ROTOR DYNAMICS OF AIRCRAFT GAS TURBINE ENGINES

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Abstract

Ever since Sir Frank Whittle built the first jet engine for aircraft propulsion, rotor dynamics became the key issue in design and continues to be most vexing problem for the designers. The rotors are gradually replaced by drum construction and the bearing supports have become very flexible owing to thin engine casings mounted on the wings. The compressor and turbine rotors are no more coupled serially through couplings as in steam turbines but are mounted one over other as spools. Now, there are several critical speeds of multi spools excited by unbalances and misalignment. The disks mounted on the rotors carrying the blades have also become very flexible with several critical speeds excited by different per rev excitations from flow path and subjected to severe resonances during start up and shut down operations. Besides, these mounted parts have become globally elastic but locally plastic structures and their lifing has become an important design problem. This paper traces the developments taken place leading to optimum designs and life estimation of gas turbine rotating parts.

Introduction

Rankine [1] is the first one to remark on the existence of a critical speed of a rotor in 1869. He defined this as a limit of speed for centrifugal whirling. It was not clear whether a rotor can cross the critical speed.

Laval built the turbine named after him in 1883 and ran his rotor upto 40,000 rpm by using an extremely flexible shaft. By a force balance, he derived the equation for whirl radius y

$$y = \frac{\omega^2 \delta}{\frac{Fg}{W} - \omega^2} \qquad (1)$$

where

δ is the eccentricity
F is the stiffness
W is weight of the disk

He defined the critical speed when the denominator in equation (1) becomes zero and when the shaft speed is greater than this critical speed the denominator becomes negative. Then y approaches the value - δ, and the wheel rotates around its CG with great stability. This is the principle of the ingenious flexible shaft of Laval, see Kearton [2].

Jeffcott Rotor Models

Even with the general knowledge of critical speeds, the shaft behavior at any general speed was still unclear until Jeffcott [3] who in 1919 formulated the rotor problem as one of forced vibration. He showed for the first time that the shaft did not primarily rotate about its rest position, but about its own centerline; while a mass vibrates about its mean equilibrium position, the shaft spins at the driver speed about its center line at ω and whirls at frequency v (akin to free vibration) about the bearing center line. This is a significant development in the understanding of rotor dynamic behavior. Rotors modeled as a single disk on a flexible mass-less shaft are named after Jeffcott. They can be physical models or models made from modal analysis of actual distributed rotors using modal mass and stiffness. A rotor on rigid bearings can be first analyzed using a standard code and its modal properties can be used to define a Jeffcott rotor. The support properties from bearings/seals/casing/steam-whirl/cracks and other appropriate effects can then be included on this modal model.

A Jeffcott rotor on 8 coefficient bearings is shown in Fig.1. Rao [4] used such models extensively to study

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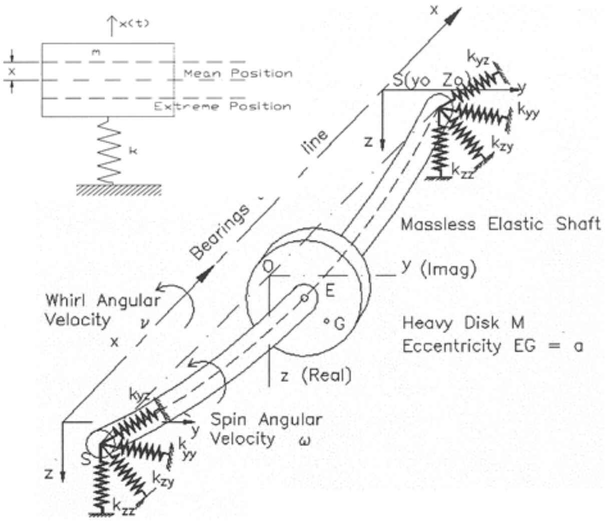

Fig.1 Jeffcott rotor model on 8 coefficient bearings

unbalance response, stability of practical rotors. The advantage of such an analysis is to gain physical understanding on the parametric variations through simple models before attempting a full solution of the rotor with the supports. Jeffcott rotor models also help us in understanding the behavior of shafts with asymmetric cross-section (the so-called gravity criticals), rotors with internal friction/ loose joints and the resulting instability, see [4] for a variety of problems dealing with such models. The effect of misalignment was also studied using two Jeffcott rotors [5].

Tabular Methods

Some of the earlier rotor failures belong to propeller shafts in torsion of steam driven war ships during I world war. The story goes thus: When a propeller shaft failed, it was felt that designers did not provide sufficient diameter of the shaft to take care of the transmitted torque, therefore its diameter was increased by 10%. The modified shaft however failed in less than half time. That is when the designers began taking rotor failures seriously to adopt dynamic design. It was found that by increasing the diameter of the shaft the natural frequency became closer to the excitation harmonic resulting in an earlier failure. Holzer [6] in 1921 presented a tabular method to determine the torsional natural frequencies of systems, which can be discretized in the form of several rigid inertias, connected by massless torsional springs. This is a simple method in which the inertia torque and torsional amplitude of each disk are calculated sequentially beginning from one end with a unit amplitude and arbitrarily chosen frequency. After completing the calculations till the end of the train, the boundary condition was checked - in this case the total inertia of the system in free vibration to be equal to zero. Obviously the first attempt will not yield the assumed frequency to be the correct natural frequency; therefore Holzer proposed an iteration method by varying frequency until a satisfactory answer is obtained. The method works well for a reasonable number of rotors in a simple table - it is so simple that it lasted into the computer age.

Determination of bending natural frequencies of practical beam type rotors has been a difficult issue since there were four state quantities as compared to torsional problem with only two state quantities with one of them assumed unity in a mode shape. It was left to Myklestad [7] who in 1944 extended Holzer's approach for bending vibrations of aircraft wing type structures. This involved in two tables to be set up and carry on tabular calculations beginning from one end of the beam. Prohl [8] adopted a similar tabular method in 1945 to determine the critical speeds of rotating shafts. This method has been used for a considerable period of time until it was replaced by transfer matrix forms for application by using computers. Lund (1930-2000) [9] developed some of the most widely used rotor bearing programs using Myklestad-Prohl approach.

Transfer Matrix Models

With the availability of computers by 1950's, both torsional and bending problems were initially expressed in Transfer Matrix form; see Pestel and Leckie [10] to speed up the calculation procedures. For torsion problems, matrices of the order 2x2 are involved and multiplication of these matrices is a simple matter. Another major advantage of transfer matrix method is in storage, the final matrix is also 2x2, thus needing very small memory. The initial main frames with 32 to 64 K RAM could handle rotors with as many as 200 or more stations. This eliminated a need to make a model with a small number of stations as in tabular methods. Eshleman [11] adopted a transfer matrix method for continuous systems thus eliminating a discretization process for the shafts. Rao et al. [12] adopted a transfer matrix method for torsional vibration under transient conditions of excitation such as electrical disturbances.

For bending problems of stationary beams without rotary inertia, the transfer matrices are merely 4x4 in size. Therefore, the transfer matrix methods have become very popular even till recently. They have been extensively adopted in solving rotor bending problems, see Rao [4].

Figure 2a shows i^{th} shaft element with bending in both the planes. The state quantities are shown in both the planes in Figs. 2b and 2c. The transfer matrices are built from one station to another station using the equilibrium relations.

For stationary beams, Rayleigh [13] pointed out the effect of rotary inertia of a disk when it is located away from any anti-nodal points. When the shaft rotates, the rotary inertia effect becomes a gyroscopic effect. The gyroscopic moment has a significant effect on the rotor dynamic behavior. Each natural frequency is split into two, one attributing to forward whirl and the second to backward whirl. The rotor whirls in a backward direction between these two critical speeds. The gyroscopic effect is explained by Den Hartog [14] by considering a cantilevered rotor. The gyroscopic moment is calculated separately and introduced as additional term in the Point transfer matrices.

For whirling rotors, we have to consider bending in two planes as given above when compared to stationary beams. Further, since the excitation from unbalance has both sine and cosine terms, we have to define four 4x4 size matrices and combine them to form general transfer matrices. For solving unbalance response of rotors, we have the excitation force and therefore, we are required to use 17x17 matrices. Fortunately, the overall transfer matrix still remains 17x17, thus making the transfer matrix most economical in the usage of memory; a plus point for a long time in rotor dynamic calculations. Many industries still use codes based on transfer matrices, because of simplicity and less CPU time.

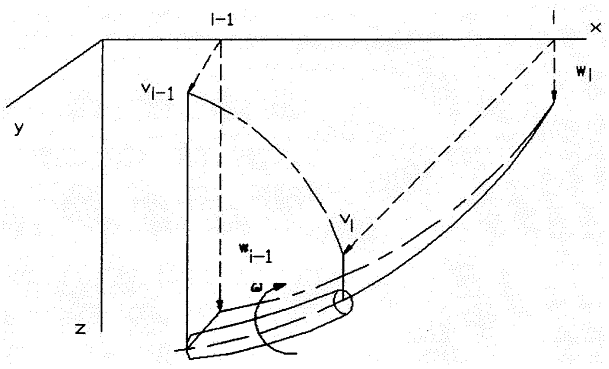

Fig.2a ith rotor shaft element

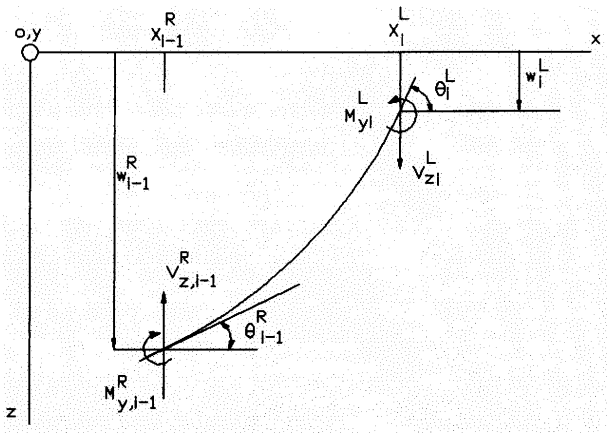

Fig.2b State quantities for tranfer matrices in plane xz

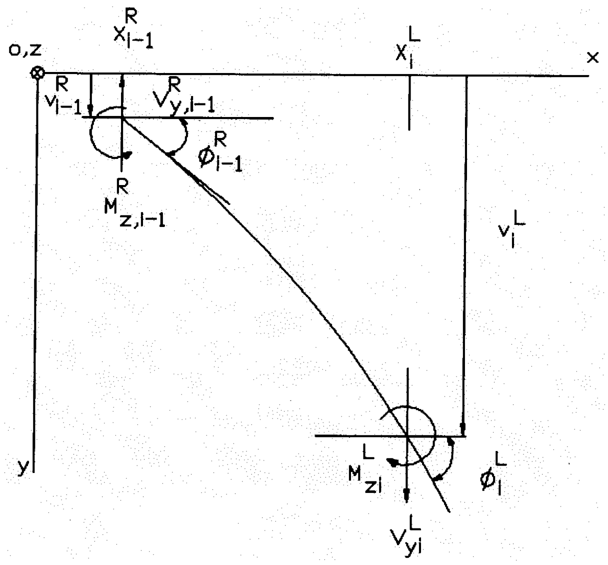

Fig.2c State quantities for transfer matrices in plane xy

Transfer matrices have been enhanced to account for twin spool rotor systems as adopted in modern aircraft engine power plants [15]. The matrix size becomes 33x33. The inter-shaft bearing conditions have to be derived for each specific case and therefore the transfer matrix method has a limitation in making the procedure general in nature.

Another serious limitation with transfer matrix method is to consider the gear mesh stiffness in a train. Gearmesh provides a coupling between bending and torsion in a shaft system. This makes the transfer matrix formulation complicated, particularly if there is more than one gear pair in action. Another limitation is to consider the effect of casing/foundation stiffness on the rotor dynamics. Modern day design of rotors couples all these motions and it appears that the transfer matrix method will gradually be replaced by others in the very near future.

Finite Element Models

Courant [16] in 1943 suggested an approach similar to the finite element method as we know today that involved piecewise continuous functions defined over triangular regions. More than a decade later, Turner et al. [17] in 1956 presented the first paper in finite element method. Along with the development of high speed digital computers, the application of finite element method progressed at impressive speeds.

The modern trend of the application of finite element method to rotor dynamics, however, was initiated by Ruhl and Booker [18] in 1972. Nelson and McVaugh [19] presented a general finite element method for rotor dynamics taking into account Timoshenko shear correction factor [20] and gyroscopic effects. Since then a large number of papers appeared for classical one dimensional analysis of rotors and adopted for practical applications, see Lalanne [21] and Rao [4]. The general scheme adopted in deriving finite elements is illustrated in Fig. 3. The notations used in finite element derivations are different from transfer matrix methods.

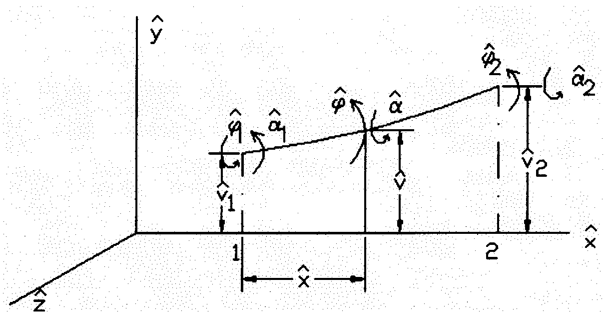

Fig.3a Shaft element in xy plane

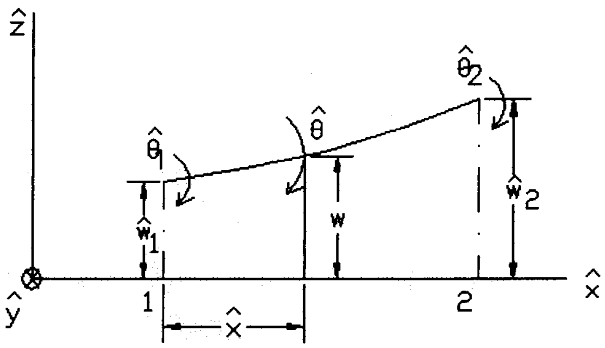

Fig.3b Shaft element in xz plane

Advantages of Finite Element Models are:

- Ease with which the assembled matrices are obtained.
- The method is more versatile than Transfer Matrix method - it can easily account for combined bending and torsion when gear tooth flexibility is taken into account, e.g., see Rao et al. [22].
- It is also general in application when dual rotor systems are considered, see [4, 23].
- The casing stiffness effects can be included by substructuring analysis, see [24].
- It is easy to adopt advanced analyses, e.g., variable stiffness effects in gearmesh, see [25], the study of chaos in geared rotor systems, see [26].

Drawbacks of One-Dimensional Models

One-dimensional bending and torsion models are traditionally employed in rotor dynamic analysis because of simplicity in deriving the governing equations of motion and adopting them to transfer matrix or finite element form. However, they suffer from many disadvantages.

- Real life rotors are not one-dimensional.
- Considerable time and effort are involved in deriving a good approximation of one-dimensional model from actual drawings of rotors.
- The influence of disks on shafts, vice versa is not possible in these rotor dynamics models - separate analyses are required.
- Centrifugal effects of distributed shafts and mounted parts cannot be accounted in the beam models.
- Gyroscopic effects are calculated as separate elements of equivalent disks and given as inputs to the one-dimensional models; otherwise the speed does not enter into the determination of Campbell diagrams and critical speeds.
- The foundation and casing effects are to be determined by sub-structuring analysis to determine their stiffness and damping effects and included in the beam models.

An accurate rotor dynamic analysis needs solid models for the rotors. In modern day designs, all components are CAD modeled and auto-meshing features in commercial codes makes meshing very accurate and accomplished in very little time compared to procedures in beam modeling. All the above disadvantages of beam models are automatically taken care of in solid rotor dynamics. However, the CPU time, RAM and hard disk requirements go up considerably. With recent advances in computers, these limitations are gradually diminishing, thus making solid rotor dynamics more attractive.

Solid Rotor Models - New Results

Analysis of solid rotor models progressed rapidly just in the last few years, see Rao [27, 28]. Consider the example of an aero-engine power plant twin spool rotor [4] shown in Fig. 4. The bearings are all considered isotropic and their stiffnesses are given in the figure.

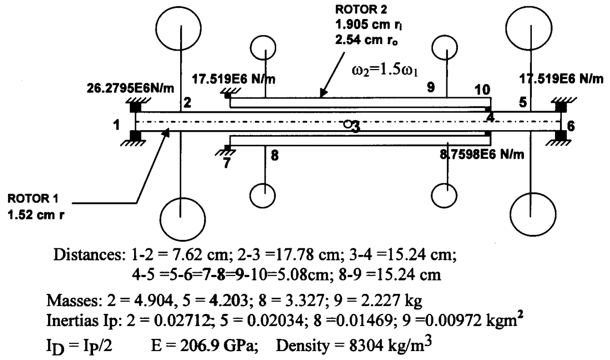

Fig.4 Example of a twin spool rotor

An equivalent solid rotor model for the above twin spool rotor is made as given in Fig. 5, which gives the same masses and inertias as in Fig.4. It may be noted that Fig.4 can represent in a unique manner an equivalent beam model of the solid model of Fig. 5, even though several other solid models can be derived for Fig. 4 beam model. This in fact is the main limitation of beam model analysis as an equivalent derived beam model may represent the dynamics of different solid models. An actual physical model in solid form eliminates this approximation.

Figure 6 shows the model with the bearings simulated by COMBIN-14 elements. Both the rotors are subjected to spin speeds and both Stress stiffening and Spin-soften-

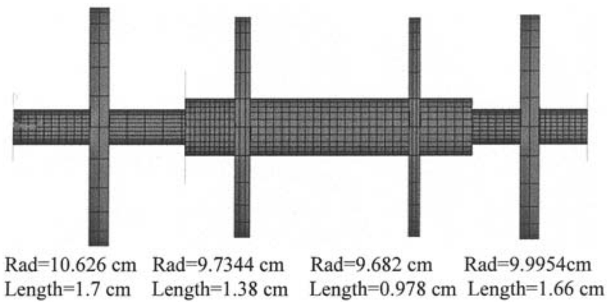

Fig.5 A solid rotor model of Fig.4

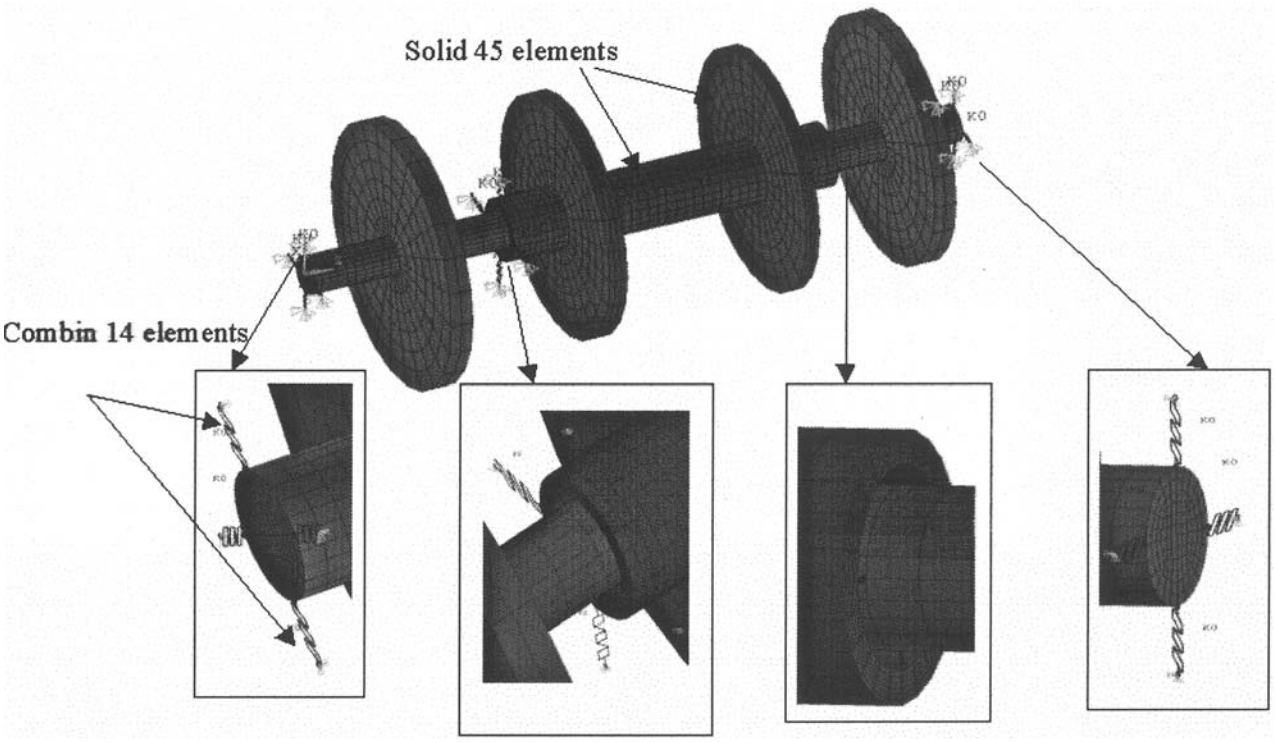

Fig.6 Full solid rotor model

ing effects are included. The elements are 12680 in all with 15367 nodes. The Campbell diagram is shown in Fig.7.

The stationary shaft system critical speed is 629.5 rad/s (100.2 Hz). At a spin speed 629 rad/s, the frequencies in the first mode obtained are 5.018 Hz for the backward whirl and 111.42 Hz for the forward whirl. The forward whirl gradually increased from 100.2 Hz to 111.42 Hz, whereas the backward whirl dropped to 5.018 Hz. At 630 rad/s spin speed, there is only one frequency detected which is forward whirl 111.4 Hz. Thus the backward whirl is dominated by spin softening effect compared to the forward whirl frequencies where the stress stiffening plays a significant role. The backward whirl disappears when the rotor speed crosses the natural frequency of stationary shaft. This is a new result. Beam models cannot predict the backward whirl modes as stress stiffening and spin softening influences cannot exist with the structure as a line element.

Aircraft Engine and Cryogenic Pump Rotor Dynamics Models

Deriving beam models as in Fig.4 for real life engines is time consuming and arguable; the casing and support structure is very light in construction and therefore participates in coupled motion with the rotor. As a first step, for civil transport aircraft, this casing may be considered rigid.

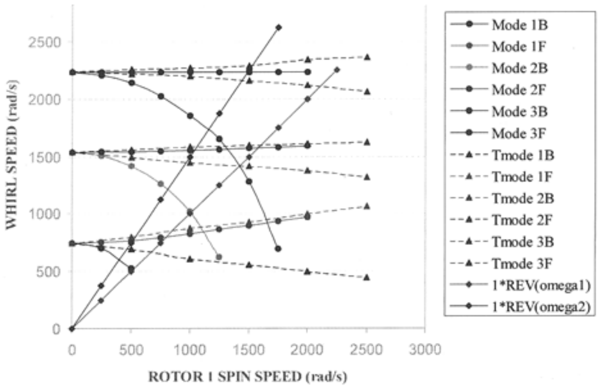

Fig.7 Campbell diagram of twin spool rotor system

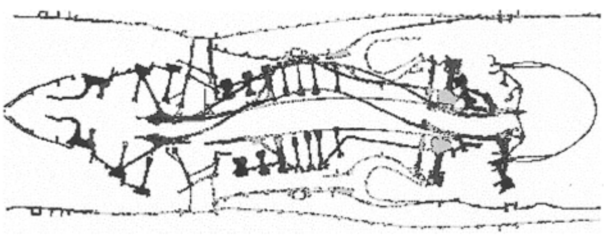

Fig.8 Typical mode shape of an aircraft engine

For modern state-of-art military aircraft engines, however, such a simplification creates problems. Recent advances give us the ability to use solid models directly resulting in Model-Mesh-Analyze approach to rotor dynamics. An aircraft engine mode shape is illustrated in Fig. 8 [29].

High speed cryogenic pumps in some cases run up to 100,000 rpm; the bearings become highly nonlinear and seals are responsible for instability regions. Internal pressures can also affect the stiffness of the flexible casing. Today it is possible to perform a nonlinear analysis and typical response of a high speed rotor as shown in Fig.9, see [30].

High speed rotors are also required to accelerate from start up to full speed in 3-5 seconds. The response at which peak response occurs can be obtained by transient analysis. The transient analysis for the unbalance response of a highly accelerating rotor is depicted in Fig.10.

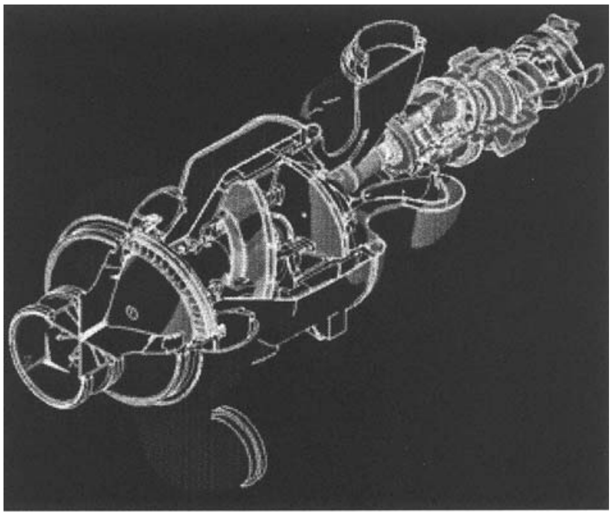

Fig.9 Unbalance response of a high speed cryogenic pump with nonlinearities at the bearings

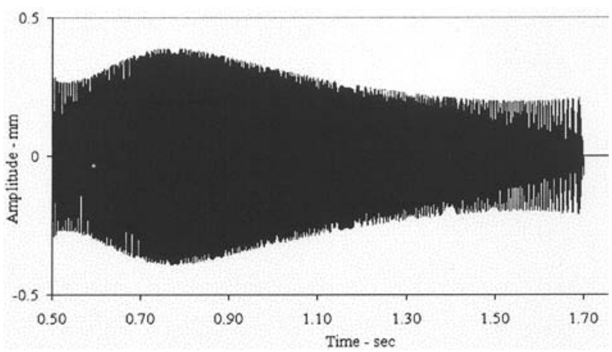

Fig.10 Coast up response of a high speed turbo pump with acceleration 1800 rad/sec2

Mounted Parts on Rotors

Bladed disks used in turbomachinery are most stressed components. First stage compressor blades in gas turbine engines and last stage turbine blades, both of them being the longest are subjected to heavy centrifugal loads. In fact these blades limit the design; many of them have been left to run with a small plastic zone in the dovetail regions (stress risers) which otherwise is globally elastic. In these regions, material flows and the blade and disk can get cold welded if such a plastic zone exists. These blades are however subjected to fatigue failures, particularly when they have a material or manufacturing defect in the stress riser regions; this is what happened to Narora nuclear turbine last stage blades in 1993 [31]. Fig.11 shows an early application giving the stress picture under elastic and elasto-plastic analysis. The peak stress 3253 MPa observed in an elastic analysis is far above ultimate stress in a fraction of an element at the notch or discontinuity. Elasto-plastic analysis of the same problem gives a peak stress 1157 MPa only. Average stress remains same in both the cases at 358 MPa. These are globally elastic but locally plastic structures. The elasto plastic analysis is not necessary since the peak stress at the notch is not exactly correct and we need to find true stress and strain there; otherwise the predominant elastic field is accurately determined by simple elastic analysis and the plastic zone is very small.

For globally elastic and locally plastic structures, Strain Based method is most appropriate to estimate life. Ludwik [32] is first to provide an explanation for the relationship between the stress and strain in plastic region. Subsequently, there were several explanations for the material behavior in the plastic region, the notable amongst them coming from Hollomon [33]. Hertzberg [34] identified the two behaviors type I and type II for a material in true stress and strain log-log plots. Martin et al. [35] suggested that a stable hysteresis loop can be described by a cyclic strain to be sum of the elastic and plastic ranges. The concept of local stress and local strain is the most promising approach to predict the crack initiation in a structure subjected to fatigue loads. Essentially, the approach [35] is based on the assumption that the local fatigue response of the material at a critical point, such as a notch or any other stress raiser that is the crack initiation location, is analogous to the fatigue response of a small smooth specimen subjected to the same cyclic strains and stresses. Neuber [36] provided the relation between the local strain and stress concentration factors and the theoretical stress concentration factor. The stress life for elastic part of the strain amplitude is determined through tests by Basquin [37]. In a similar manner, Manson [38] and Coffin [39] generated log-log plots of the stable plastic strain amplitude versus the number of reversals for failure. Together, they form basic lifing rule for crack initiation of a structure subjected to local plastic strain field.

Life estimation depends on how accurately the resonant stresses at a critical speed are determined. A major factor affecting this process until recently is an appropriate and realistic damping model. Usually an average value from a spin pit test or stationary rap tests is adopted; but

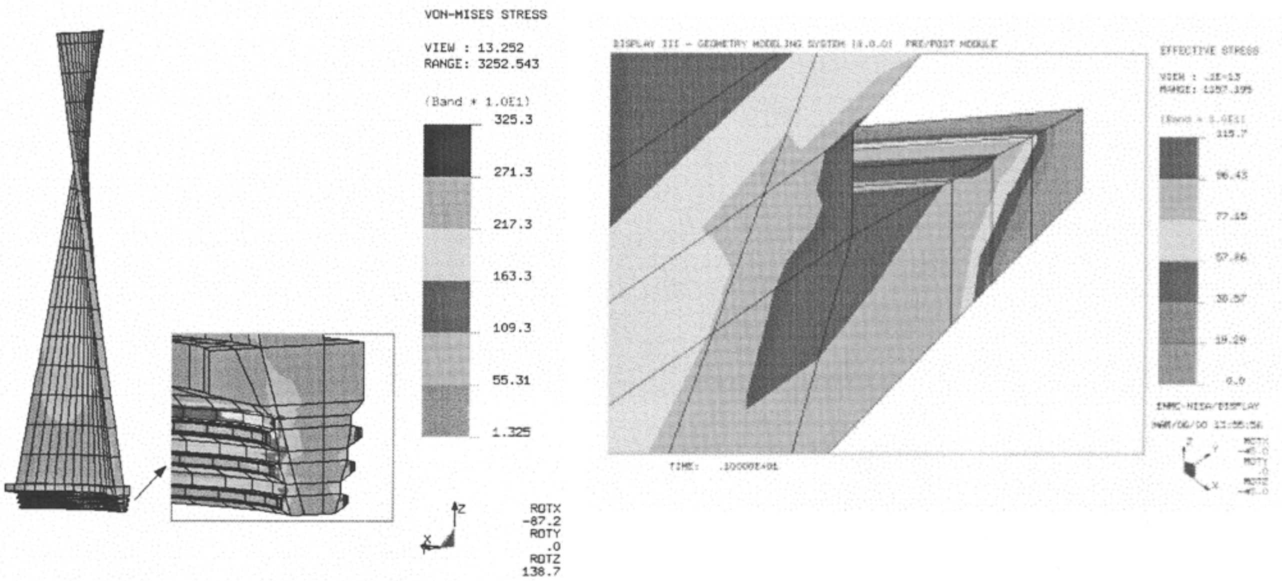

Fig.11 Elastic and elasto-plastic analysis of a last stage steam turbine blade

this requires the prototype before the design. Therefore the damping value is adopted from experience and the life is determined approximately. Fortunately, recent developments have removed this serious limitation by analytically estimating a nonlinear damping envelop [40] for a given speed of rotation and mode as a function of reference amplitude or strain, similar to experimentally observed results [41]. The material damping is determined by using Lazan's law [42] and interfacial friction is determined by transient analysis simulated in a given mode of vibration. This is a significant development as design cycle time and costs can be reduced considerably and bring accurate life estimation to simulation models.

Analytically determined material damping and combined friction and material damping nonlinear models from [40] are given in Figs.12 and 13 respectively. Friction is highly nonlinear and depends on the deformations experienced by slipping surfaces at different speeds of rotation and the analytical results in Fig.13 look as though they are generated by experiments; experimental results from friction do not follow a regular pattern. Usually one attributes this to errors in experiments, however this is not true.

The damping law thus determined analytically helps in determining life accurately at the design stage. The resonant stresses are determined following the procedure established in [43].

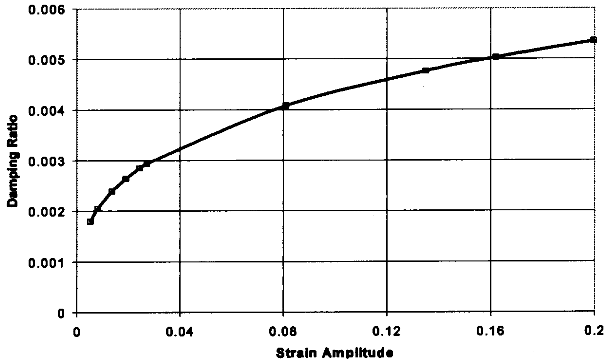

Fig.12 Analytically estimated material damping

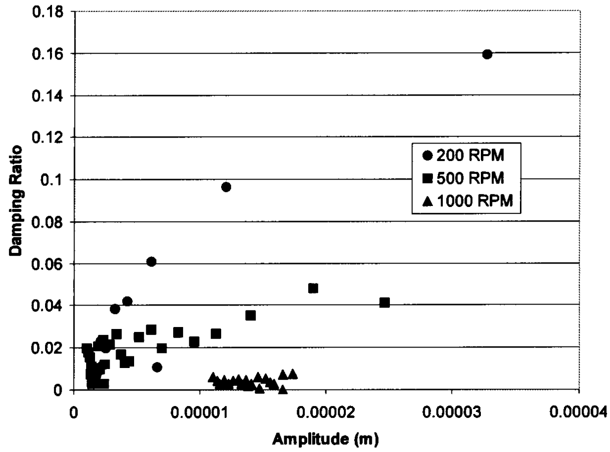

Fig.13 Analytically estimated damping in a rotating bladed-disk due to combined friction and material deformation at slipping surfaces

Life Estimation

Life estimation process is illustrated in Chart-1, see [44].

The steady load analysis in the upper left hand side of chart 1 for mean stresses is well established; the main sources are centrifugal loads, thermal loads and gas loads.

The dynamic stress module is usually complicated, particularly while estimating the alternating force definition and damping definition. Current practices of CFD allow an accurate determination of the force field in a turbine or compressor stage with moving row of blades, see [45].

Modal analysis of rotating blades is a complex subject that attracted attention of researchers over the last five decades. In the initial days of main frames in 1960's, energy methods were developed using beam models. A typical rotating blade can be depicted as shown in Fig. 14.

The governing equation of this system can be derived from basic energy principles as given in equation (2), see [46]. Contributions of each term in the equation are identified in the same equation. Solutions for such highly nonlinear equation are complicated; they have been achieved for special cases, e.g., the response in fundamental mode under transient conditions. The nonlinear terms become important for long slender blades such as helicopter blades under acceleration. Fortunately many of these nonlinear and Coriolis terms have negligible influence in aircraft engine blades as far as forced vibration at critical speeds is concerned in determination of stress field.

The natural frequencies and mode shapes of turbine and compressor blades are determined following well established Finite Element methods and commercial

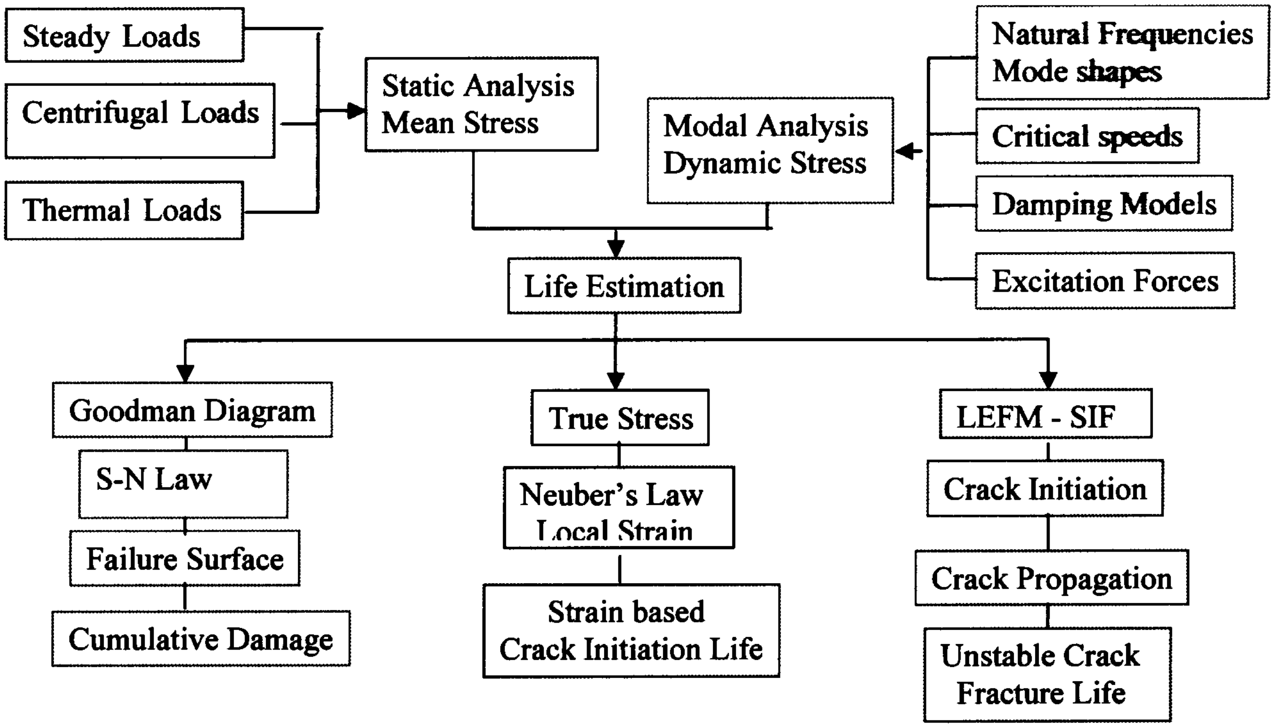

Chart-1 : Life estimation

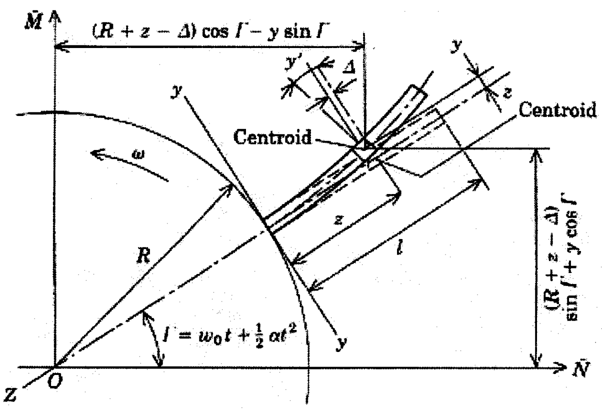

Fig.14 Rotating blade modeled as a beam

codes. Campbell diagram can then be drawn to identify the critical speeds. At these critical speeds, the damping is estimated as outlined and resonant stress determined.

The resonant stress is determined using the nonlinear damping model by an iteration process and the stress variation around the critical speed is obtained by using dynamic magnifier relation. The steady state and dynamic stresses together are used in a cumulative damage calculation process to assess the life at design stage.

The tedious and time consuming life estimation is recently developed as a user friendly code BLADE for stress based and strain based methods, see [47] as a platform that operates in Hyper Works. and for fracture mechanics, see [48]. Todays technology allows a speedy life estimation of complex bladed disk assemblies at the design stage that can reduce considerable time and save prototype making and testing costs.

Optimization

Aircraft engines work in limits; they demand as high possible life that is permissible under globally elastic and locally plastic conditions and in addition minimum weight consistent with structural integrity. In earlier practices, dedicated codes are developed to achieve a specific optimization problem but are limited to one or two objective functions and few design constraints in a real life structural problem.

Several attempts have been made in recent years to develop commercial user friendly optimization codes. One such general optimization code is Altair HyperStudy which uses global optimization methods. These methods use higher order polynomials to approximate the original structural optimization problem over a wide range of

Translation and Rotational Inertias

Beam Stiffness

Stiffness due to rotation

Hardening Softening

$$\ddot{y} + \frac{I_{xx}}{A}\ddot{y}'' + \frac{EI_{xx}}{\rho A} y'''' + \left(\omega_0 + \alpha t\right)^2 \left\{\left(R + Z\right)y' - y - R_2 y''\right\} +$$

$$+ 2\left(\omega_0 + \alpha t\right)\left\{\dot{y}y' - y''\int_0^l \dot{y}\,dz - \int_0^z y'\dot{y}'\,dz\right\} +$$

Nonlinear Coriolis Forces

$$+ \alpha\left\{yy' - \frac{1}{2}\int_0^z y'^2\,dz - y''\int_0^l y\,dz\right\} + \alpha\left(R + Z\right) = 0 \qquad (2)$$

Nonlinear terms due to acceleration

Force due to acceleration independent of y

design variables. The polynomial approximation techniques are referred to as Response Surface methods. A sequential response surface method approach is used in which, the objective and constraint functions are approximated in terms of design variables using a second order polynomial. One can create a sequential response surface update by linear steps or by quadratic response surfaces. The process can also be used for non-linear physics and experimental analysis using wrap-around software, which can link with various solvers.

Earlier practices in the design of the dovetail shapes relied heavily on testing that has given a desired life. This is costly and time consuming; today this can be achieved through simulation using, e.g., HyperStudy. In a recent study [49] demonstrating the optimization of a blade root shape, the parameters varied consistent with manufacturing constraints and available space are shown in Fig.15. Fig.16 shows the baseline and optimized root shape that reduced plastic strain at the singularity by 26%. Using strain based method of lifing this strain reduction increased the life as much as 4 times. A design optimization of all the stress raiser locations through simulation can improve the life of many existing rotating machinery components.

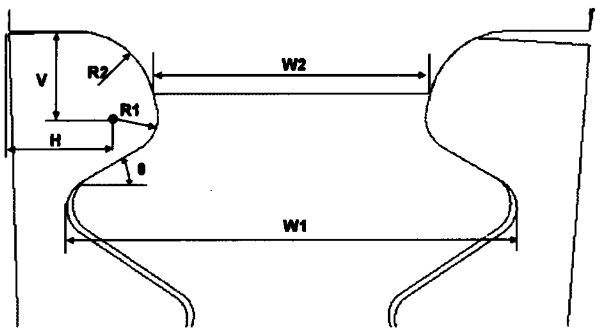

Fig.15 Parameters varied during optimization

Another significant optimization reported recently is concerned with weight optimization, see [50]. Weight is an important criterion for all military applications of fighter aircraft. As mentioned earlier, though the peak stresses go beyond yield in the small plastic zone around the singularity, the average stress in the root is low. This region can be used to remove substantial weight.

A compressor blade used for weight optimization is shown in Fig.17. The stress result under centrifugal loading is given in Fig.18. The lean stress regions are used to decrease the weight of the blade shank.

A baseline root with 8 holes and 2 cutouts in the shank is made, see Fig.19. The design variables are chosen as given in Fig.20. HyperStudy was used to optimize the weight keeping the peak stress within yield. It took 16 iterations to give the optimized result, see Tables-1 and 2.

The weight reduction achieved was 10.03%. Thus, we can achieve through simulation the blade life analytically at the design stage and then optimize the same for the required objectives, life, shape, weight.

Concluding Remarks

Rotor Dynamics models have been adopted as beam models for over a century; this has been a very tedious process for determining simplified beam models that are acceptable in producing results that are comparable to test values. During last 5-6 years, this practice has been changed to adopt directly solid models that allow using modeling, meshing and analysis concept thus reducing considerable time of skilled engineers. These solid models have been shown to accurately account for support structures, multi spool systems, bearing nonlinearities and angular acceleration effects of high speed and rapidly accelerating rotors.

Rotors carry Bladed Disks in turbines and compressors of the aircraft engine. Modern aircraft engines demand light weight in rotors and mounted parts consistent with

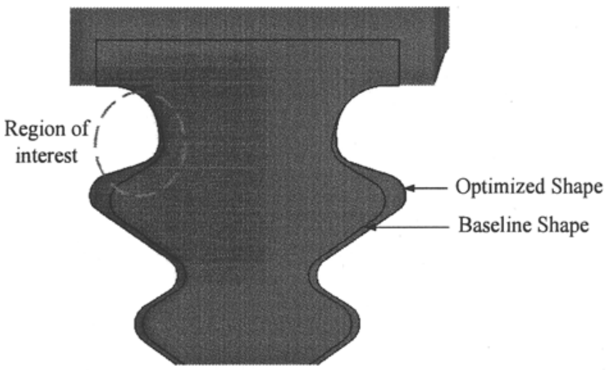

Fig.16 Blade root shape optimization

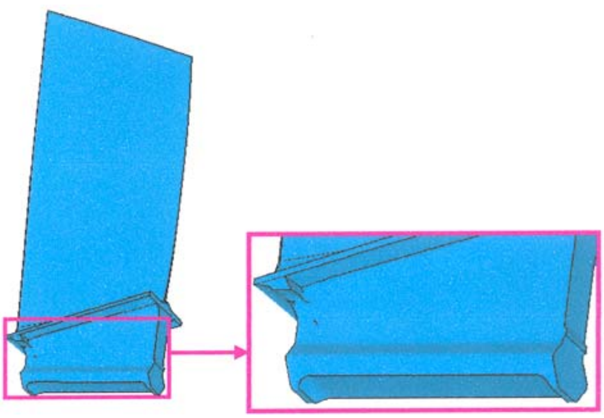

Fig.17 A compressor blade root for weight optimization

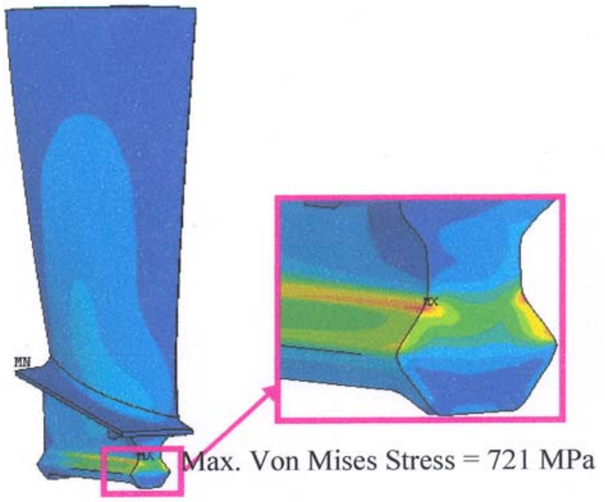

Fig.18 Baseline stress result under centrifugal loading

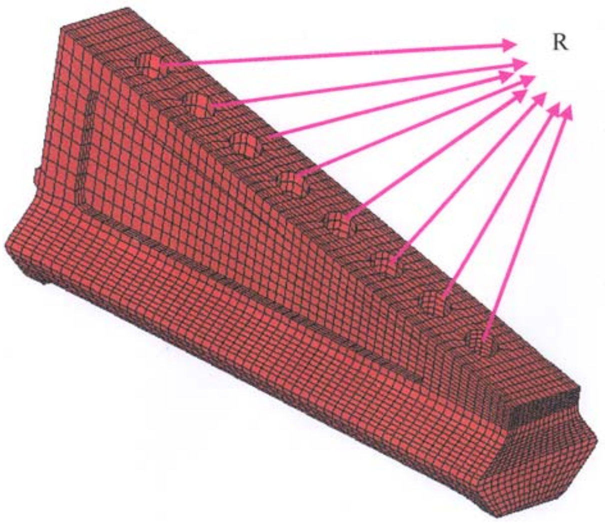

Fig.19 Blade shank baseline generated for optimization with 8 holes and two cutouts in the shank

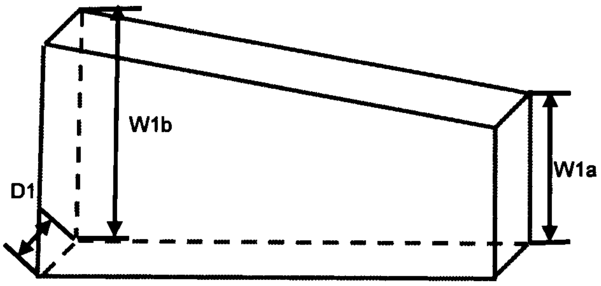

Fig. 20 Design Variables

Fig.20 Design variables

Table-1 : Range of design variables

Variable	Range mm
D1	1.1. to 2.0
D2	1.1. to 2.0
W1a	4.75 to 6.5
W1b	13.94 to 15.5
W2a	4.44 to 6.0
W1b	13.64 to 16.0
R	1.75 to 2.25

Table-2 : Optimized design variables

Variable	Value mm
D1	1.64
D2	1.64
W1a	5.09
W1b	15.35
W2a	5.57
W1b	14.56
R	2.0

advanced designs taking the structure to local plastic conditions. Life estimation, a tedious process becomes important at the design stage. Some of the road blocks in assessing damping without prototypes and costly testing have been removed by advancing analytical methods. Also the drudgery in life estimation process has been removed by developing special user friendly codes.

Besides assessing life, the modern designs demand optimized structures for the best possible life and minimum weight. Methods of shape optimization and weight optimization have now been developed.

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