BUCKLING ANALYSIS OF BEAM ON WINKLER FOUNDATION BY USING MDQM AND NONLOCAL THEORY

T. Murmu and S.C. Pradhan*

Abstract

In the present work buckling analysis of beam using Eringen nonlocal elasticity theory is being carried out. The associated governing differential equation is solved by the modified differential quadrature method (MDQM). The present MDQM employs Chebyshev polynomial for the determination of weighting coefficient matrices. The results obtained from present analysis are being validated with those reported in literature. Effect of number of interpolation points on the accuracy of the results is also investigated. It is found that seven numbers of interpolations are required to achieve reasonable accurate results for various nonlocal parameter values. It is also observed that the effect of nonlocal parameter on critical buckling load for the higher modes is higher and more nonlinear than the lower modes. Further the effect of (i) nonlocal parameter, (ii) Winkler elastic foundation moduli and (ii) boundary conditions on the critical buckling loads are being investigated and discussed.

***Keywords**: nonlocal parameter, differential quadrature method, buckling load, Winkler foundation, boundary conditions*

Nomenclature

A	= cross section area of beam
A_{ij}, B_{ij}, C_{ij}, D_{ij}	= first, second, third and fourth order weighting coefficients
C	= fourth-order elasticity tensor
E	= Youngs modulus
H	= nonlocal modulus
I	= moment of inertia
K	= winkler elastic modulus
L	= length of beam
M	= Bending moment of beam
$\overline{N}$	= critical load
N_0	= matrix of Chebyshev polynomial element
N_{cr}	= non dimensional critical load
N_{cr}_0, N_{cr}_05	= non dimensional critical loads at μ=0 and 0.05 respectively
$T(x)_i$	= i^{th} term of Chebyshev polynomial
a	= internal length
e_0	= material constant
k	= non-dimensional elastic modulus of Winkler foundation
l	= external length
n	= no. of interpolation points
q	= transverse load
$t(x)$	= stress function corresponding to classical mechanics
w	= deflection of beam
x_o, x_l, ..., x_n	= x co-ordinates of $n+1$ interpolation points
$\in_{xx}$	= strain component
κ	= bending strain
μ	= nonlocal parameter
σ	= nonlocal stress tensor
σ_{xx}	= stress component
τ	= material constant

Introduction

In nonlocal elasticity theory the stress at a point is defined as a function not only of the strain at that point (classical local mechanics) but also a function of the strain at all other points in the domain. The contribution of forces between atoms and the effect of internal and external lengths are being included in the formulation. Recently

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there has been growing interest for application of nonlocal continuum mechanics especially in the field of fracture mechanics, dislocation mechanics and micro/nano technologies (carbon nanotubes) etc.

Nonlocal elasticity theory holds an important area of research for the future structural developments and design in modern aerospace and aeronautical fields. This is due to the fact that small-size structures such as CNT, micro/nano sensors and actuators which are being applied in aerospace structures (CNT-reinforced composite, MEMS/NEMS devices (smart structures)) could not be accurately analysed by local (classical) theory. The local classical mechanics theory is assumed to be as scale free theory. Experiments and atomic simulation have shown that there is a significant size effect in mechanical properties when the dimensions of these structures become small. Applying nonlocal theory to these small structures could lead to correct prediction of mechanical behaviors.

The nonlocal elasticity theory was first reported by Eringen [1-2]. This theory also has wide application which includes wave propagation in solids, dislocation mechanics, surface tension in fluids etc. Some researchers have applied this nonlocal elasticity theory and studied bending, vibration and buckling of structural members. Peddieson et al.[3] applied the nonlocal elasticity theory to formulate a nonlocal version of Euler-Bernoulli beam theory. Based on the theory of nonlocal continuum mechanics, Sudak [4] carried out buckling analysis of multi walled nanotubes. Wang et al. [5] developed nonlocal elastic model for beam and shell and carried out buckling analysis of carbon nanotubes. Their results show that buckling solutions via local continuum mechanics are overestimated and scale effect is very much required in stability analysis of carbon nanotubes. Wang et al. [6] presented elastic buckling analysis of micro and nano rods based on nonlocal elastic theory. The governing equations and the boundary conditions were derived using the principle of virtual work. Also expressions for the critical buckling loads are derived for axially loaded rods with various boundary conditions. Recently, Reddy [7] derived the nonlocal elasticity theory for bending, buckling and vibration of one-dimensional structural members. His work showed that the inclusion of nonlocal effect increases the magnitude of deflections and decreases both buckling loads and natural frequencies.

The present paper proposes a differential quadrature formulation for buckling analysis of beams which includes Eringens nonlocal variables. These formulations will be important in the application and analysis of nonlocal theories in structural studies for small-size analysis (CNT in reinforced composites). The differential quadrature method (DQM) is a simple and efficient technique for solving partial differential equations as reported by Bellman and Casti [8] and Bellman et al. [9] .DQ researchers have applied this method in solving various engineering problems. These include Civan and Sliepcevich [10] , Bert et. al. [11], Jang et. al. [12], Sherboume and Pandey [13] and Mohammad et. al. [14]. Better convergence behaviors are observed by DQM compared to its peer numerical competent techniques viz. finite element method, finite difference method, boundary element method and meshless technique. Usually in these numerical techniques accuracy improves with *h*, *p* and *h-p* refinements. However, in case of the DQM a smaller number of interpolation points are adequate to yield reasonably accurate results. This is because all uniform or non uniform interpolation points are used to represent the each-order derivation of the function at each point. Thus accurate numerical solutions are obtained by employing few interpolation points. The present numerical technique is successfully applied in the analysis of beams, plates and shells. Further, in structural analysis Chen et al. [15] and Pradhan and Murmu [16] employed Modified Differential Quadrature Method (MDQM). In this method the weighting coefficient matrix for the first order is derived based on Chebyshev polynomial. Wang and Bert [17] implemented exactly the boundary conditions in MDQM by employing modified weighting matrix.

Engineering structures in general are often found to be resting on elastic foundation. These structures are modeled as being supported on elastic foundation along their span. For example in the analysis of runways of airports, the structure is usually modeled as a "plate resting on elastic foundation". Further mechanical fasteners in composite materials are also modeled as beams on elastic foundation. Similarly, structures (e.g. carbon nanotubes, micro/nano beams) which require nonlocal theory for accurate predictions of it behavior can also be generally found in an elastic medium. The elastic medium can be modeled as a Winkler type foundation. Structures on elastic foundation which are analyzed with the local theory (classical mechanics) are also being modeled by Winkler type foundation. In this Winkler model[18], the elastic foundation is analyzed by replacing elastic foundation with closely spaced virtual springs. The elastic foundation modulus is being represented by the equivalent stiffness of the springs K.

Studies based on local stress theory on homogenous isotropic structures resting on elastic foundation are found in literature. However, there is no work reported on buckling analysis of beams with elastic foundation and with nonlocal theory. Thus in the present work an attempt is made to include nonlocal theory in the buckling analysis of beams with Winkler foundation.

In the present work, based on Eringen [1-2] nonlocal elasticity, Euler-Bernoulli beam model is derived. Further, the nonlocal-type beam is considered to be supported on an elastic foundation. The effect of elastic foundation can be described by a Winkler-like model. The DQ formulations of critical buckling load of the column are extended to the beam resting on Winkler elastic foundation. The associated governing differential equations based on nonlocal elasticity theory for stability are derived and solved using the modified differential quadrature method. Results obtained by the present method are validated with those reported in literature. Finally, the effect of (i) nonlocal parameter, (ii) Winkler foundation elastic moduli and (iii) boundary conditions on critical loads of beam are investigated and discussed.

Formulation

Nonlocal Theory

The stress field at a point **x** in an elastic continuum depends on strains at all other points of the body as mentioned by Eringen [1] . This is in accordance with atomic theory of lattice dynamics and experimental observations on photon dispersion. The most general form of the constitutive equation for nonlocal elasticity involves an integral over the whole body. Thus, the nonlocal stress tensor σ at a point **x** is expressed as

$$\sigma = \int_V H(|X' - X|, \tau)\, t(X')\, dx' \tag{1}$$

The terms t(x) and $H(|x' - x|, \tau)$ are the classical stress at point x and the nonlocal modulus; respectively. $|x' - x|$ represents the distance in Euclidean form. τ is a material constant which depends on internal (e.g. lattice spacing) and external characteristics lengths (e.g. wavelength). The constitutive relation is written similar to the generalized Hooke's law

$$t(x) = C(x) : \varepsilon(x) \tag{2}$$

C is the fourth-order elasticity tensor; : denotes the double dot product. According to Eringen [2] the integral constitutive relation is equivalent to a differential form

$$\left(1 - \tau^2 l^2 \nabla^2\right)\sigma = t, \quad \tau = \frac{e_o a}{l} \tag{3}$$

where ∇ is the Laplacian operator, e_o is a constant for adjusting the model in matching some reliable results by experiments or other models. The parameter e_o is estimated such that the relations (3) of the model could provide satisfied approximation of atomic dispersion curves of plane waves with those of atomic lattice dynamics. The terms *a* and *l* are the internal (lattice parameter, granular size, or molecular diameters), and external lengths, respectively.

For the case of one-dimensional structures such as a beam, the Laplacian operator is reduced to one dimensional form and the strains in the y and z directions are negligible. Hence a uniaxial stress state is established in the one dimensional nonlocal theory. Thus the nonlocal constitutive relation for the macroscopic stress is given as Reddy [7].

$$\sigma_{xx} - \left(e_o a\right)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E\, \varepsilon_{xx} \tag{4}$$

The constitutive relation in Euler-Bernoulli beam theory is expressed as

$$M - \mu \frac{\partial^2 M}{\partial x^2} = E I \kappa \tag{5}$$

E and *l* are the Youngs modulus and moment of inertia, respectively. And μ is the nonlocal parameter and is equal to $(e_o a)^2$. The inclusion of the nonlocal parameter in the above equation takes into account the effects of "scale-factor", usually smaller size. The parameter transforms the classical local equation into a nonlocal mechanics equation. Classical continuum elasticity, which is a scale free theory, cannot predict the size effects. The small size analysis using local theory over predicts the results. Thus the consideration of nonlocal parameter is necessary for correct prediction of micro/nano structures. It should be noted that when the nonlocal parameter μ is zero, the equation (5) reduces to that of classical mechanics one. For a specific material or structure, the corresponding nonlocal parameter can be estimated experimentally or approximated by matching the dispersion curves of plane waves with those of atomic lattice dynamics.

The Euler Lagrange equation without time dependent terms is expressed as

$$\frac{\partial^2 M}{\partial x^2} + q - \frac{\partial}{\partial x}\left(\bar{N}\frac{\partial w}{\partial x}\right) = 0 \tag{6}$$

where $\overline{N}$ and q are the buckling load and transverse load, respectively. Substituting the second derivative of equation (6) into equation (5) and then substituting the expression of M into equation (6), the governing differential equation for buckling is expressed as

$$\frac{\partial^2}{\partial x^2}\left(-EI\frac{\partial^2 w}{\partial x^2}\right) + \bar{N}\mu\,\frac{\partial^4 w}{\partial x^4} - \bar{N}\frac{\partial^2 w}{\partial x^2} = 0 \tag{7}$$

The governing differential equation for buckling with Winkler type foundation (on elastic medium) can be expressed as

$$\frac{\partial^2}{\partial x^2}\left(-EI\frac{\partial^2 w}{\partial x^2}\right) + \mu\left(\bar{N}\frac{\partial^4 w}{\partial x^4} + K(x)\frac{\partial^2 w}{\partial x^2}\right) - \bar{N}\frac{\partial^2 w}{\partial x^2} - K(x)\,w = 0 \tag{8}$$

where K is the elastic modulus of Winkler foundation.

Modifed Differential Quadrature Method for Nonlocal Theory

In the differential quadrature method (DQM) partial derivatives (appearing in partial differential equation) of a function with respect to a space variable at a given interpolation point is approximated as a weighted linear summation of function values at all chosen interpolation points.

$$f_x'(x_i) = \sum_{j=1}^{n} A_{ij}\,.\,f(x_j), \qquad j = 1, 2 \ldots, n \tag{9}$$

Thus, DQM transforms the governing differential equation into a set of equivalent simultaneous equations. This is done by replacing the partial derivative with equivalent weighting coefficients. For example, the first partial derivative is equivalent to a weighting coefficient matrix.

$$\frac{\partial}{\partial x} \equiv [A]_x \tag{10}$$

Similarly, second, third, and fourth order partial derivative are expressed as

$$\frac{\partial^2}{\partial x^2} \equiv [B]_x = [A]_x\,[A]_x$$

$$\frac{\partial^3}{\partial x^3} \equiv [C]_x = [A]_x\,[A]_x\,[A]_x$$

$$\frac{\partial^4}{\partial x^4} \equiv [D]_x = [A]_x\,[A]_x\,[A]_x\,[A]_x \tag{11}$$

In this manner, the original governing differential equation is transformed into a set of distinct simultaneous algebraic equations. In the present analyses modified differential quadrature method is employed. The implementation of this MDQM technique depends on how accurately the weighting coefficient matrix is computed and the interpolation points (grid points) are distributed in the domain. The weighting coefficients matrix for the MDQM is determined as follows. The function at any point within the computational domain is expressed as

$$f(x_i) = c_o\,T_o(x_i) + c_1\,T_1(x_i) + c_2\,T_2(x_i) + \ldots + c_n\,T_n(x_i) \tag{12}$$

Here T_o, T_1, T_2 .... T_n are the Chebyshev polynomials terms of first kind. They are defined as

$$T_1(x_i) = 1\,,\;\; T_2(x_i) = x_i\,,\;\; T_2(x_i) = -1 + 2x_i^2\,,$$
$$T_3(x_i) = -3x_i + 4x_i^3 \;\ldots. \tag{13}$$

In matrix form, equation (12) is written as

$$\{w\} = \left[N_o\right]\{c\} \tag{14}$$

For any point into the computational domain, the equation (14) can be written as

$$\{\overline{w}\} = [N]\{c\} \tag{15}$$

Differentiating equation (15) and using equation (14),

$$\frac{d}{dx}\{\overline{w}\} = \left(\frac{d}{dx}[N]\right)[N_o]^{-1}\{w\} + \frac{[N]\,\frac{d}{dx}\left([N_o]^{-1}\{w\}\right)}{(=0)}$$

$$= \left(\frac{d}{dx}[N]\right)[N_o]^{-1}\{w\} \tag{16}$$

From equation (16) we obtain,

$$\frac{d}{dx}\{w\} \equiv [A]\{w\} \tag{17}$$

where

$$\left[A\right] \equiv \left[N_o^{'}\right]\left[N_o\right]^{-1} \tag{18}$$

$[N_o]$ is a matrix developed from Chebyshev polynomials of first kind and is defined as

$$\begin{bmatrix} T_o(x_o) & T_1(x_o) & T_2(x_o) & \cdots & T_{n-1}(x_o) & T_n(x_o) \\ T_o(x_1) & T_1(x_1) & T_2(x_1) & \cdots & T_{n-1}(x_1) & T_n(x_1) \\ T_o(x_2) & T_1(x_2) & T_2(x_2) & \cdots & T_{n-1}(x_2) & T_n(x_2) \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ T_o(x_{n-1}) & T_1(x_{n-1}) & T_2(x_{n-1}) & \cdots & T_{n-1}(x_{n-1}) & T_n(x_{n-1}) \\ T_o(x_n) & T_1(x_n) & T_2(x_n) & \cdots & T_{n-1}(x_n) & T_n(x_n) \end{bmatrix} \tag{19}$$

Here, $x_o , x_1 , x_2 , ... x_n$ are the interpolation points. For choosing interpolation points one simple way are to divide the computational domain into equal spaces. However, Chen et. al. [15] observed that uniform spacing among the interpolation points did not result in accurate results. Instead, Pradhan and Murmu [16] reported that accurate and stable results are being obtained by employing Chebyshev-Gauss-Lobatto interpolation points. Thus, in the present study these interpolation points are being considered. Locations of these interpolations are determined by

$$x_i = \frac{1}{2}\left[1 - \cos\left(\frac{\pi\, i}{n}\right)\right], \quad i = 0 , ... ,n \tag{20}$$

where $n+1$ is the number of interpolation points. The functional values w_i at the specific interpolation points x_i are computed as

$$w_i = w\,(x_{i-1}) , \quad i = 1 , 2 , ... , n+1 \tag{21}$$

Using the above differential quadrature approach, the derivatives in the governing differential equation for buckling analysis of beam with nonlocal parameter and without Winkler foundation (equation (7)) are replaced by corresponding matrices of weighting coefficients. Thus equation (7) is derived as

$$-\,E\,I\sum_{j=1}^{n+1} D_{ij}\,w_j + \mu\,\overline{N}\sum_{j=1}^{n+1} D_{ij}\,w_j - \overline{N}\sum_{j=1}^{n+1} B_{ij}\,w_j = 0$$

$$i = 1 , 2 , ... , n+1 \tag{22}$$

Thus equation (22) is a simple set of equivalent simultaneous equations. By this approach any complex differential equation with nonlocal elasticity theory can be transformed into similar differential quadrature discretised form. Similarly, differential quadrature discretisation equation for buckling analysis of beam with nonlocal parameter and Winkler foundation is written as

$$-\,E\,I\sum_{j=1}^{n+1} D_{ij}\,w_j + \mu\,\overline{N}\sum_{j=1}^{n+1} D_{ij}\,w_j + \mu\,(K)_i\sum_{j=1}^{n+1} B_{ij}\,w_j$$

$$-\,\overline{N}\sum_{j=1}^{n+1} B_{ij}\,w_j - K_i\,w_i = 0 \quad i = 1 , 2 , ... , n+1 \tag{23}$$

It should be noted that when the value of $\mu = 0$, the equation is reduced into classical Euler theory. While imposing various boundary conditions the modified weighting coefficient matrix (MWCM) approach is considered. One may refer to Ref. [17] for details. For the column buckling analysis MWCM approach imparted more accurate results as compared to other DQM methods for same number of interpolation points. In MWCM approach the boundary conditions are being imposed during the computation of weighting coefficient matrix for inner interpolation points. For simply supported simply supported case $w = 0$ at $x = 0$ and $x = L$.This leads to elements of first and last columns being replaced by zeros. Thus, A_{ij} is updated as $\overline{A}_{ij}$ where $i, j = 1, 2, ... , n+1$. The second order weighting coefficients is thus written as

$$\bar{B}_{ij} = A_{ik}\,\bar{A}_{kj} \tag{24}$$

During the formulation of third order derivative coefficient matrix for simply supported - simply supported case, the boundary conditions, $w'' = 0$ at $x = 0$ and $x = L$ are implemented. Similarly, third and fourth order weighting coefficients are computed as

$$\bar{C}_{ij} = \bar{A}_{ik}\,\bar{B}_{kj}$$
$$\bar{D}_{ij} = A_{ik}\,\bar{C}_{kj} \tag{25}$$

The other types of boundary conditions are also implemented in the similar way. Thus, the differential quadrature analogous of equations (15) and (16) with appropriate boundary condition are expressed as

$$-\,E\,I\sum_{j=1}^{n+1}\bar{D}_{ij}\,w_j + \mu\,\bar{N}\sum_{j=1}^{n+1}\bar{D}_{ij}\,w_j - \bar{N}\sum_{j=1}^{n+1}\bar{B}_{ij}\,w_j = 0 \tag{26}$$

$$-\,E\,I\sum_{j=1}^{n+1}\bar{D}_{ij}\,w_j + \mu\,\bar{N}\sum_{j=1}^{n+1}\bar{D}_{ij}\,w_j + \mu\,(K)_i\sum_{j=1}^{n+1}\bar{B}_{ij}\,w_j$$
$$-\,\bar{N}\sum_{j=1}^{n+1}\bar{B}_{ij}\,w_j - K_i\,w_i = 0 \tag{27}$$

It should be noted that the above equations are solved for interior points.

Results and Discussions

Validations

Modified Differential Quadrature Method

Computer code based on modified differential quadrature method (MDQM) is developed to predict the critical buckling loads of one dimensional structural member. In this method the boundary conditions are implemented using the modified weighting coefficient matrix (MWCM). Non dimensional critical buckling load is defined as

$$N_{cr} = \bar{N} \times (L^2 / E\,I) \tag{28}$$

where $\bar{N}$ is the critical buckling load. L, E and I represent beam length, Youngs modulus and moment of inertia, respectively. A simply supported - simply supported beam is being considered and non dimensional critical loads as mentioned in equation (28) for first four modes are being computed and results are listed in Table-1. These results are restricted to local theory. From this table one could note that the computed critical loads are in good agreement with those computed employing distributed transfer function method (DTFM) of Yang[19] .

Including Moduli of Winkler Elastic Foundation

Further, non dimensional critical loads mentioned in equation (28) are determined for a simply supported - simply supported beam resting on Winkler elastic foundation. Non-dimensional Winkler modulus is defined as $k = K\,L^4 / E\,I$. Five values of non-dimensional Winkler elastic moduli are considered viz. 1, 5, 10, 50 and 100. The non dimensional critical loads for first four modes and various Winkler elastic moduli are listed in Table-2. For the same parameters results are also obtained employing distributed transfer function method [DTFM] of Yang [19] and compared with the results obtained by present MDQM analysis. The comparative study is listed in Table-2. From this comparison it is observed that present MDQM results do agree with the DTFM results. Thus the developed code based on MDQM is said to yield accurate results with nine interpolation points. Further, employing the developed code buckling analysis of beams with nonlocal theory and various boundary conditions are carried out and discussed.

Including Nonlocal Theory

Developed computer code is extended to include effect of nonlocal parameter μ. Employing the developed code critical buckling loads is computed for a simply supported - simply supported isotropic beam. Non dimensional critical buckling load mentioned in equation (22) are obtained for the first four modes of buckling. The values of nonlocal parameter μ are considered to be 0.0 to 0.05 and the computed critical loads are plotted in Fig.1. From this figure it is observed that present results are in good agreement with those reported by Reddy [7]. Further, it is found that reasonably accurate results are being obtained by

Table-1 : Comparison of nondimensional critical loads

Buckling Mode	1st	2nd	3rd	4th
Yang [19]	9.8696	39.4784	88.8260	157.9102
Present Analysis	9.8696	39.4784	88.8264	157.9140

employing only seven interpolation points in the analysis. The critical load decreases with increase in value of nonlocal parameter μ. Similar trend with nonlocal parameter is also observed by Reddy[7].

Convergence Study with Number of Interpolation Points

Critical buckling loads for a simply supported-simply supported isotropic beam with various nonlocal parameter μ are being computed. Critical loads are obtained employing various numbers of interpolation points. Relative difference percent between the Reddys result and present analysis result is defined as 100 x (present result - result of Reddy [7])/result of Reddy [7] . Results are depicted in Fig.2. From this figure it is interesting to note that with seven interpolation points reasonably converged results are obtained for all nonlocal parameter μ values considered. Thus in the rest of the nonlocal analysis nine inter-

Table-2 : Non-dimensional critical buckling loads of simply supported beams on Winkler Foundation

k	1	5	10	50	100
1st Mode					
Yang [19]	9.9709	10.3762	10.8828	14.9357	20.0017
Present Analysis	9.9709	10.3762	10.8828	14.9357	20.0017
2nd Mode					
Yang [19]	39.5037	39.6051	39.7317	40.7449	42.0114
Present Analysis	39.5037	39.6050	39.7317	40.7449	42.0114
3rd Mode					
Yang [19]	88.8377	88.8827	88.9392	89.3893	89.9522
Present Analysis	88.8353	88.8803	88.9367	89.3870	89.49925
4th Mode					
Yang [19]	157.9200	157.9450	157.977	158.2301	158.5471
Present Analysis	158.0931	158.1184	158.1500	158.4030	158.7193

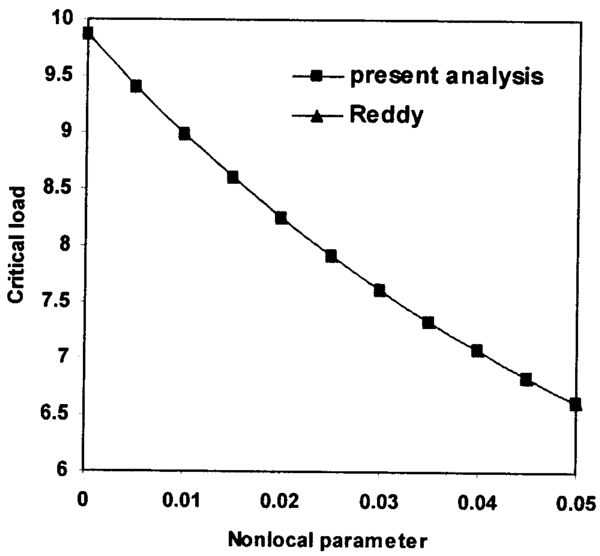

Fig.1 Critical loads with various nonlocal parameter values

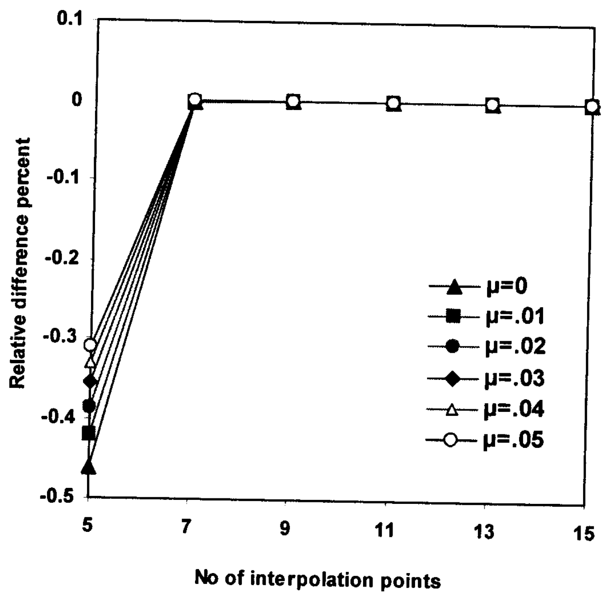

Fig.2 Convergence study with various nonlocal parameter

polation points are being employed. It is also observed that the relative difference percent is less for larger values of nonlocal parameter μ as compared to smaller values of nonlocal parameter μ.

Effect of Nonlocal Parameter and Higher Modes

Non dimensional critical buckling loads for first four modes of a simply supported-simply supported beam with nonlocal theory are determined. The variation of non dimensional critical loads for the first, second, third and fourth modes with various nonlocal parameter μ values are plotted in Fig. 3. From this figure one could observe that the influence of nonlocal parameter on non dimensional critical load for the higher mode is larger and more non-linear as compared to the lower modes. Further, the non-dimensional critical load for the fourth mode drops rapidly for nonlocal parameter μ varying from 0.0 to 0.01. This drop is gradual for nonlocal parameter μ varying from 0.01 to 0.05. From Fig. 3 one could infer that effect of nonlocal parameter on the critical load is larger for the fourth mode as compared to first, second and third modes. Further it is observed that for the first mode there is insignificant change of critical load with increase in nonlocal parameter μ from 0.0 to 0.05.

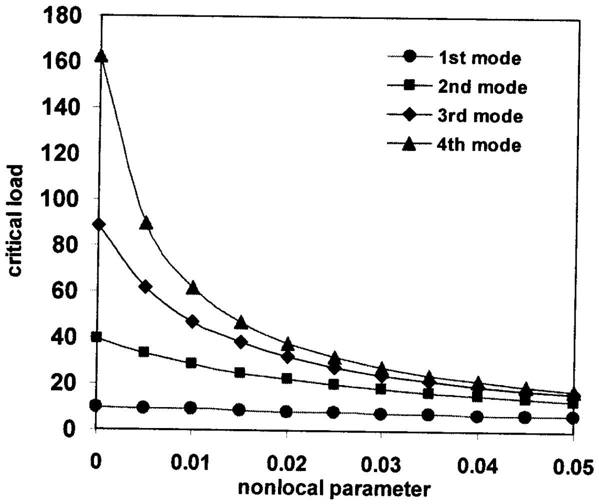

Fig.3 Effect of nonlocal parameter and higher modes on critical buckling load

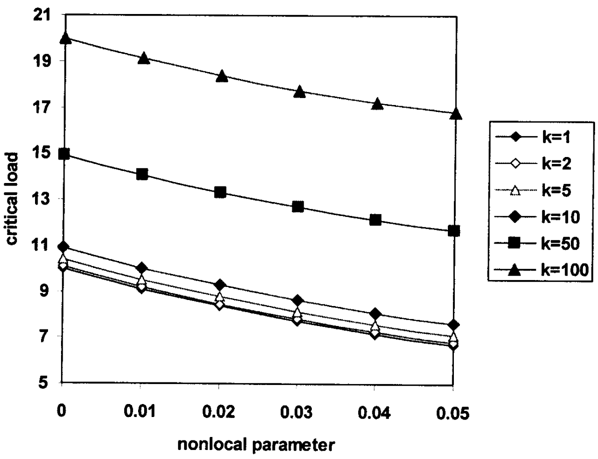

Fig.4 Effect of nonlocal parameter and elastic modulus of Winkler foundation on critical buckling load

Effect of Nonlocal Parameter and Elastic Modulus of Winkler Foundation

Non dimensional critical buckling loads for a simply supported-simply supported beam with nonlocal theory and various Winkler elastic moduli are determined. The variation of non dimensional critical loads for non dimensional elastic moduli of 1, 2, 5, 10, 50 and 100 and various nonlocal parameter μ are plotted in Fig. 4. From this figure one could observe that non dimensional critical loads are decreasing with increasing value of nonlocal parameter μ. This trend is same or all the Winker elastic moduli considered. Further, percent change of non dimensional critical buckling load is defined as

$$100 \times \left[(N_{cr}\_0) - (N_{cr}\_05)\right] / N_{cr}\_05 \tag{29}$$

where N_{cr}_0 and N_{cr}_05 are non dimensional critical loads at nonlocal parameter $\mu = 0$ and 0.05, respectively. This percentage change in critical load for various moduli of Winkler foundation is plotted in Fig. 5. From this figure

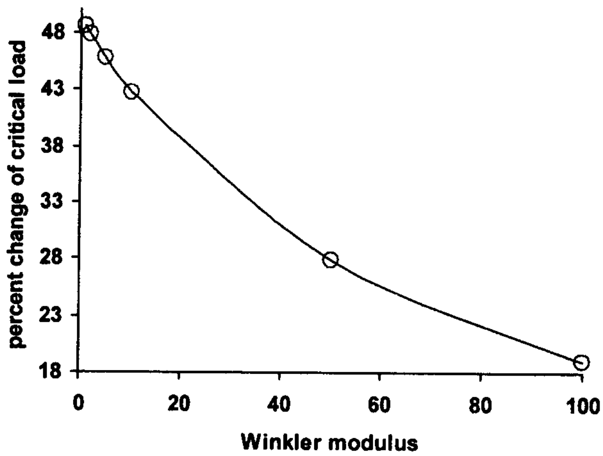

Fig.5 Percent change of critical load with various Winkler elastic modulii

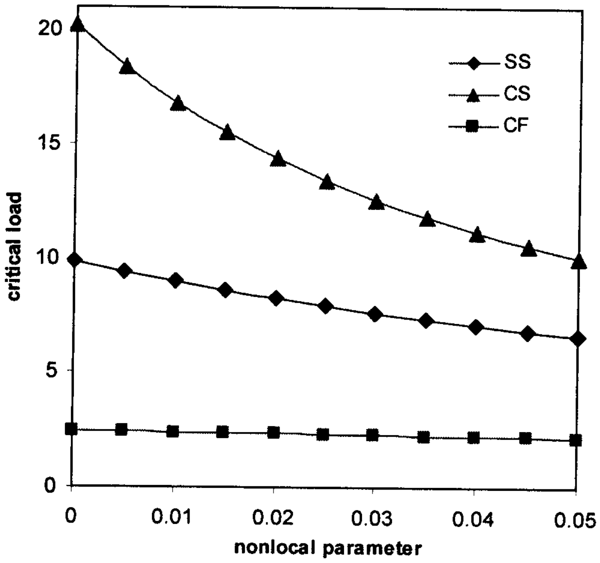

Fig.6 Effect of nonlocal parameter and boundary conditions on critical buckling load

one could observe that the percentage change of critical load is larger for low modulus of Winkler foundation and smaller for high modulus of Winkler foundation. This percentage change of critical load is 48 percent for the case of no Winkler foundation (k=0). Further this percentage change of critical load is around 19 percent for the case of non dimensional Winkler foundation modulus of 100. Thus it is concluded that change of critical load decreases exponentially with increase in magnitude of Winkler elastic moduli.

Effect of Nonlocal Parameter and Boundary Conditions

Effect of nonlocal parameter on non dimensional critical buckling loads for three different boundary conditions are investigated and results are shown in Fig.6. These boundary conditions are (i) simply supported - simply supported (SS), (ii) Clamped - simply supported (CS) and (iii) clamped - free (CF). From Fig. 6 it is noted that effect of nonlocal parameter on CS boundary condition is larger as compared to SS boundary condition. Similarly, it is also observed that effect of nonlocal parameter on SS boundary condition is larger as compared to CF boundary condition. Further it is note that that effect of nonlocal parameter for CF boundary condition is insignificant and could be ignored.

Conclusions

Computer code based on modified differential quadrature method (MDQM) is developed for the buckling analysis of one dimensional member. Developed code includes nonlocal theory, modulus of Winkler elastic foundation and three boundary conditions. Non dimensional critical buckling loads for a simply supported simply supported beam (i) with elastic foundation (ii) without elastic foundation and (iii) with nonlocal theory are being computed. These results are found to be in good agreement with the corresponding results found in the literature. It is observed that with seven interpolation points reasonably converged results are obtained for various nonlocal parameter μ values, Winkler foundation moduli and boundary conditions.

Further, it is found that the influence of nonlocal parameter on non dimensional critical buckling load for the higher mode is larger and more nonlinear as compared to the lower modes. It is also observe that non dimensional critical loads decrease with increase of nonlocal parameter μ. From the present investigation it is concluded that change of critical load decreases exponentially with increase in magnitude of Winkler elastic foundation moduli. Furthermore, it is found that effect of nonlocal parameter on clamped - simply supported (CS), simply supported - simply supported (SS) and clamped - free (CF) are in decreasing order. It is also noted that effect of nonlocal parameter on clamped - free boundary condition is not significant.

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