ON THE VIBRATIONS AND THERMAL BUCKLING OF FUNCTIONALLY GRADED SPHERICAL CAPS

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Abstract

Here, the axisymmetric free flexural vibrations and thermal buckling characteristics of functionally graded spherical caps are investigated employing a three-noded axisymmetric curved shell element based on field consistency approach. The formulation is based on first-order shear deformation theory and it includes the in-plane and rotary inertia effects. The material properties are graded in the thickness direction according to the power-law distribution in terms of volume fractions of the constituents of the material. The effective material properties are evaluated using homogenization method. A detailed numerical study is carried out to bring out the effects of shell geometries, power law index of functional graded material and base radius-to-thickness on the vibrations and buckling characteristics of spherical shells.

Keywords *: Functionally graded, Vibration, Spherical shell, Thermal buckling, Power law index*

Introduction

The demand for improved structural efficiency in space structures and nuclear reactors has resulted in development of a new class of materials, called Functionally graded materials [1-3] (FGMs). FGMs are microscopically inhomogeneous, in which the material properties vary smoothly and continuously from one surface of the material to the other surface and thus, distinguish FGMs from conventional composite materials. Typically, these materials are made from a mixture of ceramic and metal, or a combination of different materials. Further, varying the properties in FGMs in a continuous manner is achieved by gradually changing the volume fraction of the constituent materials. The advantages of using these materials are that they are able to withstand high-temperature gradient environment while maintaining their structural integrity, and they avoid the interface problem that exists in homogeneous composites. Furthermore, a mixture of ceramic and metal with a continuously varying volume fraction can be easily manufactured [4-6]. Although these materials are initially designed as thermal barrier materials for aerospace structural applications and fusion reactors, they are now employed for general use as structural elements for different applications [7]. For example, a common structural element for such applications is the rectangular plate, for which several recent studies on static buckling, vibration and dynamic behaviors have been performed [8-12].

Among various structural elements, shell elements form an important class of structural components with many significant engineering applications such as vessels or vessel's enclosures. Studies pertaining to FGMs shell structures are mainly limited to thermal stress, deformation, and fracture analysis in the literature [13-19]. Makino et al. [13] Obata and Noda [14], and Takezono et al. [15] have investigated thermal stress of FGM shells whereas the discs and rotors have been examined based on analytical approach by Durodola and Adlington [16], and Oh et al [17]. The elasto-plastics deformation of FGM shell is studied in the work of Dao et al. [18], and Weisenbek et al [19]. Few transient dynamic analyses of cracked FGM structural components are also reported in the literature [20-22]. Li et al. [20,21] have analyzed the stress intensity factor of FGMs under dynamic situation whereas Zhang et al. [22] studied the dynamic responses of cracked FGM structural components. The parametric instability analysis of functionally graded cylindrical shells under harmonic axial loading has been carried out [23 and 24]. However, to the author's knowledge, it must be stressed that work

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on the vibrations and stability behavior of functionally graded material spherical shells is not commonly yet available in the literature, and such study is immensely useful to the designers while optimizing the designs of FGMs spherical shell structures.

In the present work, a three-noded shear flexible axisymmetric curved shell element developed based on the field-consistency principle [25,26] is employed to analyze the axisymmetric vibration and thermal buckling characteristics of functionally graded spherical caps. The material properties are graded in the thickness direction according to the power-law distribution in terms of volume fractions of the material constituents. The present formulation is validated considering isotropic case for which solutions are available. Numerical results are presented considering different values for geometrical parameter, power law index, and boundary conditions on the axisymmetric vibration and thermal stability behavior of functionally graded spherical caps.

Formulation

An axisymmetric functionally graded shell of revolution (radius a, thickness h) made of a mixture of ceramics and metals is considered with the coordinates s, θ and z along the meridional, circumferential and radial/thickness directions, respectively as shown in Fig.1. The materials in outer ($z = h/2$) and inner ($z = -h/2$) surfaces of the spherical shell are ceramic and metal, respectively. The locally effective material properties are evaluated using homogenization method that is based on the Mori-Tanaka scheme [27, 28]. The effective bulk modulus K and shear modulus G of the functionally gradient material evaluated using the Mori -Tanaka estimates [27-29] are as

$$\frac{K-K_m}{K_c-K_m} = V_c \Big/ \left[1+(1-V_c)\frac{3\,(K_c-K_m)}{3\,K_m+4\,G_m} \right] \tag{1}$$

$$\frac{G-G_m}{G_c-G_m} = V_c \Big/ \left[1+(1-V_c)\frac{(G_c-G_m)}{G_m+f_1} \right] \tag{2}$$

where,

$$f_1 = \frac{G_m\,(9\,K_m+8\,G_m)}{6\,(K_m+2\,G_m)}$$

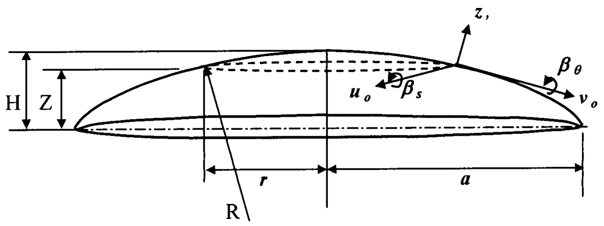

Fig.1a Geometry and the co-ordinate system of a spherical cap

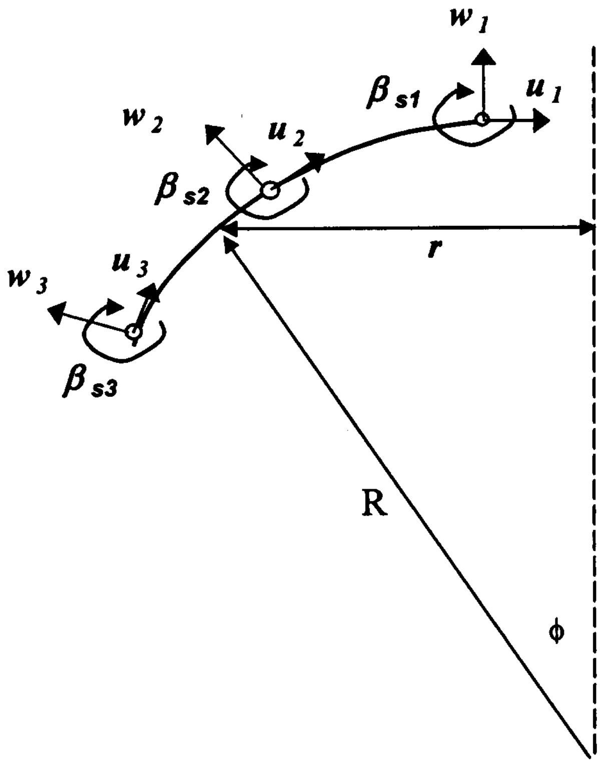

Fig.1b Curved axisymmetric quadratic shell element

Here, V is volume fraction of phase material. The subscripts c and m refer the ceramic and metal phases, respectively. The volume-fractions of ceramic and metal phases are related by $V_c + V_m = 1$, and V_c is expressed as

$$V_c(z) = \left(\frac{2\,z+h}{2h}\right)^k \tag{3}$$

where k is the volume fraction exponent ($k \geq 0$).

The effective values of Young's modulus E and Poisson's ratio υ can be found as from

$$E(z) = \frac{9KG}{3K+G} \quad \text{and} \quad \upsilon(z) = \frac{3K-2K}{2(3K+G)} \tag{4}$$

The locally effective heat conductivity coefficient κ is given as [Ref.30].

$$\frac{K-K_m}{K_c-K_m} = V_c \Big/ \left[1+(1-V_c)\frac{(K_c-K_m)}{3K_m} \right] \tag{5}$$

The coefficient of thermal expansion α is determined in terms of the correspondence relation [Ref.31].

$$\frac{\alpha-\alpha_m}{\alpha_c-\alpha_m} - \left(\frac{1}{K}-\frac{1}{K_m}\right) \Big/ \left(\frac{1}{K_c}-\frac{1}{K_m}\right) \tag{6}$$

The effective mass density ρ can be given by rule of mixture as [Ref. 32].

$$\rho(z) = \rho_c V_c + \rho_m V_m \tag{7}$$

The temperature variation is assumed to occur in the thickness direction only and the temperature field is considered constant in the *xy* plane. In such a case, the temperature distribution along the thickness can be obtained by solving a steady-state heat transfer equation

$$-\frac{d}{dz}\left[\kappa(z)\frac{dT}{dz}\right] = 0 \ , \quad T = T_c \ \ at \ \ z = h/2 \ ;$$

$$T = T_m \ \ at \ \ z = -h/2 \tag{8}$$

The solution of this boundary value problem provides the temperature distribution through the thickness of the plate [33].

By using the Mindlin formulation, the displacements at a point (s, θ, z) are expressed as functions of the mid-plane displacements u_o, v_o and w, and independent rotations β_s, and β_θ of the radial and hoop sections, respectively, as

$$u(s,\theta,z,t) = u_o(s,\theta,t) + z\,\beta_s(s,\theta,t)$$

$$v(s,\theta,z,t) = v_o(s,\theta,t) + z\,\beta_\theta(s,\theta,t)$$

$$w(s,\theta,z,t) = w(s,\theta,t) \tag{9}$$

where t is the time. The various strain components such as the membrane strains $\left\{\varepsilon_p^L\right\}$, bending strains $\left\{\varepsilon_b\right\}$, shear strains $\left\{\varepsilon_s\right\}$ are written as [34].

$$\left\{\varepsilon_p^L\right\} = \begin{Bmatrix} \dfrac{\partial u_o}{\partial s} + \dfrac{w_o}{R} \\[2ex] \dfrac{u_o \sin\phi}{r} + \dfrac{w\cos\phi}{r} \\[2ex] -\dfrac{v_o \sin\phi}{r} + \dfrac{\partial v_o}{\partial s} \end{Bmatrix};$$

$$\left\{\varepsilon_b\right\} = \begin{Bmatrix} \dfrac{\partial \beta_s}{\partial s} + \dfrac{\partial u_o}{R\,\partial s} \\[2ex] \dfrac{\beta_s \sin\phi}{r} + \dfrac{u_o \sin\phi}{R\,r} \\[2ex] \dfrac{\partial v_o}{\partial s}\dfrac{\cos\phi}{r} + \dfrac{\partial \beta_\theta}{\partial s} - \dfrac{\beta_\theta \sin\phi}{r} \end{Bmatrix};$$

$$\left\{\varepsilon_s\right\} = \begin{Bmatrix} \beta_s + \dfrac{\partial w_o}{\partial s} \\[2ex] \beta_\theta \pm \dfrac{v_o \cos\phi}{r} \end{Bmatrix} \tag{10}$$

where r, R and ϕ are the radius of the parallel circle, radius of the meridional circle and angle made by the tangent at any point in the middle-surface of the shell with the axis of revolution.

If $\{\boldsymbol{N}\}$ represents the stress resultants (N_{ss}, $N_{\theta\theta}$, $N_{s\theta}$) and $\{\boldsymbol{M}\}$ the moment resultants (M_{ss}, $M_{\theta\theta}$, $M_{s\theta}$), one can relate these to membrane strains $\left\{\varepsilon_p\right\}$ and bending strains $\left\{\varepsilon_b\right\}$ through the constitutive relations as

$$\left\{N\right\} = \begin{Bmatrix} N_{ss} \\ N_{\theta\theta} \\ N_{s\theta} \end{Bmatrix} = [A_{ij}]\left\{\varepsilon_p\right\} + [B_{ij}]\left\{\varepsilon_b\right\} - \left\{N^T\right\} \tag{11}$$

$$\{M\} = \begin{Bmatrix} M_{ss} \\ M_{\theta\theta} \\ M_{s\theta} \end{Bmatrix} = [B_{ij}]\{\varepsilon_p\} + [D_{ij}]\{\varepsilon_b\} - \{M^T\} \tag{12}$$

where the matrices $[A_{ij}]$, $[B_{ij}]$ and $[D_{ij}]$ $(i, j = 1, 2, 6)$ are the extensional, bending-extensional coupling and bending stiffness coefficients and are defined as $[A_{ij}, B_{ij}, D_{ij}] = \int_{-h/2}^{h/2} [\overline{Q}_{ij}]\,(1, z, z^2)\, dz$. The thermal stress resultant $\{N^T\}$ and moment resultant $\{M^T\}$ are

$$\{N^T\} = \begin{Bmatrix} N^T_{ss} \\ N^T_{\theta\theta} \\ N^T_{s\theta} \end{Bmatrix} = \int_{-h/2}^{h/2} [\overline{Q}_{ij}] \begin{Bmatrix} \alpha(z) \\ \alpha(z) \\ 0 \end{Bmatrix} \Delta T(z)\, dz \tag{13}$$

$$\{M^T\} = \begin{Bmatrix} M^T_{ss} \\ M^T_{\theta\theta} \\ M^T_{s\theta} \end{Bmatrix} = \int_{-h/2}^{h/2} [\overline{Q}_{ij}] \begin{Bmatrix} \alpha(z) \\ \alpha(z) \\ 0 \end{Bmatrix} \Delta T(z)\; z\, dz \tag{14}$$

where the thermal coefficient of expansion $\alpha(z)$ is given by Eq. (6), and $\Delta T(z) = T(z) - T_0$ is temperature rise from the reference temperature T_0 at which there are no thermal strains.

Similarly the transverse shear force $\{Q\}$ representing the quantities $\{Q_{sz}, Q_{\theta z}\}$ is related to the transverse shear strains $\{\varepsilon_s\}$ through the constitutive relations as

$$\{Q\} = [E_{ij}]\{\varepsilon_s\},$$

$$\text{where } E_{ij} = \int_{-h/2}^{h/2} \left[\overline{Q}_{ij}\right] \kappa_i \kappa_j\, dz \tag{15}$$

Here $[E_{ij}]$ $(i, j = 4, 5)$ are the transverse shear stiffness coefficients, κ_i is the transverse shear coefficient for non-uniform shear strain distribution through the shell thickness. $\overline{Q}_{ij}$ are the stiffness coefficients and are defined as

$$\overline{Q}_{11} = \overline{Q}_{22} = \frac{E(z)}{1-\nu^2};\ \overline{Q}_{12} = \frac{\nu E(z)}{1-\nu^2};\ \overline{Q}_{16} = \overline{Q}_{26} = 0;$$

$$\overline{Q}_{44} = \overline{Q}_{55} = \overline{Q}_{66} = \frac{E(z)}{2(1+\nu)} \tag{16}$$

where the modulus of elasticity $E(z)$ is given by Eq.(4).

The strain energy functional U is given as

$$U(\delta) = (1/2)\int_A \Big[\{\varepsilon_p\}^T [A_{ij}]\{\varepsilon_p\} + \{\varepsilon_p\}^T [B_{ij}]\{\varepsilon_b\} + \{\varepsilon_b\}^T [B_{ij}]\{\varepsilon_p\} + \{\varepsilon_b\}^T [D_{ij}]\{\varepsilon_b\} + \{\varepsilon_s\}^T [E_{ij}]\{\varepsilon_s\}$$

$$\left[-\{\varepsilon^L_p\}^T \{N^T\} - \{\varepsilon_b\}^T \{M^T\} \right] dA \tag{17-a}$$

$$U(\delta) = (1/2)\ \{\delta\}^T [K]\ \{\delta\} \tag{17-b}$$

where δ is the vector of the degree of freedom associated to the displacement field in a finite element discretisation and [K] is element linear stiffness matrix.

The kinetic energy of the shell is given by

$$T(\delta) = (1/2)\int_A \left[p\left(\dot{u}_0^2 + \dot{v}_0^2 + \dot{w}_0^2\right) + I\left(\dot{\beta}_s^2 + \dot{\beta}_\theta^2\right)\right] dA \tag{18-a}$$

$$T(\delta) = (1/2)\ \{\delta\}^T [M]\ \{\delta\} \tag{18-b}$$

where $p = \int_{-h/2}^{h/2} \rho(z)\, dz$, $I = \int_{-h/2}^{h/2} z^2(z)\, dz$ and $\rho(z)$ and $\rho(z)$ is mass density which varies through the thickness of the spherical shell and is given by Eq. (7), [M] is the mass matrix. The dot over the variable denotes derivative with respect to time.

The shell is subjected to temperature filed and this, in turn, results in-plane stress resultants (N^{th}_{xx}, N^{th}_{yy}, N^{th}_{xy}). Thus, the potential energy due to thermal pre-buckling stresses (N^{th}_{xx}, N^{th}_{yy}, N^{th}_{xy}) developed under thermal load can be written as

$$V(\delta) = \int_A \left\{ \frac{1}{2}\left[N^{th}_{xx}\left(\frac{\partial w}{\partial x}\right)^2 + N^{th}_{yy}\left(\frac{\partial w}{\partial y}\right)^2 + 2N^{th}_{yy}\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right)\right]\right.$$

$$+\frac{h^2}{24}\left[N^{th}_{xx}\left\{\left(\frac{\partial \theta_x}{\partial x}\right)^2 + \left(\frac{\partial \theta_y}{\partial x}\right)^2\right\} + N^{th}_{yy}\left\{\left(\frac{\partial \theta_x}{\partial y}\right)^2 + \left(\frac{\partial \theta_y}{\partial y}\right)^2\right\}\right.$$

$$\left.\left.\left\{\left[2N^{th}_{xy}\left\{\left(\frac{\partial \theta_x}{\partial x}\right)\left(\frac{\partial \theta_x}{\partial y}\right) + \left\{\left(\frac{\partial \theta_y}{\partial x}\right)\left(\frac{\partial \theta_y}{\partial y}\right)\right\}\right]\right]\right\} dA \qquad (19)$$

By minimization of total potential energy obtained from Eqs. (17-b) and (19), the governing equations are derived for thermal stability case as [35].

$$([\mathbf{K}] + \Delta T [\mathbf{K_G}]^{th})\{\delta\} = \{\mathbf{o}\} \qquad (20)$$

Here, $[\mathbf{K}_G]^{th}$ is the geometric stiffness due to thermal loads and ΔT ($=T_c - T_m$) is the critical temperature difference, respectively.

Similarly, substituting Eqs. (17-b) and (18-b) in Lagrange's equation of motion, the governing equation for the free vibration case is obtained as [35]

$$[M]\{\ddot{\delta}\} + [\mathbf{K}]\{\delta\} = \{0\} \qquad (21)$$

where $\{\ddot{\delta}\}$ is the acceleration vector.

For free vibration case, assuming harmonic vibration, $\{\ddot{\delta}\} = -\omega^2\{\delta\}$, Eq. (21) leads to

$$([\mathbf{K}])\{\delta\} - \omega^2 [M]\{\delta\} = \{\mathbf{o}\} \qquad (22)$$

where ω is the natural frequency.

The frequency and the critical temperature difference can be calculated using standard eigenvalue extraction algorithm.

Element Description

The axisymmetric three-noded curved shell element used here is a C^0 continuous shear flexible one and has 5 nodal degrees of freedoms. If the interpolation functions for three-noded element are used directly to interpolate the five field variables u_o , v_o , w_o , β_s and β_θ in deriving the transverse shear and membrane strains, the element will lock and show oscillations in the shear and membrane stresses. Field consistency requires that the membrane and transverse shear strains must be interpolated in a consistent manner. Thus, β_s term in the expression for $\{\varepsilon_s\}$ given in Eq. (10) has to be consistent with field function $\frac{\partial w}{\partial s}$ as shown in the works of Prathap and Ramesh Babu [25]. Similarly the w and (u_o, v_o) terms in the expression of $\{\varepsilon^L_p\}$ given in Eq. (10) have to be consistent with the field functions $\frac{\partial u_o}{\partial s}$ and $\frac{\partial v_o}{\partial s}$, respectively. This is achieved by using the field redistributed substitute shape functions to interpolate those specific terms that must be consistent as described in Refs. [25] and [26]. The element derived in this fashion behaves very well for both thick and thin situations, and permits the greater flexibility in the choice of integration order for the energy terms. It has good convergence and has no spurious rigid modes.

Results and Discussion

In this section, we use the above formulation to investigate the effect of parameters like gradient index, shell geometrical parameter on the axisymmetric free flexural vibration characteristics and thermal buckling of functionally graded material spherical caps. Since the finite element used in this study is based on field consistency approach, an exact integration is employed to evaluate all the strain energy terms. The shear correction factor, which is required in a first order theory to account for the variation of transverse shear stresses, is taken as 5/6. For the present study based on progressive mesh refinement, a 15-element idealization is found to be adequate in modeling the spherical caps.

Figure 2a shows the typical variation of the volume fractions of ceramic in the thickness direction z for the FGM spherical cap. The outer surface is ceramic rich and

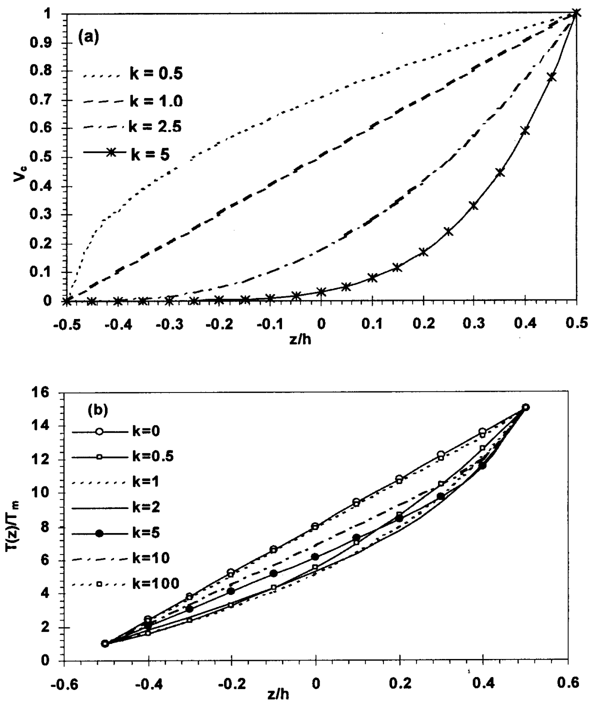

Fig.2 Variation of volume fraction of ceramic and temperature through thickness : (a) Volume fraction of ceramic; (b) Temperature

the inner surface is metal rich. The typical temperature variation through the thickness direction is presented in Fig. 2b and it can be noted that the temperature variation in the thickness of functionally graded shell is nonlinear compared to those of pure ceramic and metal cases (k=0 and k=100). The FGM spherical shell considered here consists of aluminum and alumina [36]. The Young's modulus, conductivity and the coefficient of thermal expansion for alumina is E_c = 380 Gpa, K_c = 10.4 W/mK, $\alpha_c = 7.4 \times 10^{-6}$ 1/°C, and for aluminium is E_m =700 Gpa, K_m = 204 W/mK, $\alpha_m = 23 \times 10^{-6}$ 1/°C, respectively. The shell is of uniform thickness and boundary conditions considered here are :

Simply supported :

$$u = v = w = 0 \quad \text{on} \quad r = a$$

Clamped support :

$$u = v = w = \beta_s = \beta_\theta = 0 \quad \text{on} \quad r = a\,.$$

Before proceeding for the free flexural vibration characteristic study of FG spherical cap, the formulation developed herein is simplified for pure metallic case

Table-1a : Comparison of fundamental frequency for isotropic shallow spherical shell

$\omega\,(a/h)\,(\rho a^2/E)^{1/2}$

H/h	Fundamental frequency	
	Ref. [37]	Present
2	6.14	6.42
5	13.08	13.37

Table-1b : Comparison of critical buckling thermal strain ε_T for isotropic hemi-spherical shell

h/R	Critical buckling thermal strain		
	Ref. [38]	Present	% diff
0.01	0.00407	0.00424	-4.18%
0.02	0.00844	0.00840	0.47%
0.03	0.01204	0.01251	-3.90%
0.04	0.01624	0.01657	-2.03%
0.05	0.02034	0.02052	-0.88%
0.06	0.02379	0.02446	-2.82%
0.07	0.02764	0.02844	-2.89%
0.08	0.03164	0.03222	-1.83%
0.09	0.03530	0.03584	-1.53%
0.10	0.03927	0.03944	-0.43%

and validated against the available clamped isotropic spherical shells results pertaining to the free vibrations and thermal buckling cases in Table-1a and 1b respectively. Here, the nondimensional frequency Ω is defined as $\omega\,(a/h)\,(\rho a^2/E)^{1/2}$, where ρ and E are the mass density and Young's modulus of metal, respectively. The results are found to be in good agreement with the existing solutions [37,38].

Next, the detailed investigations for free flexural vibrations of spherical caps are carried out for different geometrical parameters and material power law index, k. Fig.3 highlights the non-dimensional fundamental frequencies of simply supported FGM spherical caps for different values of thickness-to-radius ratio, material power law index and different spherical angle. It is observed that the increase in material power law index value results in decrease in non-dimensional frequency value. This is attributed due to the stiffness reduction because of the increase in the metallic volumetric fraction and the

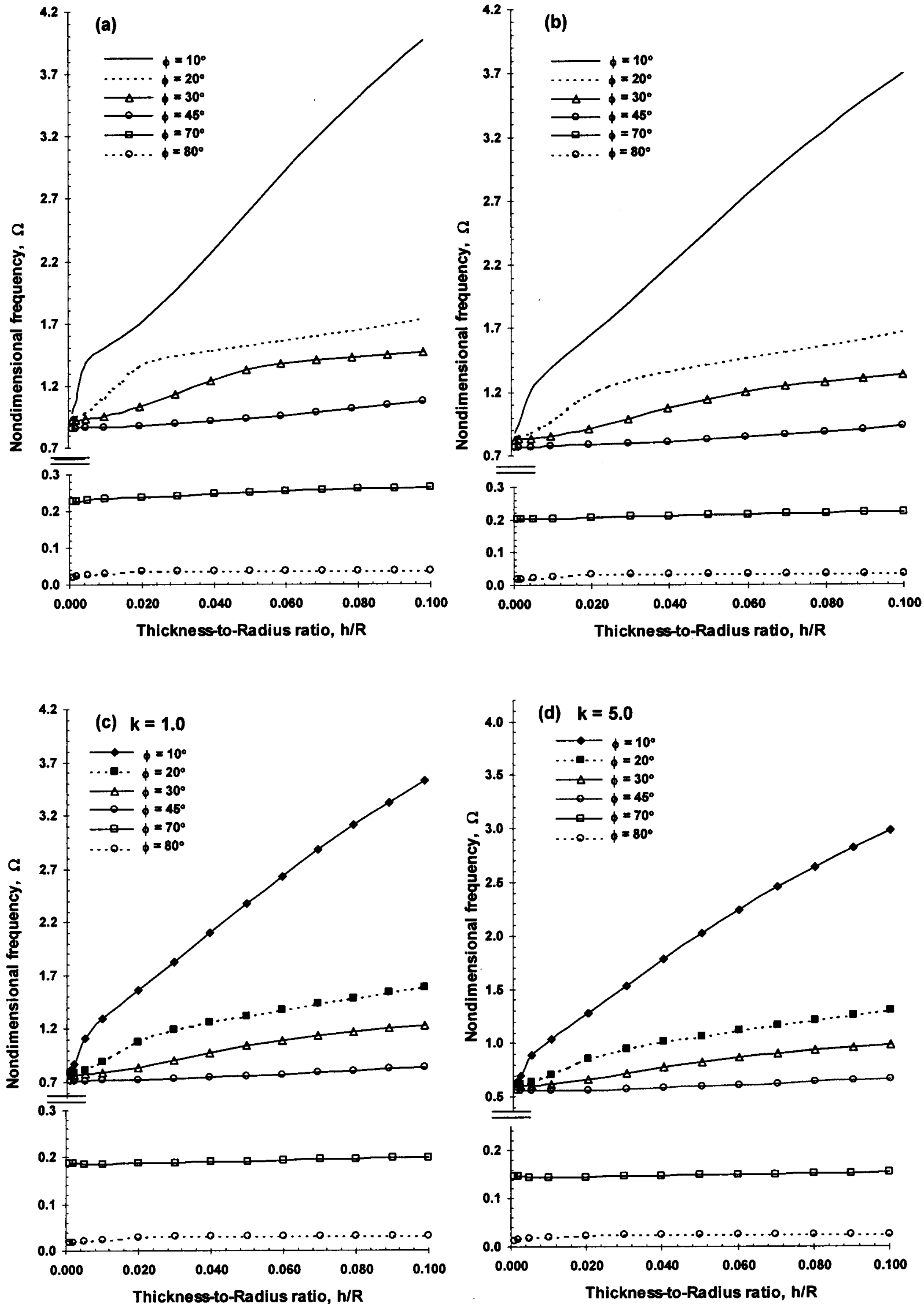

Fig.3 Variation of non-dimensional frequency Ω for a simply supported FG spherical shell for different gradient index

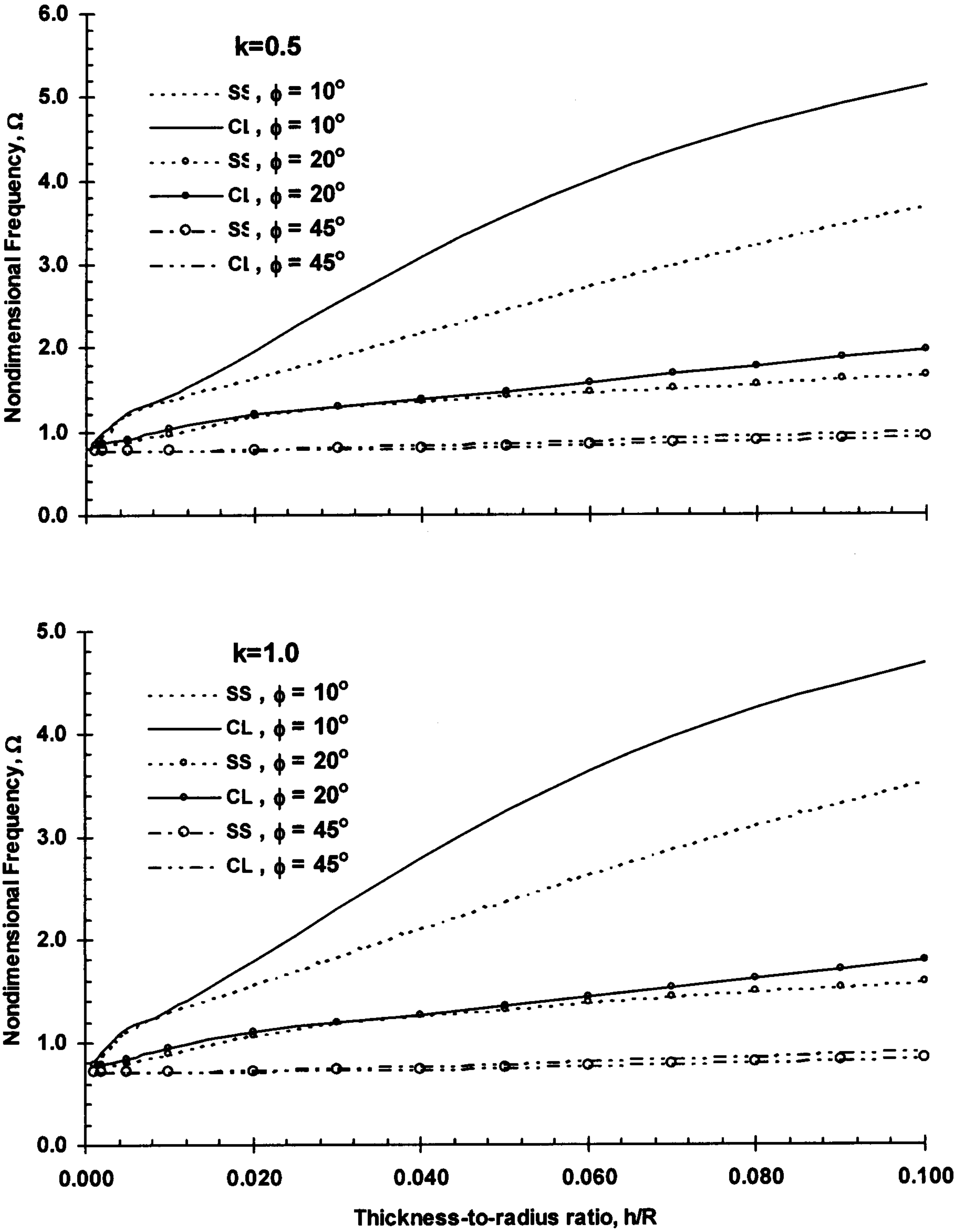

Fig.4 Comparison of non-dimensional frequency, Ω for simply supported and clamped FG spherical shells

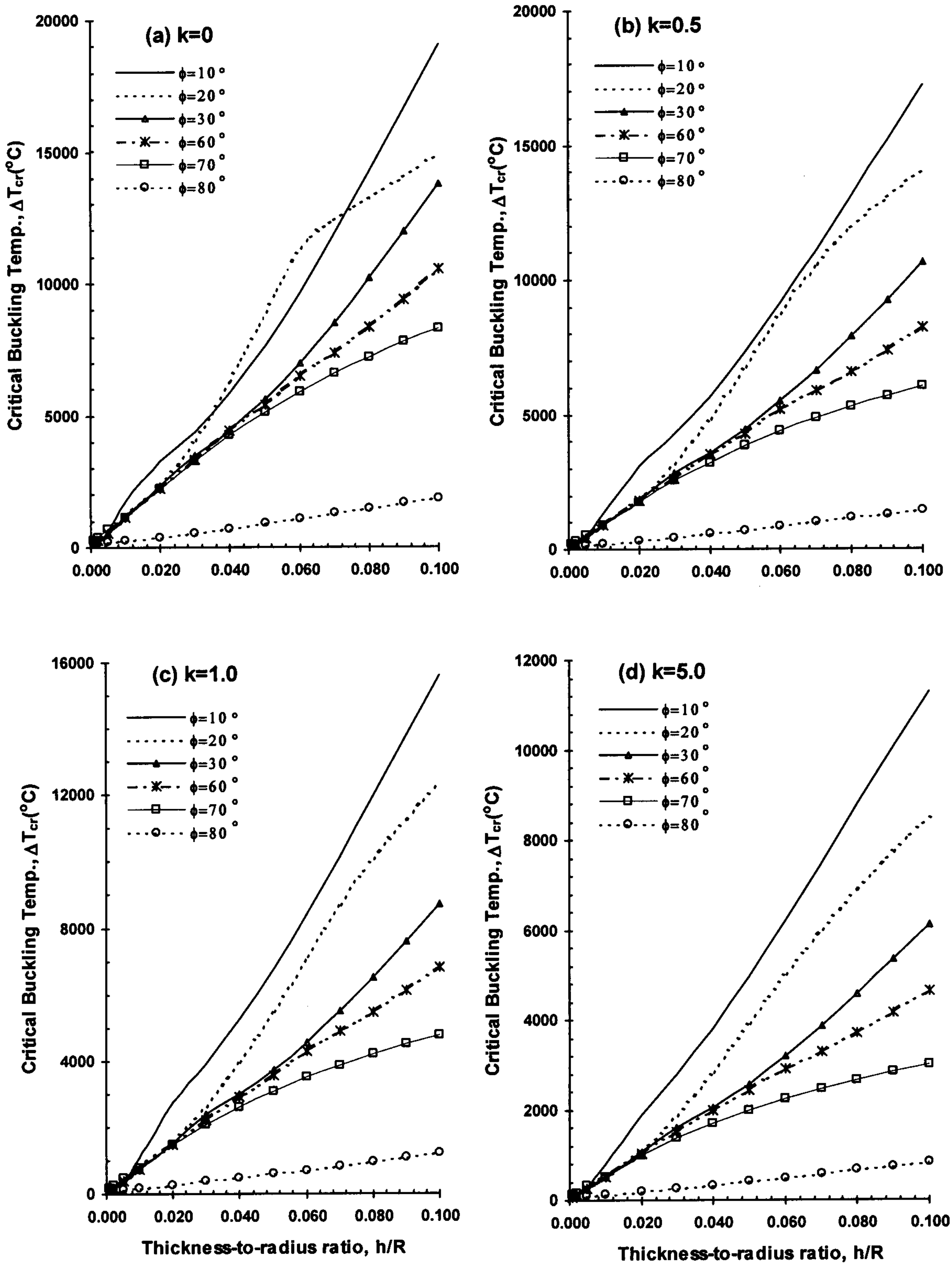

Fig.5 Variation of axisymmetric critical buckling temperature difference, ΔT_{cr} (°C) for a simply supported FG spherical shell

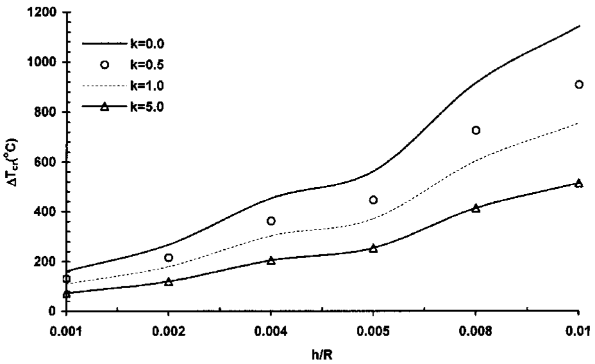

Fig.6 Variation of axisymmetric critical buckling temperature difference, ΔT_{cr} (°C) for a simply supported FG hemi-spherical shell

introduction of different stiffness couplings due to elastic properties variation through the thickness of FGM shell. It can also be opined that the frequency value increases with the increase in thickness-to-radius ratio. However, the rate of increase in non-dimensional frequency value is high for shallow spherical cases compared to deep shells. The effect of boundary conditions on frequency can be further viewed from Fig.4 for two values of material index. It is noticed from Fig.4 that the frequency values for clamped case is higher than those of simply supported shell, as expected and the difference in frequency values is with respect to thickness-to-radius however less for deep shells.

Similar analysis for the thermal buckling behavior of FGM spherical caps with simply supported boundary condition has been done considering different values of thickness-to-radius ratio and geometrical parameter. The results are plotted in Fig.5. It can be concluded that the influence of material power law index and spherical included angle on critical values is qualitatively similar to those of vibration case, i.e. reduction in thermal buckling temperature difference with increase in the values of material index and deepness of the shell. However, the critical buckling temperature difference is quite high for very shallow shells compared to those of deep cases. For moderately deep shell structures, the change in the buckling values is noticeable for higher values of thickness-to-radius ratio. The critical buckling temperature for ceramic is higher than the microscopically heterogeneous mixture of ceramic and metal, as expected and this is mainly due to the increase in metallic volumetric fraction. Furthermore, the results pertaining to FGM hemi-spherical shells are highlighted in Fig.6 and the observation is similar to those of other spherical caps.

Conclusion

Axisymmetric free flexural vibrations and thermal buckling characteristics of FGM spherical caps have been investigated using a three-noded axisymmetric curved shell element employing a field consistency approach. Numerical results obtained here for an isotropic case are found to be in good agreement with the previous findings. From the detailed parametric study, it is observed that the frequency and critical buckling temperature decrease with the increase in metallic volume fraction and spherical angle whereas they increase with the increase in thickness-to-radius ratio of the shells. It is hoped that this study will be useful for the designers while optimizing the FGM based spherical shell structures.

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