LONGITUDINAL PARAMETER ESTIMATION USING WIND TUNNEL AND SIMULATED FLIGHT DATA OF MISSILE CONFIGURATION

S. Singh* and A. K. Ghosh**

Abstract

The present study makes an attempt to estimate aerodynamic parameters from a typical flight data of a short range missile. Exhaustive wind tunnel testing was conducted to generate longitudinal force and moment coefficients at low speed. Identification methods were applied on the selected wind tunnel data to capture the general form of the aerodynamic model. During the application of Maximum likelihood (ML) method, the estimation algorithm assumed this wind tunnel identified aerodynamic model to be exact. To avoid any requirement of postulation of aerodynamic model, the Delta method was applied to estimate the aerodynamic parameters. The Delta method used measured aircraft motion and control variables as the inputs to the Feed Forward Neural Network and the aerodynamic force or moment coefficient was the output for training the Feed Forward Neural Network. The application of the Delta method results in large scatters in the estimated parameters. To overcome this problem of large scatter, the Delta method was modified by changing the training strategy. The Delta method with new training strategy will be referred as the Modified Delta method. It is expected that the proposed Modified Delta method would result in estimates with less uncertainties. Further to check the robustness of the ML, Delta and the Modified delta methods, the estimation was also carried out with flight data having known measurement noise. The effect of control input form in the accuracy of estimates obtained by ML, the Delta and the Modified Delta methods are also studied. It is observed that the Modified Delta method can advantageously be applied on the flight data of a tactical missile to estimate aerodynamic parameters. The paper progresses with the description of the generation of wind tunnel data and aerodynamic model identification using selected wind tunnel data. Finally it concludes by demonstrating applicability of ML, the Delta and the Modified Delta methods on simulated flight data of a typical short range tactical missile configuration.

Nomenclature

a_z = acceleration along body z-axis, m/s^2

AR = tail aspect ratio

C_M = modified pitching moment coefficient of body plus tail configuration

C_m = pitching moment coefficient of body plus tail configuration

C_N = normal force coefficient of body plus tail configuration

C_x = force coefficient in the *x* direction

d = diameter of missile, m

g = gravity, m/s^2

h = height, m

I_x, I_y, I_z = moment of inertia about x, y and z axes of missile, kg-m^2

$K_{B(T)}$ = ratio of addition body normal force coefficient in presence of tail, at δ=0 degree

$K_{T(B)}$ = ratio of normal force coefficient of tail in presence of body, at δ=0 degree

$k_{B(T)}$ = ratio of addition body normal force coefficient in presence of tail, at α=0 degree

$k_{T(B)}$ = ratio of normal force coefficient of tail in presence of body, at α=0 degree

M_∞ = free stream mach number

M = mass of missile, kg

q = pitch rate, rad/sec

$\bar{q}$ = dynamic pressure, kg/m-s^2

T = thrust, N

u,v,w = velocity component along x, y, z body axes, m/s^2

V = airspeed, m/s^2

* Ph.D. Student   ** Associate Professor
Department of Aerospace Engineering, Indian Institute of Technology Kanpur, Kanpur-208 016, India
Email : sanjvaab@iitk.ac.in; akg@iitk.ac.in

V_∞ = free stream velocity, m/s^2

W = weight of the missile, N

X_{cg} = distance of center-of-gravity of missile from nose, m

X_{cp} = distance of center-of-pressure of missile from nose, m

X_{cp_T} = distance of center-of-pressure of tail from nose, m^3

ρ = air density, kg/m^3

θ = pitch angle, rad

α = angle of attack, rad

β = side slip angle, rad

δ = tail control surface deflection, rad

$\Lambda_{c/2}$ = tail half-chord sweep-back angle, rad

Superscript

$(\dot{\ })$ = derivative with respect to time

Introduction

Parameter estimation from flight data as applied to aircraft, missile in the linear flight regime is currently being used on routine basis [1-4]. However the linear aerodynamic models used successfully upto this time seem to be inadequate for newly introduced short range, highly maneuverable tactical missile, rocket etc. The past few years have witnessed wide spread application of a variety of techniques, e.g., semi-emperical based models [5, 6] and Computational Fluid Dynamics (CFD) methods [7], which have been instrumental in providing details of the flow field sufficiently accurately. However, the increasing dependency of the semi-emperically based models on wind tunnel test data, complexity and high computational costs of the CFD methods and the difficulties in determining full scale non-linear flow effects from subscale wind tunnel test data are some of the factors which impose limitations on the use of these approaches for routine flight analysis. Consequently, analysis guided by flight data appears to be the best recourse [7].

In several system identification problems, one is required to determine the coefficient of the aerodynamic parameters from real flight data [8]. The flight vehicle estimation problem falls into this category. The stability sand control derivatives of the vehicle are to be accurately and reliably determined from flight recorded responses to known control inputs. Several methods have been developed and varying degrees of success have been obtained under stipulated idealized conditions. Although most easy

and straight forward to use, the least square estimate are known to yield biased results in the presence of measurement noise while requiring flight measurement of all the control and state variables and their derivatives [1-4]. The output error maximum likelihood (ML) estimator and its several variants have been successfully used for estimating aircraft. stability and control derivatives from flight data [8, 9]. Such conventional methods require postulation of a model of the flight vehicle, and defining of a cost function based on the difference between the model and the measured flight vehicle response for identical control inputs. The model response requires integration of system equations, and that, in turn, requires an estimation of initial conditions of all the motion variables of the model. To begin the iterative process, ML algorithm requires reasonable guess (initial) value of the parameters. The accuracy of estimates obtained through ML estimator is generally affected by the correctness of the postulated model, the accuracy of estimates of initial conditions, reasonableness of magnitude and sign of the initial values of parameters and the amount of measurement noise in the measured flight data.

With the introduction of short range, highly maneuverable missiles, the linear model used routinely up to this time fails to model aerodynamics of such missiles accurately. There is a need to model correctly the nonlinearity associated with these missile aerodynamics. Missile aerodynamics is complex function of Mach number, angle of attack, control surface deflection, roll orientation and configurational geometry. Angle of attack requirements are driven primarily by the launcher and the end game maneuver requirement to successfully engage a target [5]. Missile can undergo fairly high angle of attack in both the surface and air launched mode due to pitch-over requirements for minimum engagement range. In a general maneuver toward a target, one can have combined angle of attack and side slip. If one assumes maneuver in pitch plane only, then planar aerodynamics can be used and can be computed with limited accuracy [5, 6]. The last weapon system requirement that effects the aerodynamics and hence the aerodynamic model selected is some what dependent on the weapon. Unguided projectiles typically have a truncated nose, due to fuze design, with a slope discontinuity along the nose where the fuze joins the ogive. Guided projectiles typically have either sharp or spherically blunt noses, depending on the type of guidance used [5].

Determining and describing the aerodynamic forces and moment on an missile is a very important subject in

atmospheric flight mechanics. Aerodynamic forces and moments are strictly speaking function of the variables associated with flight conditions. In many practical cases, these aerodynamic forces and moments can be approximated by linear terms in their Tayler series expansion, a well known approach leading to stability derivatives [10]. By far the two largest source of nonlinear aerodynamic phenomena are due to angle of attack and Mach number. At low angle of attack, the aerodynamics are mostly linear, and linearized and slender-body aerodynamics methods prove very effective in providing acceptable accuracy for approximate aero prediction codes [11, 12]. As the angle of attack increases, the linearized models yield degraded accuracy. Further, aerodynamic forces and moments are dependent not only on the instantaneous value the motion variables associated with the flight condition, but also their entire past histories. Practically, one can write the aerodynamic forces and moments as function of their variables and all their derivatives and expand them as Taylor series about some reference values. Based on this assumption, certain practical schemes have been developed for the flight mechanics and control applications [13]. For many years, the aerodynamic forces and moments were approximated by the linear expression in their Taylor series expansions, leading to the concept of stability and control derivatives [13-15]. The approximation has been found to extremely well for attached flow for low angle of attack. Furthermore, the addition of nonlinear terms, expressing the change in stability derivatives with angle of attack, can extend the range of flight applications. In this approach, however, using either linear or non-linear aerodynamics, it is assumed that the parameter appearing in the representations are time-invariant. A major part of aerodynamic research has been devoted to the determination of correct aerodynamic model, by theoretical and experimental means [16]. However, for the purpose of parameter estimation from flight data, one would need a convenient aerodynamic model structure amenable to flight test data processing using estimation method.

In designing missile control system one considers, a typical speed (or Mach number if missile is super sonic), angle of attack (α) and the height of missile will operate at. Design of control law aims at deciding right amount of control deflection (δ) required for a particular maneuver at a given Mach number and height the missile is expected to operate. To determine the α, δ map for a given regime of Mach number (M_∞), one must capture the static aerodynamics over enough α, δ, and M_∞ conditions, so that the flight envelops will be covered. As a first step, exhaus-

tive wind tunnel test are conducted to generate the longitudinal forces and moments for various combination of angle of attack and control surface deflection. The purpose is to extract a good trim aerodynamic model from selected wind tunnel data by using an identified method. Although wind tunnel testing improves the accuracy of estimation but it is a time consuming and expensive way to estimating stability and control derivatives. Precise simulation of control surfaces, power effects and flight condition is difficult. The model tested in the wind tunnel is generally slightly different from actual vehicle due to last minute changes. Other reasons for discrepancies between flight and wind tunnel results are Reynolds number discrepancies and interference due to support system. It is therefore desirable that the wind tunnel estimates be corroborated with the estimates from actual flight test data [7, 17].

The wind tunnel derived aerodynamics model (although faces few discrepancies [9] are very useful to fix the functional form of the aerodynamic model to be used in the estimation algorithm. The estimated parameters obtained of the models through flight tests are then compared with the parameters obtained through wind tunnel testing to further update the aerodynamic database to be used for flight mechanics and control studies purposes. The conventional methods (ML and its variants) for parameter estimation have one limitation: they need the a priori fix of a functional form of aerodynamic forces and moment coefficients. Generally, the aerodynamic coefficients are assumed to be linear, polynomial or spline function of unknown parameters. Such an a priori fix of aerodynamic model converts model identification problem into a parameter estimation problem, and thus imposes severe restriction on the accuracy and validity of the resulting aerodynamic model. Such an approach would be susceptible to error in dealing with large amplitude, high angle of attack and time dependent maneuvers.

In contrast, a new thrust area has emerged in the area of aircraft modeling and parameter estimation: development of techniques for flight vehicle identification using artificial neural networks (ANNs). A class of neural networks called the feed forward neural networks (FFNNs) work as a general function approximators [18]. The FFNNs can be regarded as a non-parametric modeling method; both the structure and parameters need not to be known a priori. As shown in Fig.1, a typical FFNN can be used to map missile motion/control variables to aerodynamic forces and moment coefficients without specifying functional relationship between the aerodynamic coeffi-
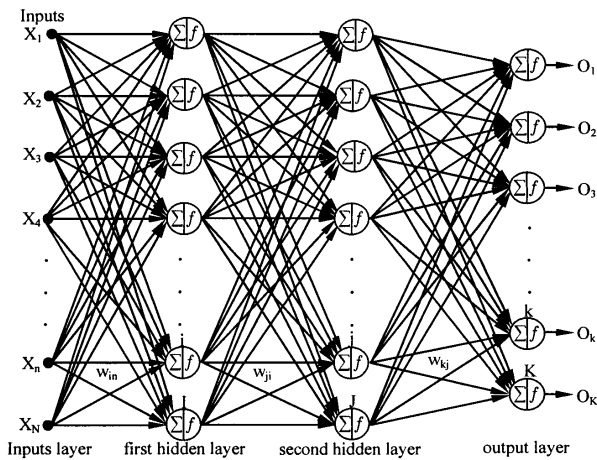
Fig.1 Schematic of feed forward neural network

cients and the motion/control variables. Further, the methods based on FFNNs do not require integration of system equations. Thus, the limitations of assuming specific form for aerodynamic coefficient and requirement of appropriate initial conditions for solution of system equation of motion are relaxed. The FFNN can be trained to predict the aerodynamic coefficient based on a suitable set of motion and control variables. This kind of modeling is akin to black-box model where in input-output relationship is established without establishing the mathematical model or the transfer function relating the input-output. It is only during the last few years that the FFNN aerodynamic modeling has been attempted. Significant contributions have been made by Hess [19], Basappa and Jategaonkar [20], Linse and Stengel [21], Youseffand Juang [22], and Raisinghani and Ghosh [23]. Hess [19] dealt with the use of FFNNs to represent aircraft aerodynamics. Basappa and Jategaonkar [20] have studied various aspects of FFNN modeling and its applicability to real flight data. Linse and Stengel [21] have shown accurate modeling of aerodynamic coefficient using system identification model composed of an extended Kalman-Bucy filter for state and force estimation and a computational neural network for aerodynamic model. Youseff [22] demonstrated the feasibility of neural modeling approach to establish a nonlinear aerodynamic model that is amenable to flight test data processing.

In some situations, an aerodynamic model alone may well serve the purpose, but as is well known, there are many applications for which explicit parameter estimation is desirable and useful. It is the usefulness of parameter estimation for purposes like validating wind tunnel or analytical predictions, analyzing aircraft stability, handling qualities and control systems, expanding flight test

envelope, updating simulators, etc. that has kept the field of parameter estimation active till date. In this context, Raisinghani, Ghosh, and Kalra [23] proposed the Delta and the Zero method. Both the methods use measured aircraft motion and control variables as the inputs to the FFNN and the aerodynamic forces or moment coefficients as the output for training the FFNN. The forces and the moment coefficients are computed using the measured linear or angular accelerations respectively. Both the delta and the zero methods were validated using simulated and real flight data of various aircraft [24-26].

The application of the Delta and the ML method on flight data of a typical tactical missile face few common difficulties for the purpose of parameter estimation. Due to operational reasons, it might not be possible to excite the missile with a multistep efficient control input [27] and that might result in having flight data with inadequate information content. Further, routine procedure for exhaustive data compatibility check may not always be possible for flight data obtained through flight tests of tactical missiles. Installation of large number of sensors for acquiring flight data for the purpose of data compatibility check may not be possible due to unavailability of space in the missile to house dedicated sensors. Although ML estimators and its several variants have been the most successfully used methods for estimating aircraft stability and control derivatives (parameters) from flight data. The data compatibility check (some time referred to as flight path reconstruction), an integral part of parameter estimation, is performed by using standard kinematic equations, and applying a Kalman filter or ML algorithms [7]. The compatibility check provides an accurate information about the aircraft states, and also estimation of biases, scale factors and time shifts in the recorded data. Elimination of such errors from flight measurements prior to estimation of parameters helps to improve accuracy of estimates. However, the process of data compatibility check for estimation of errors can itself be quite sensitive to the methodology employed, and at times, it may be as time consuming as the process of estimating parameters itself [7]. In contrast, In Ref. 28, it was demonstrated that during the application of the Delta method using FFNN, the recorded flight data directly be used without worrying about the measurement errors (except for time shifts).

In the present study, an attempt has been made to estimate aerodynamic parameters from flight data of a typical tactical missile. To avoid any explicit requirement of postulation of aerodynamic model, the Delta method was applied to estimate the aerodynamic parameters. In

application of the Delta method, it was observed that the estimated parameters showed large spread in their numerical values [23]. It was conjectured that a less than perfect match between the actual and predicted values of aerodynamic coefficients is primarily responsible for the observed spread in the estimated values [23]. To reduce such spread in the numerical values of the estimates, a new approach has been proposed by modifying the Delta method. The Delta method [23] used measured aircraft motion and control variables as input to the FFNN and the aerodynamic force or moment coefficient was the output for training the FFNN. However, in the proposed method, the input file for training the FFNN contained differential variations of motion and control variables ($\Delta\alpha$, $\Delta\alpha^2$, Δq, $\Delta\delta$, and $\Delta\alpha_T^2$) the output vector consisted of variations in aerodynamic force or moment coefficient (ΔC_N or ΔC_m). The strategy followed in the proposed method for training is pictorially presented in Fig. 2.

This proposed method will be referred to as the Modified Delta (MD) method for future reference. Further, to check the robustness of the MD method, the estimation was also carried out with the flight data having known measurement noise. The effect of control input form in the accuracy of the estimates obtained by the MD method is also studied. For the purpose of comparative study, the Delta and the ML method were also applied on the flight data to extract aerodynamic parameters. Since ML method requires a priori postulation of the aerodynamic model, exhaustive wind tunnel testing was conducted to generate longitudinal force and moment coefficients. Maximum likelihood method was applied on the selected wind tunnel data to capture the approximate form of the trim I aerodynamic model. During the application of ML method for parameter estimation using flight data, the estimating algorithm assumed the wind tunnel identified aerodynamic model to be exact. The estimated parameters ob-
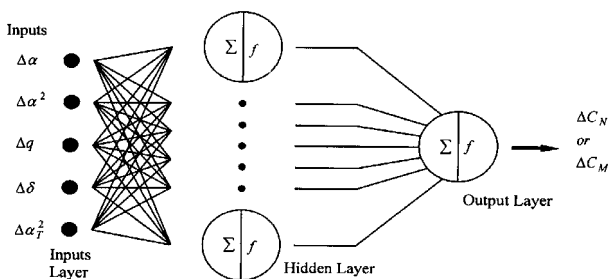
tained through the Delta, the modified Delta and the ML methods were converted into trim force and moment coefficients. These coefficients were then compared for both magnitude and accuracy with the tunnel generated force and moment coefficients. It is observed that the MD method can advantageously be applied on flight data of a typical tactical missile to estimate aerodynamic parameters. The paper progresses with the description of the generation of wind tunnel data and aerodynamic model identification using selected wind tunnel data [29]. Finally, it concludes by presenting detailed discussion on the results obtained by applying the delta, MD and ML methods on simulated flight data.

Wind Tunnel Model and Testing Arrangements

A typical configuration of a short range tactical missile as shown in Fig.3 was considered for the present study. A full scale model was tested in wind tunnel to generate aerodynamic forces and moments [29]. The surface of the model was polished and smoothened. The model was tested in a closed-circuit low speed wind tunnel with a test section 3.0 m x 1.2 m x 2.0 m. The tunnel is able to produce flow with velocity ranging from 10 to 60 m/sec at a turbulence level of less than 0.1%. The air velocity is measured with an accuracy of 0.05%. The spatial variation of mean velocity in the test section was observed to be 0.2%. The tunnel is powered by 1000 KW dc motor driving a commercial axial flow fan. Wind tunnel tests were conducted on a full scale model of a typical tail controlled missile, to generate aerodynamic forces at various angles of attack (-10 deg to + 15 deg) and tail deflec-
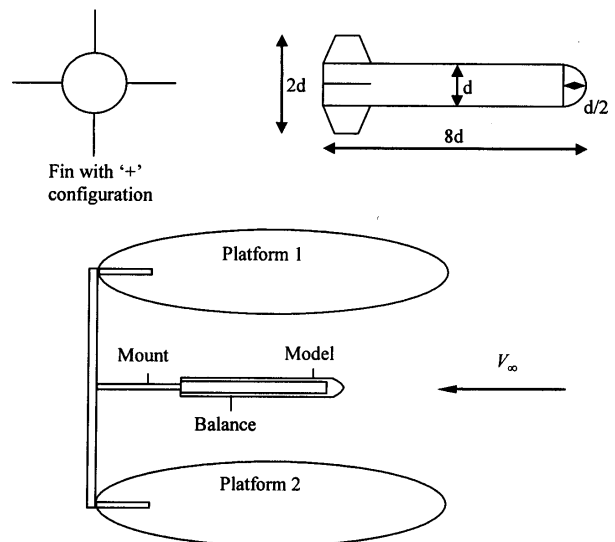

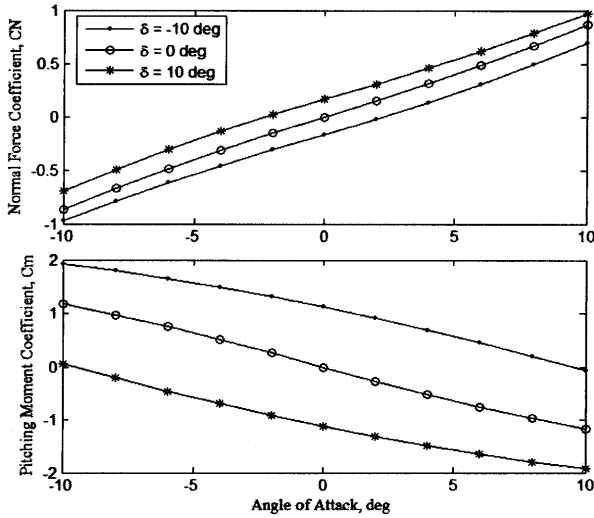Fig.3 Schematic of the model and the fitment used in wind tunnel testing

Fig.2 Schematic of feed forward neural network for proposed aerodynamic modeling

Fig.4 Variation of wind tunnel normal force coefficient and pitching moment coefficient with angle of attack

tions (-10 deg to + I 0 deg). Full scale model of the missile configuration was tested at Reynolds number of 2.0×10^6. Typical variation of C_N and C_m with α and δ are presented in Fig. 4. The reference area, length and moment reference point chosen for these sets of data are, $0.0113 \, m^2$, 1.0 m, 0.532 m from the nose tip respectively [29].

Approximate Aerodynamic Models and Model Identification using Wind Tunnel Data

Exhaustive wind tunnel testes were conducted to generate the longitudinal forces and moments at low speed for various combination of angle of attack and control surface deflections [29]. As stated earlier, the purpose was to extract a good trim aerodynamic model from the selected wind tunnel data. It was decided to apply ML method on the selected wind tunnel data to capture the trim aerodynamic model. Since ML method requires postulation of a model, a through survey of available approximate models was carried out to fix the structure of aerodynamic model for identification purpose. There are various approaches to estimate weapon aerodynamics that include, empirical estimates, aero prediction codes with Navier-Stokes solver with many invariants. Component build up approach to model the aerodynamics is commonly used in semi-empirical aerodynamic codes, Component build up of aerodynamics applies to components of aerodynamics associated with individual missile components as well as aerodynamic terms within a component. It gets its basis from linearized theory where linear solution of the equations of motion can be added together in linear sense [5]. The missile configuration chosen for the present study has a cylindrical body with hemispherical nose, and all move-

able cruciform tail [29]. There are several separate aerodynamic force and moment terms that must be considered separately in the component buildup approach. These to include body alone; tail alone and, finally the additional force on the tail as a result of being in the down wash field of the body.

The normal force and pitching moment of the total configuration can be modeled by summing the aerodynamics of the missile components

$$C_N = C_{N_B} + C_{N_T} (S_T / S_{ref}) \tag{1}$$

$$C_m = C_{m_B} + C_{m_T} \tag{2}$$

where C_{N_B} is the normal force coefficient of the body, N_{N_T} is the normal force coefficient of the tail, S_T is the tail area, S_{ref} is the reference area of missile, C_{m_B} is the pitching moment coefficient of the body and C_{m_T} pitching moment coefficient of the tail. There are several types of interference effects that occur in aerodynamics. However, for the present missile configuration, the interference effects are considered. To better understand the interference lift components, it is instructive to examine the normal force on a configuration as defined by Pitts, Nielsen, and Kaattari [30], which is given by:

$$C_N = C_{N_B} + K (C_{N_\alpha})_T (\alpha + \delta) (S_T / S_{ref}) \tag{3}$$

where K is the sum of the tail-body and body-tail interference factor. Out of the major non linearities that occur in weapon aerodynamics, are the ones that have the most influence on the body alone are the angle of attack, Mach number, cross flow Reynolds number and asymmetric vortices. All of these phenomena can be modeled in an approximate sense except for the asymmetric shedding vortices [5, 6]. Design alternatives that helped to alleviate the problems and make vortices more symmetric include blunt nose, strakes, or canards in the nose region [5, 6]. The present study is restricted to moderate angle of attack ($\alpha \leq 10^o$) of the missile having almost blunt nose. Since, the present study is restricted to moderate angle of attack, side force due to asymmetric vortex shedding will not be considered in the aerodynamic modeling.

To incorporate nonlinear effect in the normal force coefficient of the body, the body term in Eq. (3) has been

expanded as sum of linear (L) and nonlinear (NL) contributor separately.

$$C_{N_B} = (C_{N_B})_L + (C_{N_B})_{NL} \qquad (4)$$

The slender body theory [31] provides a simple means of computing $(C_{N_B})_L$. The term $(C_{N_B})_{NL}$ can be modeled by a revised method of Allen and Perkins [32, 33]. The nonlinear contribution was attributed to the generation of cross flow around the body at an angle of attack. This cross flow term is based on the drag force experienced by an element of a circular cylinder of the same diameter in a stream moving at the cross component of the stream velocity, $V_\infty \sin \alpha$. Cross flow is primarily created by the viscous effect of the fluid as it flows around the body often separating and creating a nonlinear force coefficient.

In equation form, the so called viscous cross flow theory is given by [5, 34]:

$$(C_N)_{NL} = \eta \, C_{dc} \left(\frac{S_P}{S_{ref}} \right) \alpha^2 \qquad (5)$$

where η is the drag proportionality factor, C_{dc} is the cross flow drag coefficient [34] and S_P is the planform area of missile. Using Eqs. (4) and (5), the approximate theoretical aerodynamic model for body alone can be expressed as:

$$C_{N_B} = C_1\alpha + C_2\alpha^2 \qquad (6)$$

where body alone normal force coefficient derivatives (C_1 and C_2) are given by:

$$C_1 = 2 (k_2 - k_1) \frac{S_B}{S_{ref}} \qquad (7)$$

and,

$$(C_2) = \eta \, C_{dc} \left(\frac{S_P}{S_{ref}} \right) \qquad (8)$$

where (k_2-k_1) is the munk factor and S_B is the base area of missile. Moore and McInville [6] tried a second, third and fourth order equation in angle of attack to represent the wing alone normal force coefficient as a function of angle of attack. In the present study, similar approaches have been used to model the tail alone normal force coefficient.

The all moveable tail being of low aspect ratio, only second order equation in angle of attack has been used to model the tail-alone nonlinear normal force coefficient:

$$C_{N_T} = a_0 + a_1 \, \alpha_T + a_2 \, \alpha_T^2 \qquad (9)$$

where tail angle of attack, $\alpha_T = |\, \alpha + \delta \,|$, δ is the control surface deflection and α is the angle of attack seen by the missile. Here only positive angle of attack were considered because it was assumed that the missile fin plan form had no camber and as a result, the normal force at a negative angle of attack is simply the negative of that at the same positive values of angle of attack. The chosen missile has symmetric fin surface and therefore, coefficients a_0, a_1, and a_2 and can be evaluated using the following expressions [5]:

$$a_0 = 0 \qquad (10)$$

$$a_1 = \frac{2\pi AR}{2 + \left[AR^2 \, (\beta^2 + \tan^2 \Lambda_{c/2}) + 4 \right]^{1/2}} \qquad (11)$$

$$a_2 = 34.044 \, (C_N)_{\alpha = 15^0} - 4.824 \, (C_N)_{\alpha = 35^0}$$
$$+ \, 0.425 \, (C_N)_{\alpha = 60^0} - 6.412a_1 \qquad (12)$$

The total normal force coefficient of the tail in presence of body can be expressed as

$$C_{N_T} = a_1 \left[\left(K_{T(B)} + K_{B(T)} \right) \alpha + \left(k_{T(B)} + k_{B(T)} \right) \delta \right] + a_2 \, \alpha_T^2 \qquad (13)$$

Using Eqs. (6) to (13), one can express normal force and moment coefficient of the complete missile. Although, we started from the model proposed by moore [5], Allen and perkins [32, 33], however we further reconfigure the model as given below to make it amenable for parameter estimation from flight data.

$$C_N = C_{N_\alpha} \, \alpha + C_{N_{\alpha^2}} \, \alpha^2 + C_{N_{\alpha_T^2}} \, \alpha_T^2 + C_{N_\delta} \delta \qquad (14)$$

$$C_m = C_{m_\alpha} \, \alpha + C_{m_{\alpha^2}} \, \alpha^2 + C_{m_{\alpha_T^2}} \, \alpha_T^2 + C_{m_\delta} \delta \qquad (15)$$

where

$$C_{N_\alpha} = C_1 + a_1 \left(K_{T(B)} + K_{B(T)}\right)\left(S_T/S_{ref}\right) \qquad (16)$$

$$C_{N_\alpha 2} = C_2 \qquad (17)$$

$$C_{N_\delta} = + a_1 \left(k_{T(B)} + k_{B(T)}\right)\left(S_T/S_{ref}\right) \qquad (18)$$

$$C_{N_{a_T} 2} = a_2 \left(S_T/S_{ref}\right) \qquad (19)$$

$$C_{m_\alpha} = C_{N_\alpha}\left(X_{cg} - X_{cp_L}\right)/d \qquad (20)$$

$$C_{m_\alpha 2} = C_{N_\alpha 2}\left(X_{cg} - X_{cp_{NL}}\right)/d \qquad (21)$$

$$C_{m_\delta} = C_{N_\delta}\left(X_{cg} - X_{cp_T}\right)/d \qquad (22)$$

$$C_{m_{a_T} 2} = C_{N_{a_T} 2}\left(X_{cg} - X_{cp_T}\right)/d \qquad (23)$$

For the purpose of model identification, using wind tunnel data, the aerodynamic models (Eqs. 14 and 15) were used in the estimation algorithm to extract derivatives of normal force and pitching moment coefficient namely, C_{N_α}, C_{m_α}, $C_{N_\alpha 2}$, $C_{m_\alpha 2}$, C_{N_δ}, C_{m_δ}, $C_{N_{a_T}^2}$, and $C_{m_{a_T}^2}$. The purpose was to extract a good trim aerodynamic model from selected wind tunnel data by using an identification method. Separate identification was carried out for body alone and body plus tail configurations. For body alone, we assumed non-linear model (Eqs. 14 and 15) in the estimation algorithm and subsequently the parameters namely, C_{N_α}, C_{m_α}, $C_{N_\alpha 2}$, $C_{m_\alpha 2}$ were estimated. Similar exercise was carried with wind tunnel data for body plus tail configuration. The body plus tail configuration was tested to generate forces and moments for different combination of angle of attack and tail deflections as presented in Fig. 3. The aerodynamic model given in Eqs. (14) and (15) were used in the estimation algorithm. Maximum likelihood method was applied to estimate the aerodynamic parameters, C_{N_α}, C_{m_α}, $C_{N_\alpha 2}$, $C_{m_\alpha 2}$, C_{N_δ}, C_{m_δ}, $C_{N_{a_T}^2}$, and $C_{m_{a_T}^2}$ by minimizing the error between the assumed model of C_N and C_m and wind tunnel measured C_N and C_m for various contribution of angle attack and tail deflections. This study is useful in two ways for the purpose of parameter estimation from flight data. Firstly, if we need to use ML method (for

which priori fix of model is necessary), we have a desirable form which is also amenable for parameter extraction purpose. Secondly, if we use neural method, where in no postulation of model is necessary. Such an exercise helps to select the variables to be considered in the input vector to be fed to neural model for training.

Feed Forward Neural Networks

Feed forward neural networks are composed of group of neurons that are arranged into an input layer, an output layer, and one or more hidden layers. The number of nodes (neurons) in the input and output layers are determined, respectively, by the number of input and output variables, whereas the number of neurons in the hidden layers is decided by the complexity of the problem. Each neuron of a layer is connected to each neuron of the next layer, and each connection is assigned its individual connective weight. The neurons of the hidden and the output layers have a nonlinear activation function, which provides the networks the required nonlinear decision capability for modeling. For the purpose of longitudinal aerodynamic modeling of FFNN (Fig.1), the input variables to the network are the variables α, q, α^2, α_T^2, and the control input δ. The output variables are C_N and C_M. During the training sessions of the networks, the predicted values of the total coefficients C_N and C_M are compared with the corresponding known values. The difference between the predicted and known values of the total coefficients at each time point yield the error that are back-propagated using the method called the back-propagation algorithm (BPA). The BPA essentially treats error function as a function of networks weights, and uses an iterative descent gradient algorithm in the weight parameter space to minimize the error between the predicted and the known (desired) values of the output variables. The connective weights are updated during every iterative step. Out of the two most popular BPA algorithm: 1) the batch or sweep and 2) the sequential or pattern learning, we have employed the latter wherein the network weights are updated sequentially as training data are presented. More detail about FFNNs and BPA algorithm are available in the open literature [22,35,36].

A brief study was conducted to determine whether the network should be trained to map the network input variables to all two network output variables (C_N and C_M) at a time. Because the option of one output variables at a time resulted better training, it was adopted for all of the studies reported herein. Criterion of termination of the iterative

process for training was based on computing mean square error (MSE), defmed as:

$$MSE = \frac{1}{m \times n} \sum_{j=1}^{n} \sum_{j=1}^{m} \left[Y_i(j) - X_i(j) \right]^2 \qquad (24)$$

where Y and X are, respectively, the desired (known) and the computed (predicted) outputs of the neural networks; n is the number of data points; m is the number of the output variables. If only one output variables C_N or C_M is to be trained, then MSE is defined with $m = 1$. Training sessions are continued until changes in MSE in the successive iteration are less than the prescribed value or the number of iteration exceeds the specified number.

A detail study was carried out for a few sets of simulated data to understand the influence of various network parameters (also called the influencing or tuning parameters) on the training and prediction capability of the network. A matrix of tuning parameters such as the number of hidden layers, the number of nodes in each of the hidden layer(s), the learning rate, the momentum rate, the logistic gain factor of the sigmoidal function, the initial network weights, and the scaling of input-output data was generated, where in each parameters were varied within a prescribed range [19, 20, 21] and the network was trained to arrive at the best possible set that led to minimum MSE for the given flight data. The final set of tuning parameters is chosen is as follows: number of hidden layers = 1, number of neuron in the hidden layer = 5, learning rate = 0.3, momentum rate = 0.5, logistic gain = 0.85 and number of iteration = 1000. After the training, the same input data are passed to check the prediction capability of the network. The predicted aerodynamic coefficients are deemed acceptable only if the MSE is less than the specified value. Once the network is trained satisfactorily, to map network input variables (α, q, α^2, α_T^2, and δ) to each of the network output variables (C_N and C_M), it is then used for estimation of longitudinal stability and control derivatives by using the Delta methods described next.

Delta and Modified Delta Method

Delta Method

The Delta method [23] is based on the understanding of what a stability/control derivatives stands for; the stability/control derivatives represent the variation in the aerodynamic force or moment coefficients caused by a small variation in one of the motion/control variables

about the nominal value, whereas all of the other variables are held constant. For example, C_{N_α} represents a variation in C_N with respect to α, whereas all other variables q, α^2, α_T^2, and δ are held constant. Let us consider an estimation of C_{N_α} via the Delta method. For this purpose, the neural network is first trained such that the network input variables α, q, α^2, α_T^2, and δ are mapped to C_N. Next, a modified network input file is prepared wherein α values at each time point are perturbed by $\pm \Delta \alpha$ while all of the other variables retain their original values. This modified file is how presented to the trained network and the corresponding predicted values of the perturbed C_N (C_N^+ for $\alpha + \Delta \alpha$ and C_N^- for $\alpha - \Delta \alpha$) are obtained at the output node. Now, the stability derivative C_{N_α} is given by $C_{N_\alpha} = (C_N^+ - C_N^-)/2 \Delta \alpha$. Similarly perturbing only, say δ, in the network input file will yield the control derivative C_{N_δ}. Perturbations were given in both increasing (+) and decreasing (-) directions to avoid any bias resulting from one-sided differencing.

Modified Delta Method

In application of the Delta method, it was observed that the estimated parameters showed large spread in their numerical values. It was conjectured that a less than perfect match between the actual and the predicted values of aerodynamic coefficients is primarily responsible for observed spread in the estimated values. In an attempt to improve the training, it was decided to keep differential variations $\Delta \alpha$, $\Delta \alpha^2$, Δq, $\Delta \delta$, and $\Delta \alpha_T^2$ in the input vector to be fed to FFNN. The differential variations ΔC_N or ΔC_M consisted of the output vector of the FFNN. Fig.2 schematically represent the training strategy used in Modified Delta method using FFNN. A detail study was carried out for a few sets of simulated data to understand the influence of various network parameters on training and prediction capability. The network was trained to arrive it best possible set that led to minimum MSE for given flight data. The final set of tuning parameters with proposed training scheme, is as follow: number of hidden layer = 1, number of neuron in the hidden layer = 5, learning rate = 0.35, momentum rate = 0.4, logistic gain = 0.8 and the number of iteration = 1200. After the training, the same input data are passed to check the prediction

capability of the network. The predicted ΔC_N or ΔC_M are deemed acceptable only if the MSE is less than a specified value. Once the network is trained satisfactorily, to map network input variables $(\Delta \alpha, \Delta \alpha^2, \Delta q, \Delta \delta,$ and $\Delta \alpha_T^2)$ to each of the network output variables $(\Delta C_N$ or $\Delta C_M)$, it is then used for estimation of longitudinal stability and control derivatives by using the Modified Delta method.

The proposed modified Delta method is based on interpreting the stability and control derivatives as follows: If we could obtain variation in the value of an aerodynamic coefficient due to variation in only one of the motion/control variables while the variation in other motion/control variables are identically zero, then the ratio of the variation of the aerodynamic coefficients to variation of the non-zero motion/control variable will yield the corresponding stability/control derivative. Let us say that the FFNN is trained to map the network input variables, $\Delta \alpha, \Delta q,$ and $\Delta \delta$ to the network output variable (variation), ΔC_N. Now one input (say $\Delta\alpha$) variable at a time is chosen to be at its original value while the rest of the network inputs (Δq and $\Delta \delta$) are set to zero. The predicted value of the aerodynamic coefficient (ΔC_N) corresponding to such a modified file is divided by the non-zero variation in motion/control variable to yield the corresponding stability/control derivative, C_{N_α}. Similarly all the parameters can be estimated by using suitably modified input files.

Generation of Simulated Flight Data

For the purpose of parameter estimation, flight data simulating longitudinal dynamics were generated. The six- degree-of-freedom equations of motion [7] in a body-fixed axes system were first reduced to three-degree-of-freedom equations of motion pertaining to the longitudinal dynamics and then solved using the fourth-order Runge-Kutta method to generate simulated flight data for various tail input. The following equations of motion have been used:

$$\dot{u} = (\bar{q} S_{ref}/M) C_x(\alpha, \delta) - qw - g \sin\theta + (T/M) \quad (25)$$

$$\dot{w} = -(\bar{q} S_{ref}/M) C_N(\alpha, \delta) + qu + g \cos\theta \quad (26)$$

$$\dot{q} = (\bar{q} S_{ref} d) C_M(\alpha, \delta q)/I_y \quad (27)$$

$$\dot{q} = q \quad (28)$$

$$\dot{h} = u \sin\theta - w \cos\theta \quad (29)$$

where

$$\alpha = \tan^{-1}(w/u), \ \bar{q} = (1/1) \rho V^2, \text{ and } V = (u^2 + v^2 + w^2)^{1/2}.$$

The numerical value of C_x at a corresponding values of α and δ was interpolated from the wind tunnel generated data (Fig. 4). The damping derivative (C_{m_q}) was computed using analytical expression as given below:

$$C_{m_q} = 2 C_1 \left[\left((X_{cg} - X_{cp_L})/d \right)^2 \right] + 2 \alpha_1 \left[\left((X_{cg} - X_{cp_T})/d \right)^2 \right] \quad (30)$$

The pitching moment coefficient was modified by including the pitch damping effect with the following expression:

$$C_M = C_m(\alpha, \delta) + C_{m_q} qd/2V \quad (31)$$

The example missile having $(T/W)_{boost} = 8.234$, $(T/W)_{sustain} = 1.314$, $M/d^2 = 1289.6 kg/m^2$, $I_x/I_y = 0.0219$ was chosen for generating simulated flight data.

A typical trajectory of the missile in motion is pictorially presented in Fig. 5. Referring Fig. 5, it can be observed that the missile has initial boost phase with high rate of tum (upto point a), after reaching the predetermined height, it maneuvers to a cruise or sustainer phase (a-b). The maneuver from launch upto point 'a' (Fig. 5), will be referred to as Design maneuver phase, where as the flight path from (a to b) will be referred to as sustainer phase for future reference. A typical variation of motion variables, α, q,
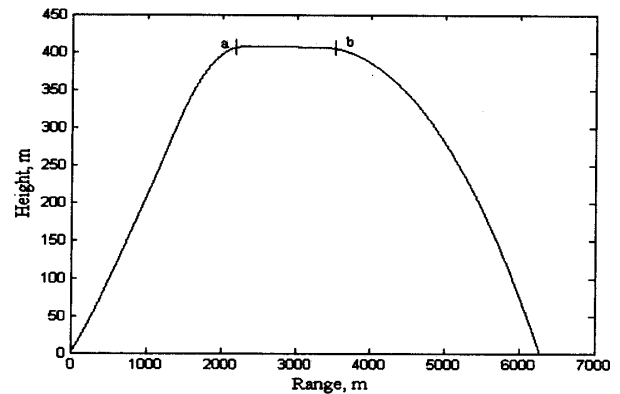

Fig.5 Typical trajectory of the missile

$\dot{q}$, a_z, V and δ are presented in Fig. 6. For the purpose parameter estimation, flight data corresponding to design maneuver was initially selected. During the design maneuver, the control input followed a pre-determined series of deflections. Further, the motion variables also showed appreciable variations as compared to sustainer phase in both magnitude and sign. To investigate the effect of control input forms on the estimated parameters, it was further decided to simulate flight trajectory by exciting the missile with multi-step control input during the sustainer phase. It may be appreciated that, such excitation may always not be possible in real situation. The flight data corresponding to this excitation at sustainer phase is presented in Fig. 7. Referring the Fig. 7, it can be observed that the multi-step control input of magnitude of around 0.1 rad could excite the missile appreciablely. The maximum angle of attack, pitch rate and acceleration were restricted to 0.174 rad, 2 rad/sec and 2g respectively.
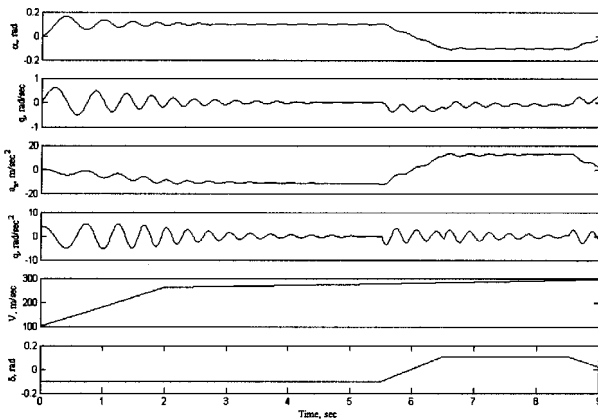

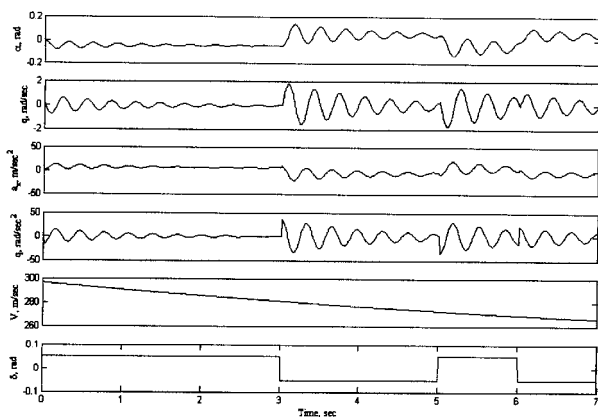*Fig.6 Flight data for design maneuver control input*

Fig.7 Flight data for 3-2-1-1 control input during the sustainer phase

For the purpose of parameter estimation, the flight data given in Fig. 6 and Fig. 7 were used. It may be clarified here that in the case of real flight data, since the true values of the parameters are not known, C_N and C_M is to be calculated from the measured values of a_z and $\dot{q}$ using the following relations:

$$C_N = -2\,Ma_z/\rho V^2 S_{ref} \tag{32}$$

$$C_M = -2\,\dot{q}I_z/\rho V^2 S_{ref}d \tag{33}$$

However, if numerical values of a_z are not reliable, an alternative method of computing C_N using time rate of change of α can be followed. The time rate of change of α may be computed by numerical differentiation of α. Using this numerical value of $\dot{\alpha}$, C_N can be computed as follows:

$$C_N = -\left(2\,M/\rho\,V^2 S_{ref}\right)(\dot{\alpha}-q) \tag{34}$$

It may be clarified here that the application of the delta or MD method do not required even an order of magnitude information about the parameters as is the case for conventional method, say the maximum likelihood estimator, where such information is required in the form of initial values. Network input-output files for several types of control inputs were separately used to train the FFNN and the parameters estimated via delta and MD method. Further, ML method were also used to estimate aerodynamic parameters using the motion (α, q, and a_z) of the same flight data used for FFNN modeling. The main fmdings are illustrated by presenting results obtained by using flight data corresponding to following cases:

Case 1: Flight data correspond to multi-step 3-2-1-1 type tail input during sustainer phase.

Case 2: Flight data correspond to pre-defined tail input during Design maneuver.

Case 3: Control input as for Case 1, but various level of noise (1%, 5%, and 10%) added to motion variables α, q, a_z, and $\dot{q}$.

Case 4: Control input as for Case 2, but various level of noise (1%, 5%, and 10%) added to motion variables α, q, a_z, and $\dot{q}$.

Result and Discussions

For the purpose of aerodynamic model identification, using wind tunnel data, the wind tunnel generated longitudinal forces and moments for body alone and body with tail configurations were considered separately. Maximum likelihood method was applied to estimate the aerodynamic parameters by minimizing the error between the assumed model (Eqs. 14 and 15) of estimated C_N and C_m and wind tunnel measured C_N and C_m for various combinations of angle of attack and tail deflections.

For body alone configuration, the parameters namely, and C_{N_α}, C_{m_α}, $C_{N_\alpha^2}$, and $C_{m_\alpha^2}$ were estimated. During the process of estimation, the numerical values of the parameters, C_{N_δ}, C_{m_δ}, $C_{N_{a_T}^2}$, and $C_{m_{a_T}^2}$, were set to zero. The estimated parameters were then compared with the parameters obtained using approximate expressions as given in Eqs. (7), (8), (20), and (21). Column 2 and 3 of Table-1 list approximate and estimated values of the parameters for body alone configuration respectively.

A fairly close match reflects confidence in assuming the chosen form of the model for subsequent parameter estimation purpose through flight data using ML method. Similar exercise was carried out with data obtained using wind tunnel model configuration data for body with tail configuration. Maximum likelihood method was applied to estimate the aerodynamic parameters, C_{N_α}, C_{m_α}, $C_{N_\alpha^2}$, $C_{m_\alpha^2}$, C_{N_δ}, C_{m_δ}, $C_{N_{a_T}^2}$, and $C_{m_{a_T}^2}$. Column 4 and 5 of Table-1 present the comparison between the numerical values obtained through approximate and ML method. A reasonable match among the parameters suggests acceptable aerodynamic structure for the purpose of estimation of trim aerodynamics of the missile at moderate angle of attack. Further, the numerical values of these parameters obtained by both the methods were used in Eqs. (14) and (15) to compute variation of C_N and C_m with respect to angle of attack and tail deflections. Fig.8 presents a comparison between estimated values of C_N and C_m with the wind tunnel generated C_N and C_m for both the configurations (body alone and body plus tail). As expected it can be seen that the estimated values of C_N and C_m lie very close to wind tunnel generated C_N and C_m. The computed values of C_N and C_m obtained using approximated method

| Table-1 : Parameters obtained using approximate method and ML method on wind tunnel model configuration data				
	Body-alone		Body with fin	
Parameter	Approximate method	ML	Approximate method	ML
C_{N_α}	1.840	1.881 (0.016)	4.815	4.223 (0.034)
$C_{N_\alpha^2}$	8.188	7.575 (0.103)	8.188	5.186 (0.110)
C_{N_δ}	-	-	1.994	1.129 (0.013)
$C_{N_{a_T}^2}$	-	-	-1.316	-0.970 (0.091)
C_{m_α}	7.539	5.859 (0.016)	-7.555	-7.821 (0.041)
$C_{m_\alpha^2}$	0.116	0.110 (0.012)	0.116	0.132 (0.002)
C_{m_δ}	-		-7.023	-7.470 (0.096)
$C_{m_{a_T}^2}$	-		4.635	6.031 (0.152)
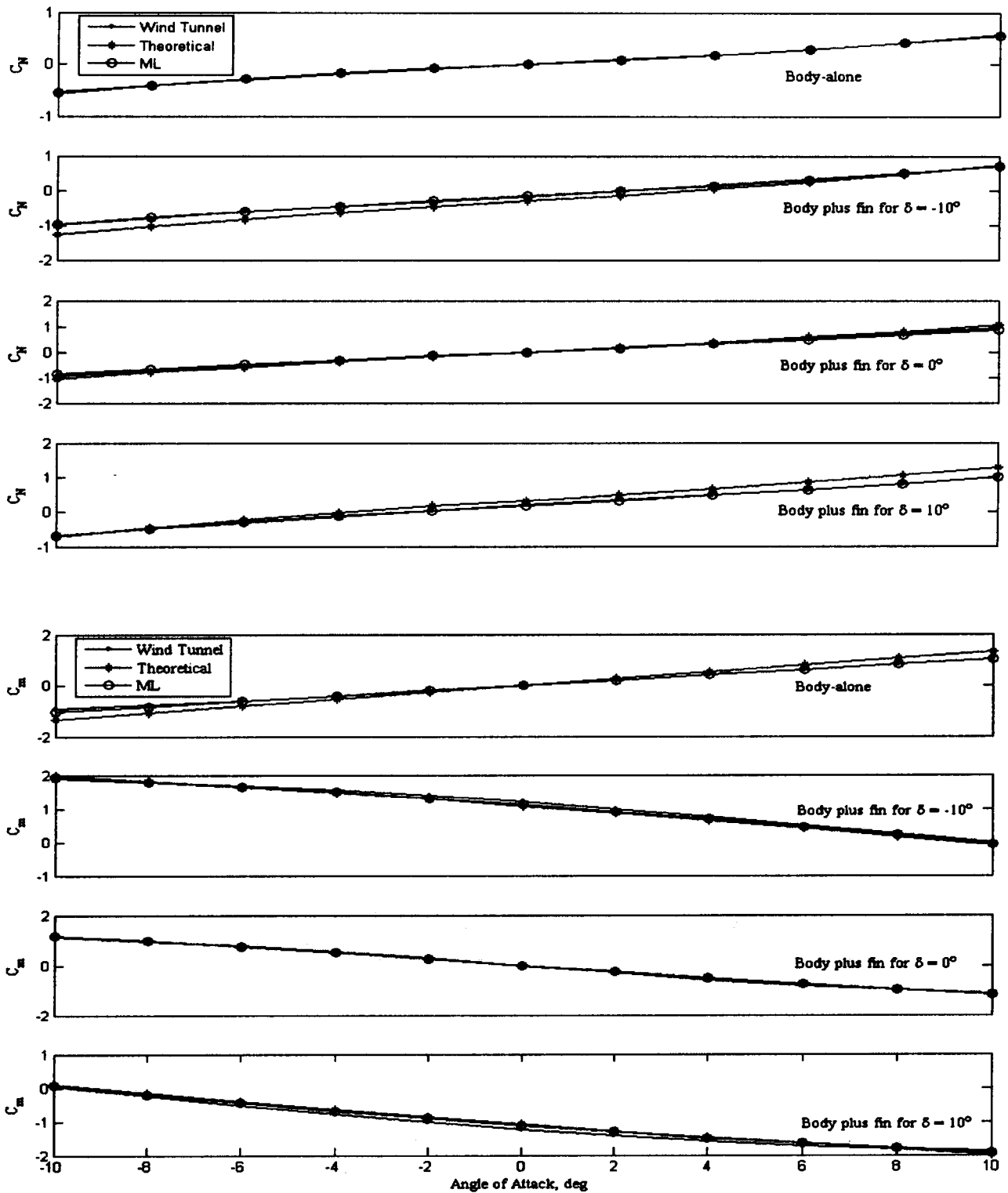
Fig.8 Comparison of normal force and pitching moment coefficients for body-alone and body plus fin configurations

also lies reasonably close to the wind tunnel generated C_N and C_m. This study is useful in two ways. Firstly, if we need to use ML method (for which a priori fix to the model is necessary), we have a convenient form of the aerodynamic model which is also amenable to parameter extraction purpose. Secondly, if we use neural method, where no postulation of model is necessary, such an exercise helps to select the input variables to consider in the input vector to be fed to FFNN model for training. This approach too some extent demystify the statement that neural model does not require, any knowledge of the form of the aerodynamic model for parameter estimation purpose.

The applicability of the Delta and MD method is tested using simulated flight data. For this purpose, the simulated flight data pertaining to longitudinal dynamics corresponding to design maneuver and sustainer phase (Fig. 6 and 7) were considered. A typical configuration of the missile shown in Fig. 2, having $(T/W)_{boost} = 8.234$, $(T/W)_{sustain} = 1.314$, $M/d^2 = 1289.6 kg/m^2$, $I_x/I_y = 0.0219$ was chosen as the example missile for this study. The numerical values of $C_N(\alpha, \delta)$ and $C_m(\alpha, \delta)$ required as input in solving equation of motion (Eqs. 4 and 5) were the wind tunnel identified C_N and C_m at the corresponding α and δ. The wind tunnel estimated values of the parameters C_{N_α}, C_{m_α}, $C_{N_\alpha^2}$, $C_{m_\alpha^2}$, C_{N_δ}, C_{m_δ}, $C_{N_{a_T}^2}$, and $C_{m_{a_T}^2}$ will be referred to as true values of the longitudinal aerodynamic parameters for the purpose of generation of simulated flight data of the chosen example missile. During the application of methods based on FFNN training network input-output files for the two chosen form of control inputs were separately used to train the FFNN and the parameters were estimated via the Delta and the modified Delta method. Further, ML method was also used to estimate aerodynamic parameters using the flight data containing information about α, q and a_z. The main findings are illustrated by presenting results for case 1 to 4 in the subsequent paragraph.

For Case 1, the flight data were generated by exciting the missile with multi-step 3-2-1-1 control input during the sustainer phase. The flight data containing the information about the motion variables are presented in Fig. 7. Column 3 and 4 of Table-2 list the numerical values of the

estimated parameters obtained using the Delta and the Modified Delta (MD) method.

It can be seen that the even the derivatives namely $C_{N_\alpha^2}$, $C_{m_\alpha^2}$, $C_{N_{a_T}^2}$, and $C_{m_{a_T}^2}$ were also well estimated with lower values of standard deviations. It is interesting to observe that in general, estimates obtained using the MD method have lower values of standard deviation as compared to estimates obtained through the Delta method. For the sake of comparison, the column 5 lists numerical values of estimates obtained using the ML method. The lower values of Cramer-Rao bound also suggest higher degree of confidence. For case 1, with no noise, it is observed that the Delta and MD method can advantageously be applied to estimate aerodynamic parameter using the flight data generated by exciting the missile at sustainer phase using a multi step 3-2-1-1 control input. However, such an excitation of the missile at sustainer phase may not always be possible. Therefore, investigations were further progressed to assess the possibility of estimating parameters using the flight data corresponding to design maneuver phase. Case 2, uses flight data corresponding to design maneuver phase as presented in Fig. 6. Columns 6, 7 and 8 of Table-2 list the numerical values of the estimated parameters obtained using the Delta, the MD and the ML method respectively. The parameters, C_{N_α}, C_{m_α}, C_{m_q}, C_{N_δ}, and C_{m_δ} were well estimated by the Delta, the Modified Delta and the ML method. However, there was marginal deterioration in the numerical values of the estimated parameters, namely $C_{N_\alpha^2}$, $C_{m_\alpha^2}$, $C_{N_{a_T}^2}$, and $C_{m_{a_T}^2}$. It is interesting to note that the estimated parameters including $C_{N_{a_T}^2}$ and $C_{m_{a_T}^2}$ obtained using the MD method showed least deterioration. To study the effect of measurement noise on parameter estimates, simulated pseudo noise of varying intensities were added to the simulated flight data. The noise was simulated by generating successively uncorrelated pseudo random numbers having a normal distribution with zero mean and an assigned standard deviation correspondingly approximately to a designated percentage (1%, 5% etc) of the maximum amplitude of the motion variables α, q, a_z, and $\dot{q}$ etc. Table-3 lists the estimated parameters obtained for Case 3 and 4. Column 3, 4 and 5 of Table-3 list the estimated parameters obtained by applying the Delta, MD and ML method on flight data (with 5% noise) generated using multi step 3-2-1-1-input. In general, as expected, there was marginal deterioration in the estimated parameters for this case also. Although most of the parameters

Table-2 : Estimated parameters obtained using 3-2-1-1 control input and design maneuver control input

Parameter	True	Case 1			Case 2		
		Noise = 0%					
		Delta	MD	ML	Delta	MD	ML
C_{N_α}	4.223	4.225 $(0.003)^+$	4.223 $(0.001)^+$	4.223 $(0.039)*$	4.229 (0.051)	4.223 (0.001)	4.203 (0.06)
$C_{N_\alpha}{}^2$	5.186	5.187 (0.021)	5.184 (0.010)	5.185 (0.015)	5.199 (0.033)	5.179 (0.007)	5.343 (0.225)
C_{N_δ}	1.129	1.128 (0.015)	1.29 (0.0001)	1.129 (0.017)	1.119 (0.053)	1.128 (0.002)	1.130 (0.046)
$C_{N_{a_T}}^2$	-0.97	-0.975 (0.014)	-0.9701 (0.011)	-0.968 (0.026)	-0.956 (0.012)	-0.9612 (0.031)	-1.136 (0.104)
C_{m_α}	-7.821	-7.801 (0.002)	-7.820 (0.004)	-7.820 (0.026)	-7.801 (0.002)	-7.829 (0.072)	-7.763 (0.029)
$C_{m_\alpha}{}^2$	0.132	0.139 (0.06)	0.129 (0.019)	0.132 (0.025)	0.141 (0.065)	0.142 (0.031)	-0.131 (0.163)
C_{m_δ}	-7.470	-7.44 (0.011)	-7.420 (0.521)	-7.469 (0.045)	-7.42 (0.032)	-7.445) (0.185)	-7.448 (0.064)
$C_{m_{a_T}}^2$	6.031	6.011 (0.0311)	6.028 (0.013)	6.027 (0.011)	5.981 (0.091)	6.115 (0.042)	5.892 (0.035)
C_{m_q}	-109.77	-109.69 (0.035)	-109.71 (0.019)	-109.77 (0.053)	-109.66 (0.091)	-109.82 (0.112)	-109.98 (0.012)

+ Sample standard deviation;   * Cramer Rao bound

were well estimated by both the Delta and the MD method. It is interesting to note that for Case 3, for noise level of 5%, unlike the ML method, the Delta and the MD methods could estimate the parameter $C_{m_\alpha{}^2}$ with correct sign and reasonable magnitude. Further, the numerical value of $C_{m_{a_T}{}^2}$ obtained using the MD method appears to have least square both in terms of magnitude and standard deviation when compared with the estimates obtained using Delta and ML method for the same flight data.

Similar observations, on the numerical values of the estimates could be extended to Case 4 also. Flight Data generated with not so efficient control input (design maneuver), could be processed by the Delta and the Modified Delta method to yield reasonably accurate values of the parameters. To see how good the estimated values are, we compared the estimated C_N and C_M, obtained by substituting the estimated values in the right hand side of Eq. (14) and (31), with the true C_N and C_M being analyzed. Fig. 9 compares true C_N and C_M, the estimated C_N and C_M (via the Delta method), the estimated C_N and C_M (via the MD method), and the estimated C_N and C_M (via the ML method). It is interesting to observe from Fig. 9 that the estimated C_N and C_M via the MD method shows a better fit with the true C_N and C_M then does the estimated C_N and C_M via the Delta method. Similar observation could be made while referring Fig. 10 representing variation of estimated C_N and C_M obtained by all the three methods using sustainer phase (with 3-2-1-1 control input) flight data. This may be due to better FFNN training obtained by replacing α, α^2, q, δ, and $\alpha\,{}^2_T$ by their variations ($\Delta\alpha$, $\Delta\alpha^2$, Δq, $\Delta\delta$, and $\Delta\,\alpha^2_T$) in the input vector used for neural training.

Table-3 : Estimated parameters obtained using 3-2-1-1 control input and design maneuver control input							
		Noise = 5%					
		Case 3			Case 4		
Parameter	True	Delta	MD	ML	Delta	MD	ML
C_{N_α}	4.223	4.205	4.211	4.260	4.021	3.981	4.617
		$(0.032)^+$	$(0.035)^+$	$(0.039)*$	(0.133)	(0.352)	(0.076)
$C_{N_\alpha}2$	5.186	5.531	5.417	4.90	6.122	5.811	8.141
		(0.084)	(0.052)	(0.152)	(0.723)	(0.111)	(0.418
C_{N_δ}	1.129	1.043	1.132	1.111	0.991	0.961	1.094
		(0.077)	(0.012)	(0.017)	(0.089)	(0.029)	(0.054)
$C_{N_{a_T}}^2$	-0.97	-1.121	-0.833	-0.775	-1.501	-0.751	2.232
		(0.068)	(0.055)	(0.027)	(0.662)	(0.231)	(1.12)
C_{m_α}	-7.821	-8.103	-7.242	-7.699	-7.314	-7.082	-7.644
		(0.119)	(0.089)	(0.026)	(0.310)	(0.515	(0.028)
$C_{m_\alpha}2$	0.132	0.441	0.337	0.148	1.922	0.0991	-2.294
		(0.095)	(0.068)	(0.022)	(0.472)	(0.307)	(0.280)
C_{m_δ}	-7.47	-7.922	-6.862	-7.405	-8.603	-6.679	-6.596
		(0.182)	(0.655)	(0.044)	(0.911)	(0.119)	(0.050)
$C_{m_{a_T}}^2$	6.031	5.839	6.581	6.251	8.371	7.720	11.173
		(0.205	(0.340)	(0.110)	(0.551)	(0.295)	(0.066)
C_{m_q}	-109.77	-108.11	-107.89	-109.01	-104.12	-102.40	-95.869
		(0.443	(0.489)	(0.536)	(0.780)	(0.102)	(1.218)
+ Sample standard deviation; * Cramer Rao bound							

Conclusion

In the present study, an attempt has been made to estimate aerodynamic parameters from flight data of a typical tactical missile. To avoid any explicit requirement of postulation of aerodynamic model, the Delta and the Modified Delta methods were applied to estimate aerodynamic parameters. In application of the Delta method, it was observed that the estimated parameters showed large spread in their numerical values. To reduce such spread in the numerical values of the estimates, a new approach has been proposed by modifying the Delta method. The Modified Delta method uses variations in motion and control variables to train the FFNN. It is observed that the Modified Delta method can advantageously be applied on flight data of a typical tactical missile to estimate aerodynamic parameters.

References

1. Hamel, P. G., "Aircraft Parameter Identification Methods and their Applications Survey and Future Aspects", AGARD, 15-104, Nov. 1979, Paper 1.

2. Maine, R. E. and Iliff, K. W., "Identification of Dynamic System - Application to Aircraft-Part 1: Output Error Approach", AGARD AG-300, Vol. 3, Part 1, Dec. 1986.

3. Klein, V., "Estimation of Aircraft Aerodynamic Parameter from Flight Data", Progress in Aerospace Sciences, Vol. 26, 1989, pp. 1-77.

4. Maine, K. E. and Iliff, K. W., "Application of Parameter Estimation to Aircraft Stability and Control", NASA RP 1168, Jan. 1986.
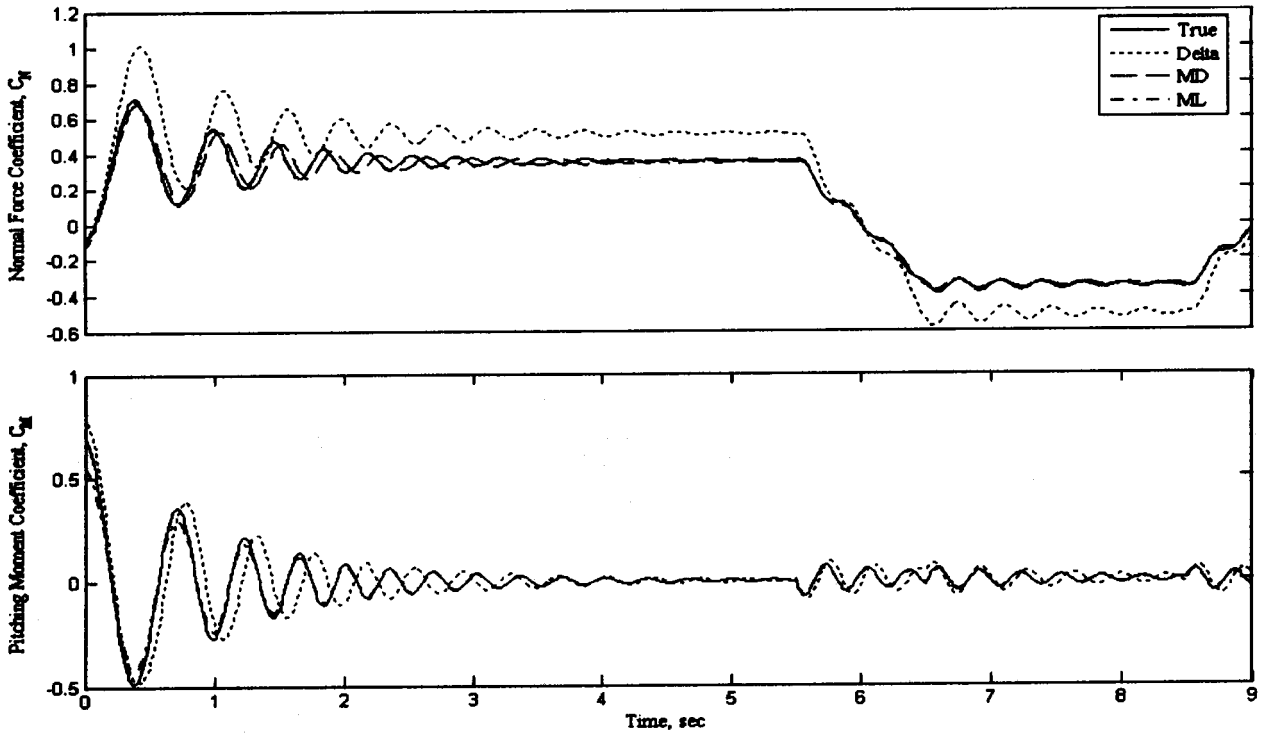
Fig.9 Comparison of estimated and true normal force and pitching moment coefficients for design maneuver control input with noise = 5%
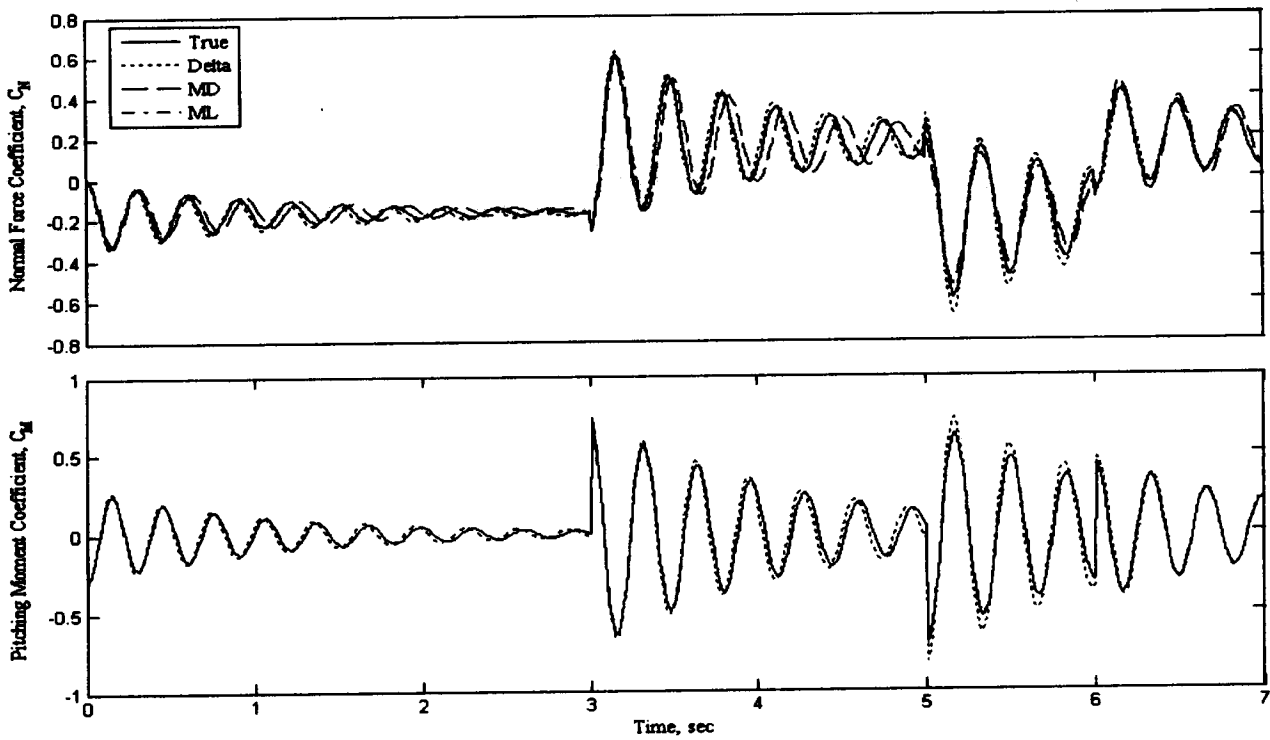

Fig.10 Comparison of estimated and true normal force and pitching moment coefficients for 3-2-1-1 control input with noise = 5%

5. Moore, F. G., Approximate Methods for Weapon Aerodynamics, Progress in Astronautics and Aeronautics, Apr. 2000. Chaps. 2, 5.

6. Moore, F. G. and McInville, R. M., "A New Method for Calculating Wing Alone Aerodynamics to Angle of Attack 180", Naval Surface Warfare Center, Dahlgren Div. Rept. NSWCDD / TR-94/3, Mar. 1994.

7. Jategaonkar, R., "Identification of the Aerodynamic Model of the DLR Research Aircraft ATTAS from Flight Test Data", DLR, DLR-FB 90-40, Braunschweig, Germany, Sept. 1990.

8. Iliff, K. W., "Parameter Estimation for Flight Vehicle", Journal of Guidance, Control and Dynamics, Vol. 12, No.5, 1989, pp. 609-622.

9. Hamel, P. G. and Jategaonkar, R. V., "The Evolution of Flight Vehicle System Identification", AGARD, DLRGermany, 8-10, May, 1995

10. Fan, Y. and Lutze, F. H., "Identification of an Unsteady Aerodynamic Model at High Angle of Attack", AIAA-96-3407-CP.

11. Moore, F. G., " Body Alone Aerodynamics of Guided and Unguided Projectiles at Subsonic, and Transonic and Supersonic Mach Numbers", Naval Weapons Lab., Now Naval Surface Warfare Center, Dahlgren Div., Rept. NWL TR-2976, Dahlgren, V A, Nov. 1972.

12. Moore, F. G., "Aerodynamics of Guided and Unguided Weapons: Part 1- Theory and Application", Naval Weapons Lab., Now Naval Surface Warfare Center, Dahlgren Diy., Rept. NWL TR-3018, Dahlgren, V A, Dec. 1973.

13. Fan Y., "Identification of an Unsteady Aerodynamic Model up to High Angle of Attack Regime", Ph.D. Dissertation, Aerospace Engineering Dept., Virginia Polytechnic Institute and State Univ., Blacksburg, Virginia, Nov. 1997.

14. Etkin, B., Dynamic of Atmospheric Flight, John Wiley and Sons, Inc. New York, 1972.

15. Etkin, B., Dynamic of Flight Stability and Control, John Wiley and Sons, Inc. New York, 1982.

16. Chambers, J. R. and Grafton, S.B., "Investigation of Lateral-directional Dynamic Stability of a Tilt-wing V/STOL Transport", NASA TND-5637, 1970.

17. Seckel, E. and Morris, J. J., "The Stability Derivatives of the Navion Aircraft Estimated by Various Methods and Derived From Flight Test Data", Rep. No. FAA-RD-71-6, Jan. 1971.

18. Haykins, S., "Neural Networks - A Comprehensive Foundation", McMaster University, Macmillan College Publishing Company, New York, 1994.

19. Hess, R. A., "On the Use of Back Propagation with Feed Forward Neural Networks for the Aerodynamic Estimation Problem", AIAA Paper 93-3639, Aug. 1993.

20. Basappa. and Jategaonkar, R. V., "Aspects of Feed Forward Neural Network Modeling and its Application to Lateral-Directional Flight-Data", DLR IB-111-95/30, Sep. 1995.

21. Linse, D. J. and Stengel, R. F., "Identification of Aerodynamic Coefficients using Computational Neural Networks", Journal of Guidance, Control and Dynamics, Vol. 16, No.6, Nov- Dec. 1993, pp. 1018-1025.

22. Youseff, H. M., "Estimation of Aerodynamic Coefficients using Neural Networks", ALAA Paper 93-3639, Aug. 1993.

23. Raisinghani, S. C., Ghosh, A. K. and Kalra, P. K., "Two New Techniques for Aircraft Parameter Estimation Using Neural Networks", The Aeronautical Journal, Jan., 1998, Vol. 102, No. 1011, pp. 25-29.

24. Jategaonkar, R. V. and Thielecke, F. , "Evolution of Parameter Estimation Methods for Unstable Aircraft", Journal of Aircraft, Vol. 31, No.3, 1994, pp. 510-519.

25. Zerweckh, S.H. , Von Flotow, A.H. and Murray, J. E., "Flight Testing of Highly Flexible Aircraft", Vol.27, No.4, 1990, pp. 342-349.

26. Waszak, M.R. and Schmidt, D. K. , "Flight Dynamics of Aeroelastic Vehicles", Journal of Aircraft, Vol. 25, No. 6, 1988.

27. Mehra, R. K., Stepner, D.E. and Taylor, J.S., "A Generalized Method for the Identification of Aircraft Stability and Control Derivatives from Flight Test Data", Proceedings of 13th Joint Automatic Control Conference of American Automatic Control Councol, PO52 A 72-38260.

28. Rao, C. S., "Effect of Measurement Errors on Parameter Estimation Via Neural Networks", M. Tech. Dissertation, Aerospace Engineering Department, Indian Institute of Technology, Kanpur, India, 1998.

29. Ghosh, A.K., Sagoo, G. and Singhal, A., "Wind Tunnel Investigation of Typical Strategic Missile Configuration at High Angle of Attack", Journal of Aerospace Sciences and Technologies, Vol. 58, No.1, Feb. 2006, pp. 75-81.

30. Pitts, W. C., Nielsen, J. N. and Kaattari, G. E., "Lift and Center of Pressure of Wing Body Tail Combinations at Subsonic, Transonic and Supersonic Speeds", NACA TN-1307, 1957.

31. The Engineering Design Handbook, American Material Command, AMCP, 706-280.

32. Allen, J. H. and Perkins, E. W., "Characteristics of Flow Over Inclined Bodies of Revolution", NACA RMA 50L07, 1965.

33. Jorgensen, L.H., "Prediction of Static Aerodynamic Characteristics for Slender Bodies Alone and With Surfaces to Very High Angle of Attack", NASA TRR-474, Sept. 1977.

34. Ericsson, L. E. and Reeling, J. P., "Asymmetric Vortex Shedding for Bodies of Revolution", Tactical Missile Aerodynamics, edited by M. J. Hemsch and J. N. Nielsen, Vol. 104, AIAA Progress in Astronautics and Aeronautics Series, AIAA, New York, 1986, pp. 243-296.

35. Chen, S. and Cowan, C.F.N., "Parallel Recursive Prediction Error Algorithm for Training Layered Networks", Int. J. Control, 1990, Vol. %0, No.6, pp. 1225-1228.

36. Hornik, K., Stinchcombe, M. and White, H., "Multi Layer Feed Forward Neural Networks are Universal Approximator", Neural Networks, Vol. 2, No.5, 1989 pp. 359-366.