{q}(k) - I_{xz}\left(p^{2}(k) - r^{2}(k)\right) - \left(I_{z}(k) - I_{x}(k)\right)p(k)r(k) - F_{eng}\cos\sigma_{eng}z_{ENCG} - F_{eng}\sin\sigma_{eng}y_{ENCG}\right]/(\bar{q}(k)S\bar{c})$$
(6)

The output vector Z(k+1) at  $(k+1)^{th}$  instant required for flight dynamic model is constructed in Eq. (7).

$$Z(k+1) = \left[ \alpha (k+1) \ \theta (k+1) \ q (k+1) \ V(k+1) \ a_x(k+1) \ a_z(k+1) \right]^T$$
(7)

Once input vector U(k) and output vector Z(k+1) are constructed using flight data, the next task is to develop the flight dynamic model using FFNN (Fig. 2). The neural network performance, that is its ability to accurately duplicate data used in training with adequate prediction capability, depends on the network tuning parameters [4, 6]. The choice of tuning parameters depends on several factors, such as number of inputs and outputs, the amount of noise in data to be matched, and the complexity of the input-output subspace. Several guidelines to tune the FFNN are available in open literature [4]. Once neural model is created, it can be used for prediction of output variable Y(k+1) for any given input variable U(k). For parameter estimation the input vector U(k) is reconstructed by keeping the same initial conditions  $(\alpha(k), \theta(k), q(k), V(k))$  used for training however,  $C_{D}\left(k\right)$ ,  $C_{L}\left(k\right)$  and  $C_{m}\left(k\right)$  are modified as per the chosen aerodynamic model in the estimating algorithm. Let, the chosen aerodynamic model for longitudinal parameter estimation be given as in Eqs.(8) - 10).

$$C_D = C_{D_0} + C_{D_\alpha} \alpha + C_{D_{\delta_e}} \delta_e$$
(8)

$$C_{L} = C_{L_{0}} + C_{L_{\alpha}} \alpha + C_{L_{q}} (\overline{q} \,\overline{c}/2 \,V) + C_{L_{\delta_{e}}} \delta_{e}$$
(9)

$$C_m^{CG} = C_{m_0} + C_{m_\alpha} \alpha + C_{m_q} (\overline{q} \, \overline{c}/2 \, V) + C_{m_{\delta_e}} \delta_e \qquad (10)$$

To start the estimation algorithm it is necessary to specify some suitable initial guess values of the unknown parameters vector  $\Theta$ , consisting of non-dimensional parameters used for the description of aerodynamic model of  $C_D$ ,  $C_L$  and  $C_m^{CG}$ .

The aim of the exercise is to estimate unknown parameter vector  $\Theta$  consisting of aerodynamic parameters namely,  $C_{D_0}$ ,  $C_{D_{\alpha}}$ , ...,  $C_{m_q}$  and  $C_{m_{\delta_{\alpha}}}$ .

For the longitudinal case let U(k) be selected as input variable vector and Y(k+1) be selected as estimated output vector. The estimated output vector can be represented as given in Eq.(12).

$$Y(k+1) = \left[ \alpha \ (k+1) \ \theta \ (k+1) \ q \ (k+1) \ V(k+1) \ a_x(k+1) \ a_z(k+1) \right]^T$$
(12)

At this point, it may be realized that during the estimation process some a priori form of the aerodynamic model has been assumed. The task is then to estimate the values of aerodynamic parameters required to characterize the model structure. In the next step the Gauss-Newton method has been applied to update the parameter vector  $\Theta$ , as per Eq.(13).

$$\Theta_{i+1} = \Theta_i + \Delta \Theta$$
, and  $\Delta \Theta = -F^{-1}G$  (13)

where, i is the iteration index, F and G and are the information (Hessian) matrix and the gradient vector matrix [4] respectively and are formulated in Eqs. (14) and (15).

$$F = \sum_{k=1}^{N} \left[ \frac{\partial Y(k)}{\partial \Theta} \right]^{T} R^{-1} \left[ \frac{\partial Y(k)}{\partial \Theta} \right]$$
(14)

$$G = -\sum_{k=1}^{N} \left[ \frac{\partial Y(k)}{\partial \Theta} \right]^{T} R^{-1} \left[ Z(k) - Y(k) \right]$$
(15)

where the definition of residual error (E(k)) and covariance matrix of the residual [4] (R) between measured and predicted output from neural model are presented in Eqs. (16) and (17).

$$E(k) = (Z(k) - Y(k))$$
 (16)

$$R = \frac{1}{N} \sum_{k=1}^{N} \left[ Z(k) - Y(k) \right] \left[ Z(k) - Y(k) \right]^{T}$$
(17)

For a small perturbation  $\delta \Theta_j$  in each of the unknown variables of the parameter vector  $\Theta$ , the perturbed response variable  $Y_{pi}(k)$  corresponding to the unperturbed variables  $Y_i(k)$  is computed. The approximate value of corresponding sensitivity coefficient [4] is obtained through Eq.(18).

$$\left\lfloor \frac{\partial Y_{pi}(k)}{\partial \Theta} \right\rfloor_{ij} = \frac{Y_{pi}(k) - Y_{i}(k)}{\partial \Theta_{j}}$$
(18)

where the perturbed response output variables  $Y_{pi}(k)$  is obtained by replacing parameter vector  $\Theta$  with the  $\Theta + \delta \Theta_j e^j$  (where  $e^j$  is a column vector with one in the  $j^{th}$  row and zeros elsewhere) in the input variable vector of the already trained neural model. The parameters are optimized by minimizing the cost function [4] ( $J(\Theta, R)$ ) as formulated in Eq.(19).

$$J(\Theta, R) = \frac{1}{2} \sum_{k=1}^{N} [Z(k) - Y(k)]^{T} R^{-1} [Z(k) - Y(k)]$$
(19)

The estimation error covariance matrix [4] P is function of model parameters  $\Theta$ , the data points being analyzed and the covariance matrix of the residuals R. The expression to compute P is given in Eq.(20).

$$F \approx \left\{ \sum_{k=1}^{N} \left[ \frac{\partial Y(k)}{\partial \Theta} \right]^{T} R^{-1} \left[ \frac{\partial Y(k)}{\partial \Theta} \right] \right\}^{-1}$$
(20)

The standard deviations of the parameter estimates or the Cramer-Rao bounds [4] ( $\sigma_{\Theta i}$ ) are the diagonal elements of the estimation error covariance matrix *P* which can be computed using Eq.(21).

$$\sigma_{\Theta i} = \sqrt{P_{ii}} \tag{21}$$

#### **Generation of Flight Data**

The final test of any scheme for parameter estimation must come from its successful demonstration on real flight data. A flight data base for identification studies was gathered from flight maneuvers with the test aircraft [22]. Typically, starting from trim flight conditions, the pilot applied control input in an attempt to excite the chosen dynamic modes. An onboard measurement system installed on the test aircraft provided measurements using dedicated sensors of a large number of signals such as aircraft motion variables, atmospheric conditions, control surface position etc. The measurements made in flight were recorded onboard using suitable interface with standard laptop. The flight data had raw data for measured  $V, \alpha, \beta, p, q, r, a_x, a_y, a_z, \phi, \theta, \psi, H, \delta_e, \delta_a$  and  $\delta_r$ , and the location of the measuring sensors. The measurement of airspeed (V), angle of attack ( $\alpha$ ), angle of side slip ( $\beta$ ) were obtained with flight log mounted on a boom fixed to the tip of the wing. The angular rates p, q and r were obtained from the measurements available from the inertial platform. The accelerations along the three body axes were measured using an accelerometer triad located near the CG of the aircraft. The angular rates  $\dot{p}$ ,  $\dot{q}$  and  $\dot{r}$  were obtained by numerical differentiation of the corresponding angular rates. The control surface deflections  $(\delta_e, \delta_a, \delta_r)$  were measured using potentiometer. The temperature *T* was recorded using the standard cockpit outside air temperature (OAT) gauge. Two sets of flight data simulating short period longitudinal dynamics were generated at an altitude 6000 feet. The cruise speed at which the perturbations were initiated was fixed at nearly 56 m/s.

The longitudinal flight data FLT1 (Fig.3) was generated using multi step elevator input ( $\delta_{e_{\text{max}}} = 7 \text{deg}$ ) having total duration of 4s only. Another flight data set FLT2 (Fig.4) was generated with two similar looking double pulse input having almost same magnitude. These two pulses although look similar but has opposite elevator







Fig.4 Longitudinal Flight Data : FLT2

deflections to excite the longitudinal dynamics. The proposed NGN method was applied on flight data FLT1 and FLT2 to estimate longitudinal aerodynamic parameters. Further these flight data were also used for model verification using proof-of-match technique [4].

## Application of the Proposed Method to Real Flight Data

The flight data FLT1 and FLT2 were generated by exciting the longitudinal dynamics of HANSA-3 test aircraft. The longitudinal short period dynamics was excited about a steady state trim at V = 56 m/s at a cruise altitude of 6000ft. Day and time of the experiment was carefully chosen to ensure fairly calm weather. The flight data acquired were preprocessed and reconstructed after carrying out exhaustive data compatibly check. Next, using these flight data, the input and output vectors for neural mapping were constructed. The input vector U(k)consisted of time histories at  $k^{th}$  instant of  $\alpha(k), \theta(k), q(k), V(k), C_D(k), C_L(k)$  and  $C_m(k)$ whereas the output vector Z(k+1)had  $\alpha$  (*k*+1),  $\theta$  (*k*+1), *q* (*k*+1), *V* (*k*+1), *a<sub>x</sub>* (*k*+1) and  $a_z$  (k+1) as its elements. A typical FFNN structure used for building flight dynamic model has already been presented in Fig.2. Using U(k) as network input vector and Z(k) as the output vector, the FFNN was trained as discussed earlier. The FFNN used a log-sigmoid and linear transfer function as the activation function and Levenberg-Marquardt [17, 18] algorithm was used for updating the neural network weights. The mean square error (MSE) criterion or the number of iteration decided the termination of the iterative process. A range of values of the network parameters was tried to arrive at the final architecture of the FFNN used for parameter estimation. The network parameters varied were the number of hidden layers (1-3), the number of hidden neurons in the hidden layers (2-15), the learning rate (0.1-0.8), the momentum rate (0.1-0.8), and the num-

The network parameters finally chosen gave a good match between the true and the predicted values of the time histories of the variables. The final FFNN used had one hidden layer with ten neurons, the learning rate = 0.3.

ber of iterations (100-2000).

The NGN method has then been used to predict the stability and control derivatives (parameters) of the aircraft. Filter error method [4] (FEM) was also applied on flight data FLT1 and FLT2 to estimate longitudinal aerodynamic parameters. The values of the estimated

parameters namely  $C_{D_0}$ ,  $C_{D_{\alpha}}$ ,  $C_{D_{\delta_e}}$ ,  $C_{L_0}$ ,  $C_{L_{\alpha}}$ ,  $C_{L_{\delta_e}}$ ,  $C_{m_0}, C_{m_{\alpha}}, C_{m_q}$  and  $C_{m_{\delta_a}}$  along with their Cramer-Rao bounds are presented in Table-1. Column 3 and 4 of Table-1 present the values of the estimated parameters obtained by applying NGN and FEM respectively on flight data FLT1. Similarly, column 5 and 6 present parameter estimates obtained by applying NGN and FEM on flight data FLT2. Column 2 lists the numerical values of the parameters obtained using wind tunnel tests [22]. It could be seen in Table-1 that the values of the parameters obtained using the NGN method are in fairly close agreement with the values obtained using wind tunnel tests [22] and FEM. It may also be seen that few aerodynamic parameters namely  $C_{D_{\delta_{e}}}$ ,  $C_{L_{\delta_{e}}}$ ,  $C_{L_{0}}$ ,  $C_{m_{0}}$  are not well estimated. Their values differ significantly, from the estimates obtained through wind tunnel tests. This may be due to lack of information content in the flight data (FLT1 and FLT2). It may be mentioned here that despite the best effort of the pilot, it was not possible to generate flight data pertaining

It may be mentioned here that while applying FEM, the estimated values showed significant dependence on the choice of initial guess values of the parameters. In contrast the values of the parameters estimated through the NGN method did not depend either on the magnitude or sign of the initial guess values of the parameters and choice of initial process noise matrix. This is expected as the propose NGN method bypasses the requirement of solving equations of motion. To check for the robustness of the NGN method with respect to choice of initial guess values of the parameters and presence of measurement noise in the flight data, the proposed algorithm was applied on simulated flight data of unstable aircraft to estimate the aerodynamic parameters. Although, not reported in this paper, the proposed NGN method could successfully estimate the values of all of the parameters with correct signs and magnitudes.

[4] to 3-2-1-1 type elevator input excitation.

Next, we carried out the model verification exercise using flight data. Both the flight data FLT1 and FLT2 were alternatively used for parameters estimation and model verification using proof-of-match [4] procedure. In the first step, parameters were estimated using flight data FLT1. Next, using these estimates the estimated model was constructed using Eq. (8), (9) and (10). This estimated model along with the initial conditions and control input used in generating real flight data FLT2 were fed to the already trained neural model (used for estimation) to pre-

	Tab	ole-1 : HANSA-3 Lo	ongitudinal Paramo	eters	
	Wind Transl				
Parameter	Value	FLT1		FLT2	
		NGN FEM		NGN	FEM
$C_{D_0}$	0.035	0.03645 (0.001284)*	0.036197 (0.00182)	0.0286 (0.0015)	0.03068 (0.00189)
$C_{D_{lpha}}$	0.0859	0.06091 (0.012539)	0.070236 (0.01828)	0.0374 (0.0149)	0.00717 (0.01834)
$C_{D_{\delta_e}}$	0.0258	0.1515 (0.011986)	0.15304 0.01754)	0.1867 (0.01477)	0.16409 (0.01861)
$C_{L_0}$	0.354	0.229404 (0.004058)	0.22091 (0.00627)	0.2722 (0.00414)	0.25843 (0.00505)
$C_{L_{lpha}}$	4.711	4.8856 (0.030049)	4.7658 (0.05823)	4.7145 (0.03199)	4.6248 (0.04203)
$C_{L_q}$	-	37.25858 (1.1391)	40.392 (1.9943)	37.1543 (0.3079)	42.1 (1.5673)
$C_{L_{\delta_e}}$	0.2653	0.37609 (0.050703)	0.54365 (0.08072)	0.31503 (0.0497)	0.5879 (0.06029)
$C_{m_0}$	0.5214	0.0905124 (0.003072)	0.083389 (0.00158)	0.089796 (0.00304)	0.08183 (0.00143)
$C_{m_{\alpha}}$	-0.3372	-0.412282 (0.026244)	-0.40089 (0.01461)	-0.3777 (0.02237)	-0.3672 (0.01153)
$C_{m_q}$	-	-8.79214 (0.98729)	-7.1649 (0.51061)	-9.4836 (0.92061)	-8.0692 (0.4426)
$C_{m_{\delta_e}}$	-0.6941	-0.73488	-0.65859	-0.78167	-0.69625

\* Values in parentheses indicate sample standard deviation (Cramer-Rao bound)

dict the estimated responses (Est-NGN-NN). These responses were then compared with the measured flight data FLT2. A fairly close comparison among the measured and estimated responses (Est-NGN-NN) is presented in Fig. 5. Further, equations of motions pertaining to longitudinal motion [4] were also solved using estimated values of parameters obtained through the NGN method and FEM to compute the estimated responses Est-NGN-EQM, Est-FEM-EQM respectively corresponding to elevator used input in FLT2. These were then compared with the flight measured variables FLT2. A fairly close agreement was observed as given in Fig.5. The estimated responses computed with FEM and NGN estimates are indistinguishable in the scale of the Fig.5. Further, to see how good the estimated values are, we compared the estimated  $C_D$ ,  $C_L$ and  $C_m$  obtained by substituting the estimated values of the parameters obtained using NGN method in the right hand side of Eq.(8) - (10), with the flight derived  $C_L$ ,  $C_D$ and  $C_m$  being analyzed. Fig.6 presents comparison between,  $C_L$ ,  $C_D$  and  $C_m$  being analyzed and estimated  $C_L$ ,  $C_D$  and  $C_m$ . The estimated model of  $C_L$ ,  $C_D$  and  $C_m$ matches closely with the flight derived values of  $C_L$ ,  $C_D$ and  $C_m$  used in developing flight dynamic model using neural network. Similar exercise was carried out wherein parameters were estimated using flight data FLT2 and for validation the estimated responses computed with the initial conditions and the control input used in generating FLT1 were compared with the real flight data FLT1. In this case also excellent matching was observed as can be seen from Fig.7. Further Fig.8 presents a comparison between estimated model of  $C_L$ ,  $C_D$ ,  $C_m$  and flight derived values of  $C_L$ ,  $C_D$ ,  $C_m$  (FLT2). In this case, also the match-

(







(\_\_\_\_ measured; -.-.-. estimated)

ing is excellent confirming the validation of the proposed NGN method.

#### Conclusion

A new method christened the Neural-Gauss-Newton (NGN) method have been proposed for estimating aircraft parameters from flight data using feed forward neural networks. The proposed method advantageously uses the universal function mapping characteristics of feed forward neural network and optimization capability of Gauss-Newton algorithm. The results obtained for real flight data pertaining to longitudinal dynamics of Hansa-3 aircraft have shown the success and the potential of the proposed method. Since the proposed method does not require solving of equations of motion, it presents itself as straight forward method in which the FFNN is trained to capture



Fig.7 Model Verification : Proof-of-Match, FLT1 \_\_\_\_\_measured; -.-.-. Est-NGN-NN; ..... Est-NGN-EQM; ----- Est-FEM-EQM)



the flight dynamic model of an aircraft, and the parameters along with Cramer-Rao bounds are estimated in few iterations. The NGN method, which bypasses the requirement of solving the equation of motion, may have special significance in handling flight data of unstable airplane.

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