NONLINEAR BEHAVIOR OF FGM SKEW PLATES UNDER IN-PLANE LOAD

T. Prakash*, M.K. Singha* and M. Ganapathi+

Abstract

Nonlinear behavior of Functionally Graded Material (FGM) skew plates under in-plane load is investigated here using a shear deformable eight noded iso-parametric plate bending finite element. The material is graded in the thickness direction according to a power-law distribution in terms of volume fractions of the constituents. The effective material properties are estimated using Mori-Tanaka homogenization method. The nonlinear governing equations for the FGM plate under in-plane load are solved by Newton-Raphson technique to obtain the out-of-plane central deflection and in-plane displacement of the loaded edge. The existence of bifurcation-type of buckling for FGM plates is examined for different boundary conditions, constituent gradient index, and skew angle.

Keywords: FGM plate, Volume fraction index, in-plane load, Extension-bending coupling, Bifurcation buckling, Secondary instability, Finite element.

Introduction

Functionally Graded Materials (FGM) with continuously changing thermal and mechanical properties at the microscopic level have recently received considerable applications in thin walled structural components of space vehicles, nuclear reactors, and other high thermal application areas [1-2]. The structural components, i.e., beams, plates and shells under such applications may be subjected to in-plane compressive loads. Hence the knowledge of stability characteristics of FGM plates and shells are of much practical importance for the design of such structures. It is observed from the existing literature, that the buckling and postbuckling characteristics of isotropic and composite plates have received considerable attention of the researchers. However, limited work has been focused on the stability behavior of FGM plates.

The notable recent contributions pertaining to the buckling analysis of rectangular FGM plates under in-plane load are available in Refs. [3-7]. Najafizadeh and Eslami [8] studied the buckling loads of circular FGM plates under uniform radial compression. Sharjat et al. [9] studied the buckling of imperfect rectangular FGM plates under in-plane compressive load. It is observed that most of the available work has been dealt with the evaluation of the critical buckling loads of rectangular / circular FGM plates using eigenvalue approach.

FGM plates are, in general, non-symmetric-through-thickness as the material properties vary through the plate thickness. The bifurcation type of instability is examined by Leissa [10], Qatu and Leissa [11] for unsymmetric cross-ply laminated plates and by Aydogdy [12] for FGM plates subjected to in-plane load. It is observed that, the clamped plates exhibit the bifurcation type of buckling and can be studied based on eigenvalue analysis. For non-symmetric situation with boundary conditions other than clamped one, nonlinear analysis is required because of the bending behavior of plate due to the extension-bending coupling. However, such problems for FGM plates under mechanical load have been investigated employing eigen-buckling analysis [3-9], which may not reveal the actual behavior. Recently, Liew et al. [13] and Shen [14] have discussed the existence of bifurcation type of instability for the piezoelectric FGM plates under thermo-electro-mechanical loading. It may be inferred from available work that a comprehensive insight into the postbuckling behavior of FGM plate structures based on an appropriate model seems to be scarce in the literature. In view of these, a nonlinear analysis including extension-bending coupling is necessitated to understand the actual characteristics of FGM plates. Furthermore, to the best of author's knowledge, the work on the postbuckling behavior of FGM skew plates that find wide applications in the different industries is not yet commonly available in the literature.

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In the present work, an eight-noded shear flexible plate bending element, developed based on field consistency approach [15, 16], is used to analyze the nonlinear behavior of FGM skew plates subjected to in-plane compressive load. The material properties are graded in the thickness direction according to a power-law distribution in terms of volume fractions of the constituent materials. Mori-Tanaka homogenization method is used to estimate the effective material properties from the volume fractions and the properties of the constituent materials. The nonlinear governing equations are solved using Newton-Raphson technique to study the nonlinear behavior of FGM skew plates. The influences of material gradient index, thickness of plate, boundary condition and skew-angle on the stability characteristics of functionally graded skew plates are highlighted.

Formulation

The postbuckling behavior of a Functionally Graded Material (FGM) skew plate of thickness h made by mixing two distinct materials - ceramic and metal - is considered here (Fig.1). The coordinates x,y are along the in-plane directions, and z along the thickness direction. The top surface ($z = h/2$) material is ceramic rich and the bottom surface ($z = -h/2$) material is metal rich. The volume fractions of ceramic (V_c) and metal (V_m) at any point of the plate are expressed as

$$V_c(z) = \left(\frac{2z+h}{2h}\right)^k \quad \text{and} \quad V_m(z) = 1 - V_c(z) \tag{1}$$

where, k is the volume fraction index ($k \geq 0$). The effective bulk modulus K and shear modulus G of the FGM evaluated using the Mori Tanaka estimates [17,18] are as follows

$$\frac{K-K_m}{K_c-K_m} = V_c \Big/ \left[1 + (1-V_c)\frac{3(K_c-K_m)}{3K_m+4G_m}\right] \tag{2}$$

$$\frac{G-G_m}{G_c-G_m} = V_c \Big/ \left[1 + (1-V_c)\frac{(G_c-G_m)}{G_m+f_1}\right] \tag{3}$$

where,

$$f_1 = \frac{G_m(9K_m+8G_m)}{6(K_m+2G_m)}$$

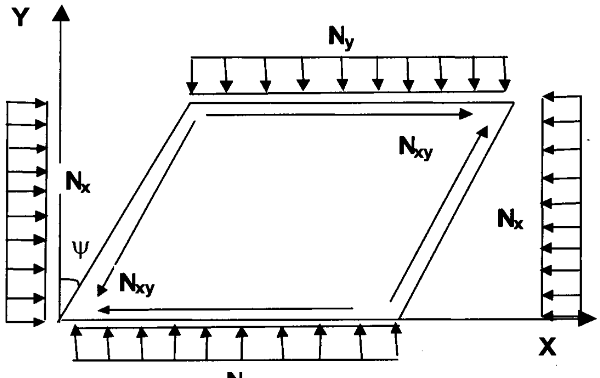

Fig.1a Geometry and Loading Condition of the Skew Plate

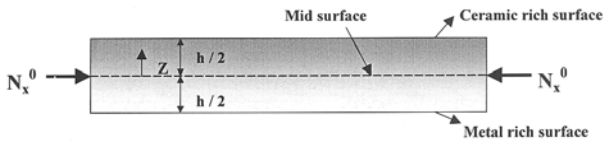

Fig.1b Gradation through Thickness

Here, the subscripts m and c refer the ceramic and metal phases, respectively. The effective values of Youngs modulus E and Poissons ratio υ can be found from

$$E = \frac{9KG}{3K+G} \quad \text{and} \quad \upsilon = \frac{3K-2G}{2(3K+G)}$$

Using Mindlin formulation, the displacements u, v, w at a point (x, y, z) in the plate are expressed as functions of mid-plane displacements u_0, v_0, w and two independent rotations θ_x and θ_y of the normal to the mid-surface as

$$u(x,y,z) = u_0(x,y) + z\theta_x(x,y)$$

$$v(x,y,z) = v_0(x,y) + z\theta_y(x,y)$$

$$w(x,y,z) = w_0(x,y) \tag{4}$$

von Karman's assumptions for moderately large deformation allows Green's strains to be written in terms of mid-plane displacements for a plate as,

$$\{\varepsilon\} = \{\varepsilon^L\} + \{\varepsilon^{NL}\} \tag{5}$$

The linear and nonlinear strain components at any point can be expressed as

$$\left\{\varepsilon^{L}\right\} = \left\{\begin{matrix}\varepsilon_m^L \\ 0\end{matrix}\right\} + \left\{\begin{matrix} z\varepsilon_b \\ \varepsilon_s\end{matrix}\right\} \text{ and } \left\{\varepsilon^{NL}\right\} = \left\{\begin{matrix}\varepsilon_m^{NL} \\ 0\end{matrix}\right\} \tag{6}$$

The mid-plane strains $\left\{\varepsilon_m^L\right\}$, bending strains $\left\{\varepsilon_b\right\}$, shear strains $\left\{\varepsilon_s\right\}$ and the nonlinear components of in-plane strains $\left\{\varepsilon_m^{NL}\right\}$ in Eqn. (6) are written as

$$\left\{\varepsilon_m^L\right\} = \left\{\begin{matrix} u_{o,x} \\ v_{o,y} \\ u_{o,y} + v_{o,x}\end{matrix}\right\}, \quad \left\{\varepsilon_b\right\} = \left\{\begin{matrix}\theta_{x,x} \\ \theta_{y,y} \\ \theta_{x,y} + \theta_{y,x}\end{matrix}\right\}$$

$$\left\{\varepsilon_s\right\} = \left\{\begin{matrix}\theta_x + w_{o,x} \\ \theta_y + w_{o,y}\end{matrix}\right\} \quad \text{and} \quad \left\{\varepsilon_m^{NL}\right\} = \left\{\begin{matrix}(1/2)\, w_{o,x}^2 \\ (1/2)\, w_{o,y}^2 \\ w_{o,x}\, w_{o,y}\end{matrix}\right\} \tag{7}$$

where the subscript comma denotes the partial derivative with respect to the spatial coordinate succeeding it.

The membrane stress resultants $\{N\}$ and the bending stress resultants $\{M\}$ can be related to the membrane strains $\{\varepsilon_m\}$ (= $\left\{\varepsilon_m^L\right\} + \left\{\varepsilon_m^{NL}\right\}$) and bending strains $\{\varepsilon_b\}$ through the constitutive relations by

$$\{N\} = \left\{N_{xx}\ N_{yy}\ N_{xy}\right\}^T = [A_{ij}]\left\{\varepsilon_m\right\} + [B_{ij}]\left\{\varepsilon_b\right\}$$

and

$$\{M\} = \left\{M_{xx}\ M_{yy}\ M_{xy}\right\}^T = [B_{ij}]\left\{\varepsilon_m\right\} + [D_{ij}]\left\{\varepsilon_b\right\} \tag{8}$$

where the matrices $[A_{ij}]$,$[B_{ij}]$, and $[D_{ij}]$ $(i, j = 1, 2, 6)$ are the extensional, bending-extensional coupling and bending stiffness coefficients and are defined as

$$[A_{ij}, B_{ij}, D_{ij}] = \int_{-h/2}^{h/2} [\overline{Q}_{ij}]\,(1, z, z^2)\, dz.$$

Similarly the transverse shear force $\{Q\}$ representing the quantities $\left\{Q_{xz}, Q_{yz}\right\}$ is related to the transverse shear strains $\{\varepsilon_s\}$ through the constitutive relations as

$$\{Q\} = [E_{ij}]\left\{\varepsilon_s\right\} \qquad i, j = 4, 5 \tag{9}$$

where $E_{ij} = \int_{-h/2}^{h/2} \left[\overline{Q}_{ij}\right] \kappa_i\, \kappa_j\, dz$

Here $[E_{ij}]$ $(i, j = 4, 5)$ are the transverse shear stiffness coefficients, κ_i is the transverse shear correction factor taken as $\sqrt{5/6}$ for non-uniform shear strain distribution through the plate thickness. $\overline{Q}_{ij}(z)$ are the stiffness coefficients and are defined as

$$\overline{Q}_{11}(z) = \overline{Q}_{22}(z) = \frac{E(z)}{1-\upsilon^2}\,;\ \overline{Q}_{12}(z) = \frac{\nu E(z)}{1-\upsilon^2}\,;$$

$$\overline{Q}_{16}(z) = \overline{Q}_{26}(z) = 0\,;\ \overline{Q}_{44}(z) = \overline{Q}_{55}(z) = \overline{Q}_{66}(z) = \frac{E(z)}{2(1+\upsilon)} \tag{10}$$

The strain energy of the plate can be expressed in terms of the field variables u_0, v_0, w_o, θ_x, θ_y and their derivatives. The potential energy due to external in-plane mechanical forces $\left(N_{xx}^o, N_{yy}^o, N_{xy}^o\right)$ can be evaluated in terms of $\left(w_o, \theta_x, \theta_y\right)$. Following standard procedure (minimization of total potential energy) the nonlinear finite element equations may be written as [19]

$$[K + K_{NL} + \lambda K_G]\{\delta\} = \{F\} \tag{11}$$

where [K] and $[K_{NL}]$ are the linear and nonlinear stiffness matrices respectively, $[K_G]$ is the geometry stiffness matrix due to unit in-plane compressive load, $\{F\}$ is the force vector, $\{\delta\}$ is the vector of the degrees of freedom associated to the displacement field in a finite element discretization. λ is the magnitude of the in-plane compressive load.

In the present work, an eight-noded C° continuous shear flexible plate bending element with five degrees of freedom per node is used here to study the nonlinear behavior of FGM skew plates. By employing field consistency approach, as described in Prathap et al. [15] and Ganapathi et al. [16], the element is found to be free from locking syndrome and has good convergence properties. For skew plate, the element matrices corresponding to global axes are transformed to local axes using transformation rules [20].

Solution Procedure

For the case of pure ceramic or metallic plate under compressive in-plane load, the critical buckling load at which Euler type of buckling occurs is found from solving the following linear eigenvalue problem:

$$[\, K + \lambda K_G \,] \{\delta\} = \{0\} \tag{12}$$

Here, the force vector $\{F\}$ is zero as the in-plane compressive load is accounted for in the geometric stiffness matrix. The lowest eigenvalue λ is the critical buckling load. To get the post-buckling paths, the following non-linear eigen value problem is solved iteratively [21] as described below.

$$[\, K + K_{NL} + \lambda K_G \,] \{\delta\} = \{0\} \tag{13}$$

After the eigenvector is obtained from the linear buckling analysis (Eq. 12), it is normalized and scaled up so that the maximum displacement is equal to the desired amplitude, say w/h = 0.2 (w is the maximum lateral displacement, h is the thickness of the plate). The iterative solution procedure for the non-linear analysis starts with this initial vector. Based on this initial mode shape, the non-linear stiffness matrix that depends on displacement is formed. Subsequently, the updated eigenvalue and its corresponding eigenvector are obtained from Eq.(13). This eigenvector is further normalized, and scaled up by the same amplitude (w/h), and the iterative procedure adopted here continues till the convergence of buckling load (eigenvalue).

In case of FGM plates bending-extension coupling produces bending curvature under in-plane load. In such cases, the nonlinear equilibrium Eq. (11) is written as

$$[\, K + K_{NL} \,] \{\delta\} = \{F\} \tag{14}$$

where $\{F\}$ is the force vector consisting of in-plane compressive loads $\left(N_{xx}^{o}\ N_{yy}^{o}\ N_{xy}^{o}\right)$.. Eq.(14) is solved by Newton-Raphson technique to get the force-displacement curves.

Results and Discussion

Here, the study is focused on the nonlinear behavior of functionally graded skew plates under in-plane load. The boundary conditions considered here are

a) Simply Supported :

$u = w = \theta_y = 0$ on x = 0 ; $w = \theta_y = 0$; u = constant on x = a

$v = w = \theta_x = 0$ on y = 0 ; $w = \theta_x = 0$; v = constant on y = b

b) Clamped Support :

$u = w = \theta_x = \theta_y = 0$ on x = 0 ; $w = \theta_x = \theta_y = 0$; u = constant on x = a

$v = w = \theta_x = \theta_y = 0$; on y = 0 ; $w = \theta_x = \theta_y = 0$; v = constant on x = b

c) Two opposite edges simply supported and other two free :

$u = w = \theta_y = 0$ on x = 0 ; $w = \theta_y = 0$; u = constanton x = a

u = constant and v = constant along the edges x = a and y = b respectively is enforced during assembly of the finite element equations.

In order to validate the efficacy of the present formulation, two examples are considered for which solutions are available in the literature. Linear buckling problem of simply supported thin isotropic skew plates is solved with different mesh sizes and the non-dimensional buckling load λ_{cr} $(= a^2 \lambda_{cr}/\pi^2 D)$ is reported in Table-1 along

Table-1 : Mesh Convergence of Mechanical Buckling Parameter for Thin Simply Supported Isotropic Skew Plate

Skew Angle	Present			Wang [22]
	4*4	6*6	8*8	
0°	4.0055	4.0023	4.0006	4.0000
15°	4.4130	4.4020	4.4003	4.3937
30°	6.0454	5.9735	5.9725	5.8963
45°	10.5686	10.4526	10.3031	10.1026

with those of Wang [22]. The critical buckling loads are obtained from the eigenvalue analysis (Eq.12). It is observed from Table-1 that the convergence is monotonic and the present results compare well with the existing solutions. Based on progressive mesh refinement, an 8 x 8 mesh is found to be adequate to model the skew plate for this analysis. The present formulation is also tested by studying the postbuckling equilibrium paths of a simply supported isotropic square plate (*a*/*h* = 50). Postbuckling paths are obtained through eigenvalue solution, as it is applicable to the chosen problem and it is found to be in excellent agreement with the solution of Sundaresan et al. [23] in Table-2. Furthermore, it is observed from Table-2 that there is a drop in the postbuckling resistance at *w*/*h*=1.0 and this is possibly due to change in stiffness leading to the redistribution of mode shape at higher level of compressive load. The redistributed mode shape looses its symmetry and the maximum displacement shifts towards one side of the plate as shown in Fig.2. This "mode redistribution" or "sudden decrease of nonlinear stiffness" is reported earlier by Singha et al. [21] while investigating the thermal post-buckling behavior of composite plates.

Next, nonlinear behaviors of aluminum / alumina FGM skew plates subjected to in-plane load are investigated. The top surface of the plate is ceramic rich and the bottom surface is metal rich. The Youngs modulus, for alumina and aluminum are E_c = 380 GPa, E_m = 70 GPa, respectively and Poissons ratio (υ) 0.3 is for both ceramic and metal. The variation of the volume fraction of ceramic (V_c) in the thickness direction (z) for the functionally graded plate is shown in Fig.3.

Postbuckling paths for a thin clamped FGM square (a/h = 100, a/b = 1) plate under uni-axial compression are presented in Fig.4 for various values of material gradient

Table-2 : Comparison of Buckling and Postbuckling of Uniaxially Compressed Simply Supported Isotropic Plate (a/h = 50, a/b = 1)

w/h	λ/λ_{cr}	
	Present	**Sundaresan et al. [23]**
0.2	1.0138	1.0137
0.4	1.0549	1.0547
0.6	1.1235	1.1233
0.8	***1.2200***	1.2198
1.0	1.2101	1.3445
λ_{cr}	3.9930	4.0081

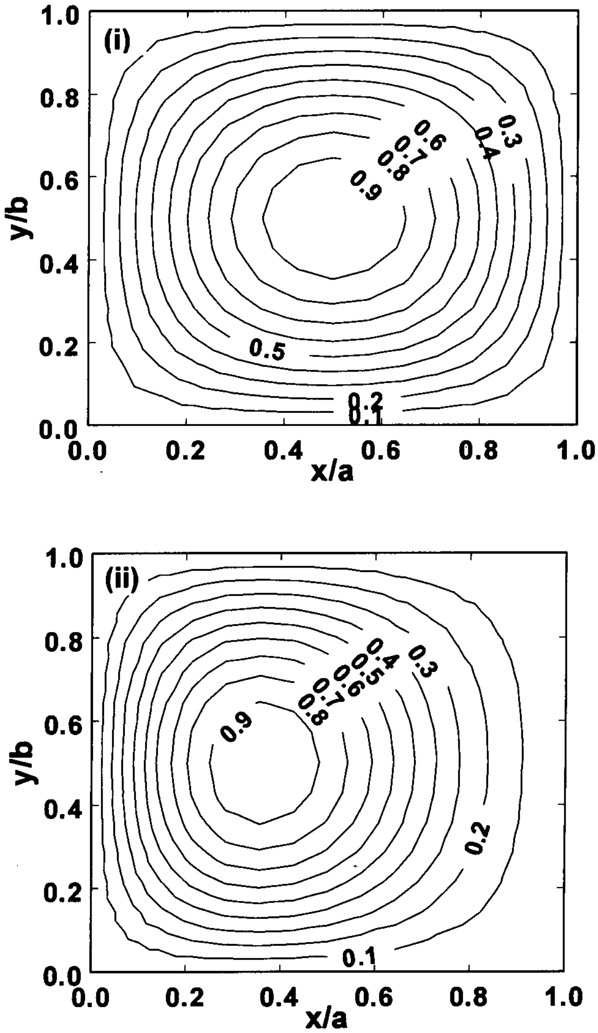

Fig.2 The Redistribution of Normalised Mode Shape Contours of Simply Supported Isotropic Plate (a/h = 50, a/b = 1) Before and After Secondary Instability : (i) w/h = 0.8, (ii) w/h = 1.0

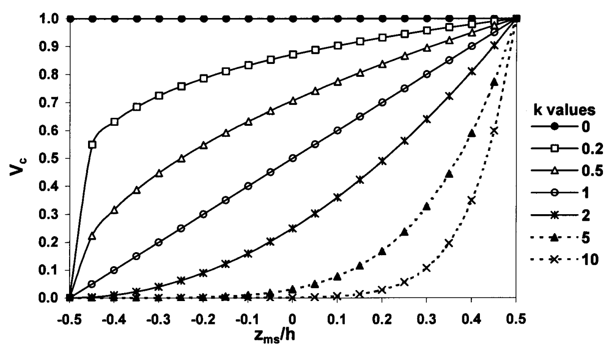

Fig.3 Variation of Volume Fraction of Ceramic Through the Plate Thickness

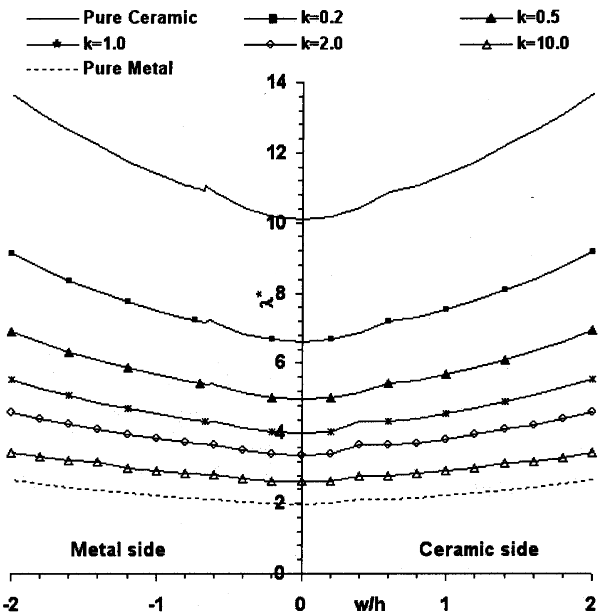

Fig.4 Effect of Gradient Index, k on Postbuckling Paths of Clamped Thin FGM Square Plate Under Uniaxial Compression (a/h = 100, a/b = 1)

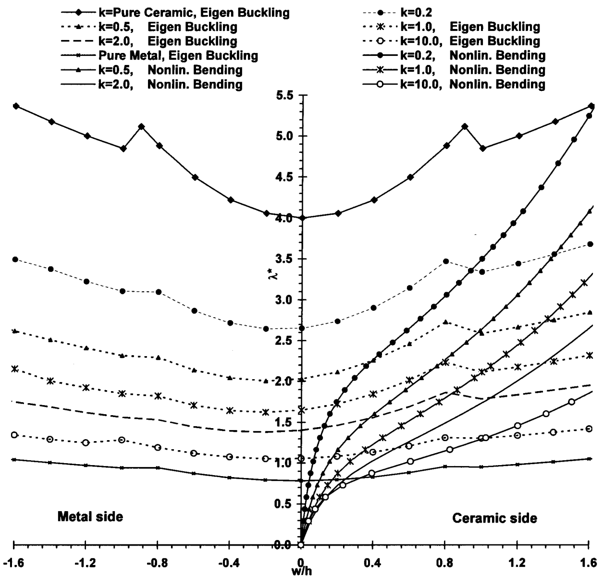

Fig.5a Effect of Gradient Index, k on Postbuckling Paths of Simply Supported Thin FGM Square Plate Under Uniaxial Compression (a/h = 100, a/b = 1)

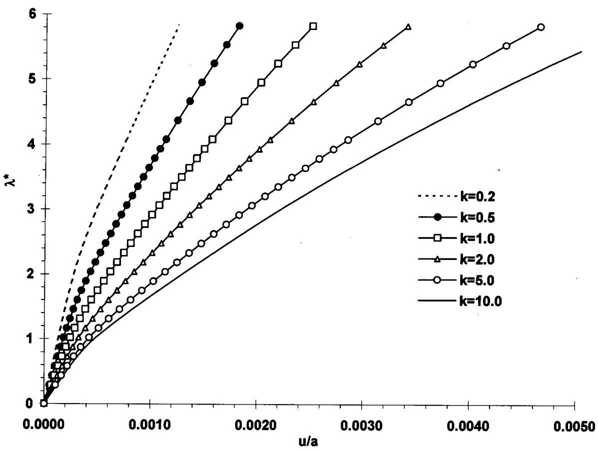

Fig.5b End Shortening of Simply Supported Thin FGM Square Plate Under Uniaxial Compression (a/h = 100, a/b = 1)

index (k). The clamped plate exhibits a bifurcation-type of instability as the extension-bending coupling is neutralized by the supports, and the postbuckling paths are obtained through eigenvalue approach. It is observed from the figure that after the bifurcation buckling, the postbuckling load increases monotonically with the increase in out-of-plane deformation. The postbuckling paths show a drop near about $w/h = 0.6$, and it corresponds to secondary instability and a redistribution of the mode shape. Furthermore, the load-displacement curves are found to be symmetric with respect to the vertical axis (mid-surface) for all the values of material gradient index, i.e. irrespective of the direction of the buckling deformation. It is also observed that, with the increase in material gradient index (k), the resistance of the plate reduces, i.e., for a specified in-plane load the out-of-plane displacement increases, and this is because of the stiffness reduction due to the more metal inclusion in FGM plate.

Next, the nonlinear behavior of simply supported thin square FGM plates (a/h = 100, a/b = 1) under uni-axial compression is investigated in Fig.5 for various values of material gradient index (k). The variations of non-dimensional out-of-plane central deflection (w/h) and in-plane displacement of the loaded edge (u/a) with non-dimensional uni-axial load $\left(\lambda^* = a^2 \lambda_{cr} / \pi^2 D_c \,,\, D_c = E_c\, h^3 / 12\,(1-\upsilon^2)\right)$ are presented in Fig.5 (a) and Fig.5 (b) respectively. The isotropic plates exhibit bifurcation buckling and the corresponding postbuckling paths for pure ceramic and pure metal cases are traced by the eigenvalue analysis. For FGM plates (k = 0.2, 0.5, 1.0, 2.0, and 10.0) the non-linear governing equation (14) is solved by Newton-Raphson technique to include the effects of extension-bending coupling and the corresponding load-displacement curves are shown by solid lines in Fig.5 (a). It is observed that with the increase in the compressive load, the FGM plate starts bending towards the ceramic side of the plate. For comparison purpose, the problem is also solved based on eigenvalue approach and the corresponding postbuckling paths are highlighted in the figure by dotted lines. It is further observed that the postbuckling curves are symmetric about the Y-axis for pure ceramic (k = 0) and pure metal cases (k = infinite), whereas, it looses its symmetry and is shifted towards ceramic side for FGM plates (k = 0.2, 0.5, 1.0, 2.0, and 10.0). This is attributed to the extension-bending coupling and a shift in the neutral surface towards the high stiff ceramic side of the plate. Furthermore, for the chosen deflection, the compressive load evaluated based on nonlinear analysis is in general low compared to those of buckling study and the nonlinear characteristics obtained from two different approaches are quite different from each other. The difference between solid curves (nonlinear analysis) and dotted curves (eigenvalue analysis) clearly indicate the effect of extension-bending coupling, which was neglected in the earlier investigations [3-9]. Hence, it can be opined that the bifurcation type of instability may not take place in actual situation when the postbuckling response curve is not symmetric. The in-plane displacements (u/a) of the loaded edge are found to increase monotonically with the increase in compressive edge load in Fig.5 (b). However, the slopes of the end shortening curves decrease with increase of compressive load, indicating a softening behavior. This softening behavior at higher load level is observed for all values of material gradient index *k*.

Similar feature is also observed for simply supported thick square plates, (a/h = 20, a/b = 1) under uni-axial compression and thin square plates (a/h = 100, a/b = 1) under bi-axial compression as shown in Fig.6 and 7 respectively. Isotropic plates do exhibit bifurcation buckling, whereas, FGM plates under compressive edge load behaves like eccentrically loaded plate and start bending towards ceramic side (radius of curvature being on the metal side) with the increase in compressive edge load.

The postbuckling paths of thin square FGM plates (a/h = 100, a/b = 1) with two opposite sides simply supported and other two edges free are presented in Fig. 8 for different values of material gradient index *k*. Here nonlinear load-displacement curves are obtained by Newton-Raphson method and the variation of out-of-plane displacement (w/h) and end shortening (u/a) with respect to edge compression is shown in Figs.8 (a) and (b) respectively. It is observed here that the load carrying capacity of the FGM plate is relatively less compared to the case of all sides simply supported boundary condition and the slope of the load-displacement curves reduces and becomes almost flat with the increase in compressive load. The same trend is observed for all the values of material gradient index *k*.

Finally, the effect of skew angle on the postbuckling paths of simply supported FGM skew plate under uni-axial compression (N_x or N_y) is studied in Fig.9. The postbuckling paths obtained for different skew angles are qualita-

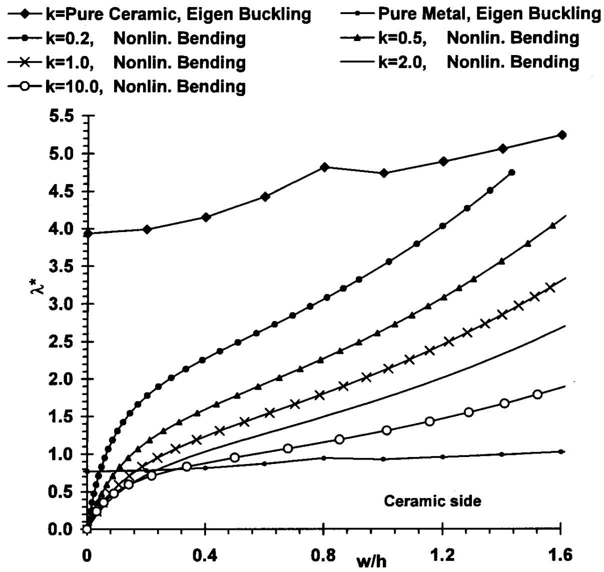

Fig.6 Postbuckling Paths of Simply Supported Thick FGM Square Plate Under Uniaxial Compression (a/h = 20, a/b = 1)

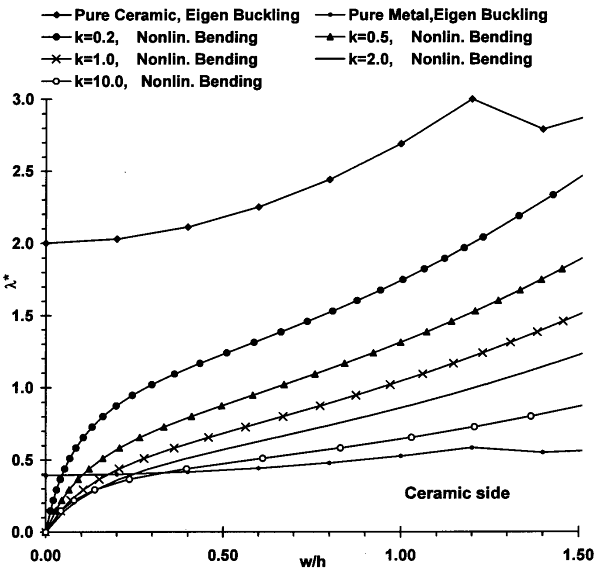

Fig.7 Postbuckling Paths of Simply Supported Thin FGM Square Plate Under Biaxial Compression (a/h = 100, a/b = 1)

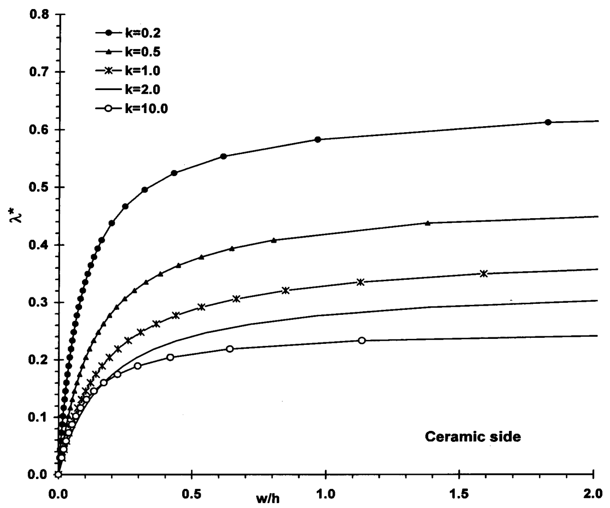

Fig.8a Post Buckling Paths of Thin FGM Square Plate with Two Opposite Edges Simply Supported and Other Two Edges Free Under Uniaxial Compression (a/h = 100, a/b = 1)

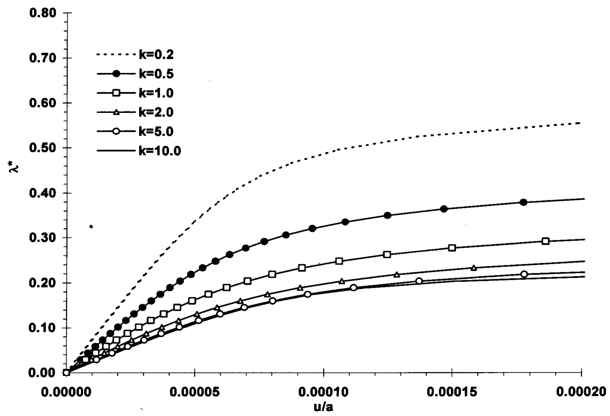

Fig.8b End Shortening of Thin FGM Square Plate with Two Opposite Edges Simply Supported and Other Two Edges Free Under Uniaxial Compression (a/h = 100, a/b = 1)

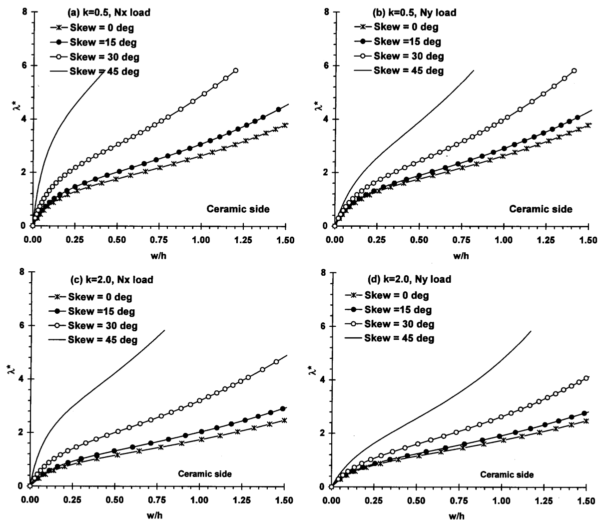

Fig.9 Postbuckling Paths of Simply Supported Thin FGM Skew Plate (a/h = 100, a/b = 1) Under Uniaxial Compressions Nx and Ny : (a) k = 0.5, Nx Load (b) k = 0.5, Ny Load (c) k = 2.0, Nx Load (d) k = 2.0, Ny Load

tively similar with respect to the deflection. It is further observed that, with the increase in skew angle, the out-of-plane deflection (w/h) reduces for a specified compressive load, i.e., the load carrying capacity increases with the increase in skew angle. It is further observed that the compressive load in the Y-direction (N_y) produces more deflection compared to the compressive load in X-direction (N_x).

Conclusions

Nonlinear behavior of aluminum-alumina FGM skew plates under in-plane compressive load is studied here using an eight-noded shear flexible plate bending finite element. The nonlinear governing equations are solved by Newton-Raphson technique to study the nonlinear behavior of FGM plates under edge compression. It is observed that FGM plates with clamped boundary condition exhibit bifurcation type of instability under in-plane load and the corresponding postbuckling path can be obtained from eigenvalue analysis. However, FGM plates with simply supported boundary condition bend towards the ceramic side of the plate due to the extension-bending coupling and the corresponding postbuckling paths can be obtained through nonlinear analysis. Further, the load carrying capacity of FGM plates increases with the increase in skew angle.

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