



## ANALYSIS OF SMART COMPOSITE PLATES - A FINITE ELEMENT APPROACH

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### Abstract

*A 4-noded finite element with seven degrees of freedom per node, based on higher-order shear deformation theory, is used for the analysis of smart composite plates. This model does not include voltage/electric potential as a degree of freedom and is valid for both surface-mounted and embedded piezoelectric elements either distributed or placed in patches. The effect of actuator voltage on the static and dynamic behaviour of such plates is studied. It is seen that the reduction in deflection is proportional to the actuator voltage and the actuator voltage does not have significant influence on the frequency of vibration. An active control of vibration is achieved by suitably amplifying the voltage sensed by the sensor.*

### Notation

$a$	= length of the plate
$b$	= width of the plate
$h$	= total thickness of the plate
$q$	= intensity of load
$u_o, v_o, w_o$	= mid-plane displacements in x,y and z directions
$w$	= central transverse deflection of the plate
$\bar{w}$	= non-dimensional central deflection
$E_1, E_2$	= Young's moduli along and transverse direction of the fibre
$G_{12}, G_{13}, G_{23}$	= in-plane and transverse shear moduli
$\xi, \eta$	= natural co-ordinates
$\nu_{12}, \nu_{21}$	= Poisson's ratios
$\bar{\sigma}_x, \bar{\tau}_{xz}, \bar{\tau}_{yz}$	= non-dimensional stresses
$\theta_x, \theta_y$	= total slopes in x and y directions

### Introduction

A large number of studies have been reported on smart or adaptive structures and structural components of laminated form involving piezoelectric patches, layers or elements. Reviews of such studies have been presented by different authors [1-4]. The fundamental work of Tiersten [5] gave much of the necessary theoretical base for the static and dynamic analysis of a single-layer piezoelectric plate. Analytical solutions for simply supported piezoelectric plates have been presented by Heyliger [6]. Lee [7] presented the theoretical background for laminated piezoelectric plates with distributed actuators or sensors. Crawley and Lazarus [8] developed a 'consistent plate

model' for the analysis of induced strain actuator systems. They presented a few exact solutions and Ritz approach to find approximate solutions. The models and solutions were also verified experimentally.

Detwiler et al. [9] used the first-order shear deformation theory (FSDT) to develop a finite element model for the static and dynamic behaviour of laminated composites containing distributed actuators and sensors. They used a 4-noded isoparametric element with five structural degrees of freedom per node and one additional electric degree of freedom per element for each piezoelectric layer. Bhattacharya et al. [10] developed a finite element model for the free vibration analysis of smart laminated composite beams and plates, using FSDT. A finite element model for the static analysis of plates with piezoelectric actuators was presented by Lin et al. [11]. This model contained an actuator element, an adhesive interface element and an eight-node isoparametric plate element based on first-order shear deformation theory. An analytical solution was also presented by them. Qu and Tong [12] presented a finite element formulation based on FSDT for modeling a laminated composite plate with distributed piezoelectric sensors/actuators.

It has been established that a higher-order shear deformation theory (HSDT) is necessary to predict the responses of a laminated composite plate accurately, particularly in the case of thick plates. But, available literature on the application of a higher-order theory to smart composite plates is very little. Mitchell and Reddy [13] developed a refined hybrid theory wherein an

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equivalent single-layer theory (third-order shear deformation theory) was used for the mechanical displacement field and a layer-wise discretisation was used to model the electric potential function for piezoelectric laminae. This work was closely related to the earlier theoretical work of Lee [7]. Exact solutions and finite element solution were presented by Ray et al. [14, 15] for the static analysis of composite plates with distributed actuator and sensor layers. They developed an 8-noded isoparametric element based on higher-order shear deformation theory.

Ha et al. [16] presented a three-dimensional finite element formulation for the static and dynamic analyses of laminated composites containing distributed piezoceramic sensors and actuators. Three-dimensional elements are computationally expensive and inefficient because of very large number of elements required, particularly in the case of thin plates. Cen et al. [17] developed a new quadrilateral finite element for the analysis of laminated composite plates containing distributed piezoelectric layers. The mechanical part of the element formulation was based on first-order shear deformation theory. Layer-wise linear theory was applied to deal with electric potential.

Early works on vibration control of laminated composite plates were based on the classical laminated plate theory (CLPT). Lam et al. [18] developed a finite element model based on the CLPT for the active vibration control of a composite plate containing distributed piezoelectric sensors and actuators. The negative velocity feedback control algorithm was used in a closed control loop to couple the direct and converse piezoelectric effects. Chandrashekhara and Agarwal [19] used the FSDT to develop a finite element formulation for composite plates with integrated piezoelectric sensors and actuators. In contrast to the earlier models, this formulation did not use voltage as an additional degree of freedom. Wang et. al [20] developed an isoparametric finite element based on FSDT and used the negative velocity feedback control for the active vibration control analysis of smart composite plates with bonded or embedded distributed sensors and actuators.

The authors have already reported the results of a study on the performance of four different shear deformable plate bending finite elements when used for the static and free vibration analysis of laminated composite plates [21, 22]. It was concluded that a 4-noded element with seven degrees of freedom per node, based on higher-order shear

deformation theory (HSDT), was required to predict the deflection as well as stresses accurately.

To the best of this authors' knowledge, there is no published work on the development of a finite element model based on HSDT for the active vibration control of composite plates. An attempt is made to use the simple 4-noded element based on higher-order shear deformation theory for studying the linear static and dynamic behaviour, including active vibration control, of smart composite plates.

**Finite Element Formulation**

A 4-noded finite element model based on higher-order shear deformation theory [23] with seven degrees of freedom per node, viz.,  $u_o, v_o, w_o, \partial w_o / \partial x, \partial w_o / \partial y, \theta_x$  and  $\theta_y$ , is used to analyse laminated composite plates with piezoelectric elements. Standard shape functions for isoparametric formulation are used to interpolate  $u_o, v_o, \theta_x$  and  $\theta_y$  and a non-conforming shape function based on Hermitian interpolation is used to interpolate transverse displacement 'w' as

$$w(\xi, \eta) = \sum_{i=1}^4 \frac{1}{8} \left[ (1 + \xi \xi_i)(1 + \eta \eta_i) (2 + \xi \xi_i + \eta \eta_i - \xi^2 - \eta^2) \right] (w_o)_i + \sum_{i=1}^4 \frac{a_1}{8} \left[ \xi_i(1 + \xi \xi_i)^2 (1 + \eta \eta_i) (\xi \xi_i - 1) \right] \left( \frac{\partial w_o}{\partial x} \right)_i + \sum_{i=1}^4 \frac{b_1}{8} \left[ \eta_i(1 + \eta \eta_i)^2 (1 + \xi \xi_i) (\eta \eta_i - 1) \right] \left( \frac{\partial w_o}{\partial y} \right)_i \tag{1}$$

where  $a_1$  and  $b_1$  are half the length of the element in X- and Y- directions respectively.

Voltage is not introduced as an additional degree of freedom as is done by some researchers [14-16]. This simplifies the finite element modelling. This model is valid for both continuous and segmented piezoelectric elements that are either surface-bonded or embedded in the laminated plate.

Geometry of a laminated plate with surface-bonded piezoelectric actuators is shown in Fig.1. The piezoelectric

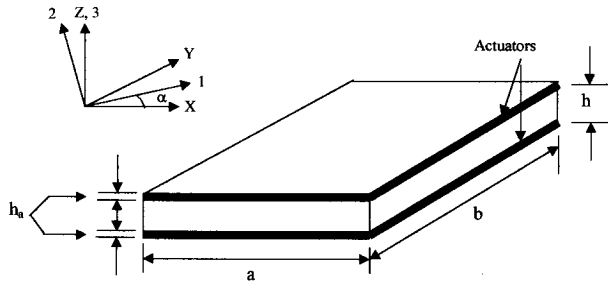


Fig. 1 Geometry of a laminated plate with piezoelectric elements

elements are assumed to be sufficiently thin and perfectly bonded to the substrate.

The linear constitutive equations coupling the elastic and the electric fields in a piezoelectric medium are given by the direct and the converse piezoelectric equations respectively

$$\{D\} = [e] \{\varepsilon\} + [\epsilon] \{E\} \quad (2)$$

$$\{\sigma\} = [C_a] \{\varepsilon\} - [e]^T \{E\} \quad (3)$$

where  $D$ ,  $e$ ,  $\varepsilon$ ,  $\epsilon$ ,  $E$ ,  $\sigma$  and  $C_a$  are the electric displacement vector, piezoelectric stress matrix, strain vector, piezoelectric permittivity, electric field vector, stress vector and constitutive matrix of the actuator layer, respectively.

The total potential energy of an actuator element,  $\Pi_a^{(e)}$ , is equal to the strain energy,  $U_a^{(e)}$ , and is given by

$$\Pi_a^{(e)} = U_a^{(e)} = \int_V U_{ao}^{(e)} dV \quad (4)$$

where  $U_{ao}^{(e)}$  is the strain energy density of the element given by

$$U_{ao}^{(e)} = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \quad (5)$$

Substituting Eq.(3) in Eq.(5) and integrating, we get

$$U_{ao}^{(e)} = \frac{1}{2} \{\varepsilon\}^T \left( [C_a] \{\varepsilon\} - 2 [e]^T \{E\} \right) \quad (6)$$

When the piezoelectric stress matrix,  $[e]$ , is unavailable, it can be expressed in terms of the piezoelectric strain matrix,  $[d]$ , as

$$[e] = [d] [C_a]$$

Hence, Eq. (6) is rewritten as

$$U_{ao}^{(e)} = \frac{1}{2} \{\varepsilon\}^T [C_a] \left( \{\varepsilon\} - 2 [d]^T \{E\} \right) \quad (7)$$

Substituting Eq.(7) in Eq.(4), we get

$$U_a^{(e)} = \int_V \frac{1}{2} \{\varepsilon\}^T [C_a] \left( \{\varepsilon\} - 2 [d]^T \{E\} \right) dV \quad (8)$$

In Eq. (8), the first term represents the stiffness of the actuator element and the second term represents the equivalent nodal load vector induced by the electric field.

That is,

$$\{R_a\} = \int_V \{\varepsilon\}^T [C_a] [d]^T \{E\} dV \quad (9)$$

Equation (9) is rewritten as

$$\{R_a\} = \int_A [\bar{B}]^T [\bar{N}] dA = \int_{-1}^1 \int_{-1}^1 [\bar{B}]^T [\bar{N}] |J| d\xi d\eta \quad (10)$$

where  $\bar{B}$  is the strain-displacement matrix and

$$[\bar{N}] = \sum_{k=1}^{N_a} \int_{z_k}^{z_{k+1}} [C_a]_k [d]^T \{E\} dz \quad (11)$$

In Eq. (11),  $N_a$  is the number of actuator layers,  $[C_a]$  is the constitutive matrix of the actuator layer transformed to global coordinates and  $\{E\}$  is the electric field vector equal to  $\{0 \ 0 \ V/h_a\}^T$  where 'V' is the voltage applied to the actuator in the thickness direction only and  $h_a$  is the thickness of the actuator layer.

Since voltage is not introduced as an additional degree of freedom, this finite element model is simple and straightforward and makes the analysis essentially the same as that of an ordinary laminated composite plate, with the piezoelectric elements also contributing to the

stiffness of the plate. The load vector includes equivalent nodal loads induced by the electric field, as given in Eq.(10), in addition to nodal loads due to external forces. For the model based on higher-order shear deformation theory, Eq.(11) is written as

$$[\bar{N}] = [P] \{d\} \text{ in which } [P] = \begin{bmatrix} N_{\Delta} \\ M_{\Delta} \\ M_{\Delta}^* \end{bmatrix} \text{ and}$$

$$\{d\} = [d_{31} \ d_{32} \ d_{33} \ 0 \ 0 \ 0]^T$$

Also,

$$N_{\Delta} = \sum_{k=1}^{N_a} \int_{z_k}^{z_{k+1}} E [C_a]_k dz, \quad M_{\Delta} = \sum_{k=1}^{N_a} \int_{z_k}^{z_{k+1}} E [C_a]_k z dz,$$

$$M_{\Delta}^* = \sum_{k=1}^{N_a} \int_{z_k}^{z_{k+1}} E [C_a]_k z^3 dz$$

**Numerical Results**

**Static Analysis**

All the results presented in this paper are obtained using a reduced integration scheme (ie., 2x2 integration for both bending and transverse shear terms) for the evaluation of stiffness matrix and 16 x16 mesh division for the full plate [21].

To test the correctness of the program, a thin cantilever laminated composite plate, of size 200mm x 200mm and thickness 1mm with surface-bonded actuators of thickness 0.1mm, for which results were reported by Lam et al. [18] using classical laminated plate theory, is analysed. The stacking sequence of the layers is PZT/-45/45/-45/45/PZT. The material properties used are:

Substrate:

$$E_1 = 150 \text{ GPa}, E_2 = 9 \text{ GPa}, \nu_{12} = 0.3, G_{12} = G_{13} = 7.1 \text{ GPa}, G_{23} = 2.5 \text{ GPa}$$

Actuators :

$$E = 63.0 \text{ GPa}, \nu = 0.3, G = 24.2 \text{ GPa}, d_{31} = d_{32} = 254.0 \times 10^{-12} \text{ m/V}$$

Equal amplitude voltages with opposite signs are applied across the thickness of the upper and lower piezoelectric layers, which are used as actuators. The deflected shapes of the centre-line of the plate under a uniform load of 100N/m<sup>2</sup> and different actuator voltages are given in Fig. 2. Results of the present analysis agree very well with those given by Lam et al. [18].

Numerical studies are carried out to determine the effect of actuator field on the response of laminated plates with piezoelectric elements, having different boundary conditions and width-to-thickness ratios. In all cases, actuators are assumed to be surface-bonded, having a layer thickness of 1% of the total thickness of the substrate. The properties of substrate and piezoelectric materials are assumed to be the following in all cases, unless otherwise specified.

Substrate:

$$E_1 = 175 \text{ GPa}, E_2 = 7.0 \text{ GPa}, \nu_{12} = 0.25, G_{12} = G_{13} = 3.5 \text{ GPa}, G_{23} = 1.4 \text{ GPa}$$

Actuator:

$$E = 63.0 \text{ GPa}, \nu = 0.3, d_{31} = d_{32} = 254.0 \times 10^{-12} \text{ m/V}$$

The size of the plate considered is 1m x 1m.

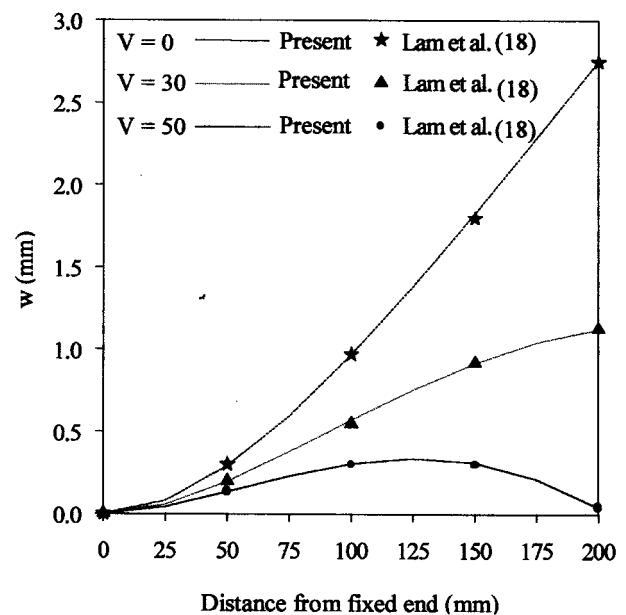


Fig. 2 Centre-line deflection under uniform load and different actuator voltages

Symmetric 4-layer cross-ply and angle-ply laminates with piezoelectric elements [(PZT/0/90/90/0/PZT) and (PZT/45/-45/-45/45/PZT)], simply supported at all edges and subjected to a uniformly distributed load, are considered to study the effect of actuator voltage with  $b/h$  ratio. A non-dimensional load,  $Q = qb^4/E_2h^4 = 10$ , is applied. The percentage reduction in deflection, with an actuator voltage of 50 volts, for different width-to-thickness ratios, is presented in Fig.3. It is observed that for a voltage of 50

volts, the reduction in deflection is negligible in the case of thick plates ( $b/h \leq 20$ ). But as the plate becomes thinner, there is a very steep increase in the reduction in deflection. For  $b/h = 30$ , the reduction is only 1% whereas it is nearly 30% for  $b/h = 100$ . It is also seen that the percentage reduction is slightly more in cross-ply laminates than in angle-ply laminates. This can be attributed to the smaller stiffness of cross-ply laminates. The percentage reduction in stresses for the same plates is given in Figs.4-6. From

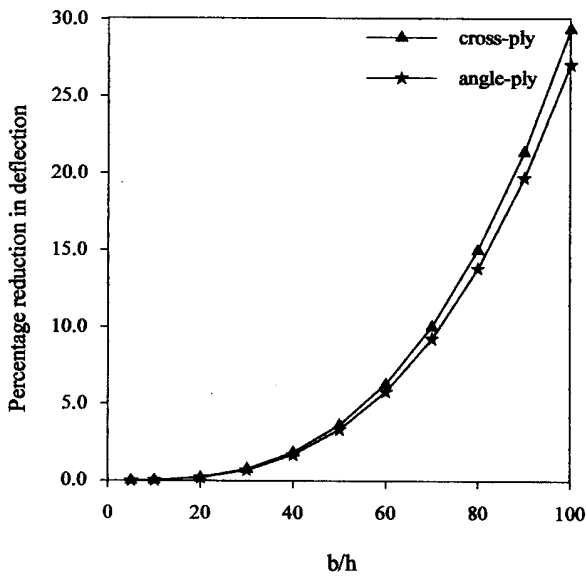


Fig. 3 Variation of percentage reduction in deflection with  $b/h$  ratio

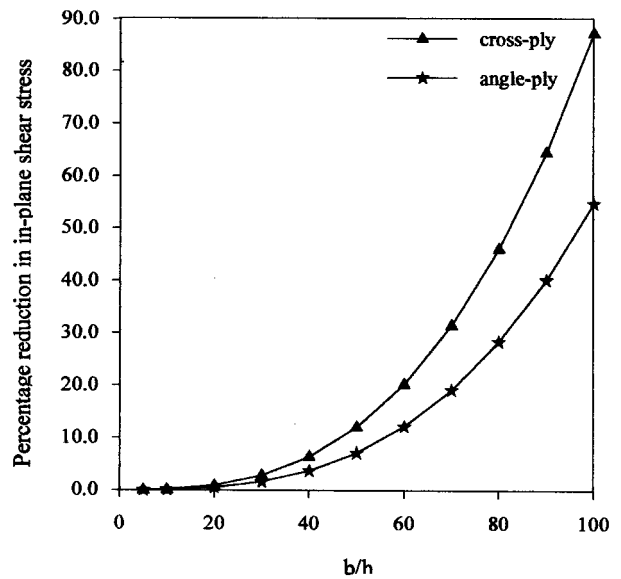


Fig. 5 Variation of percentage reduction in in-plane shear stress with  $b/h$  ratio

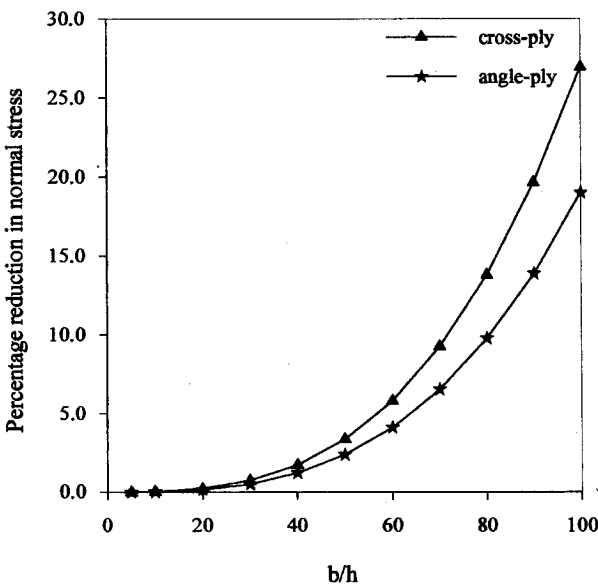


Fig. 4 Variation of percentage reduction in normal stress with  $b/h$  ratio

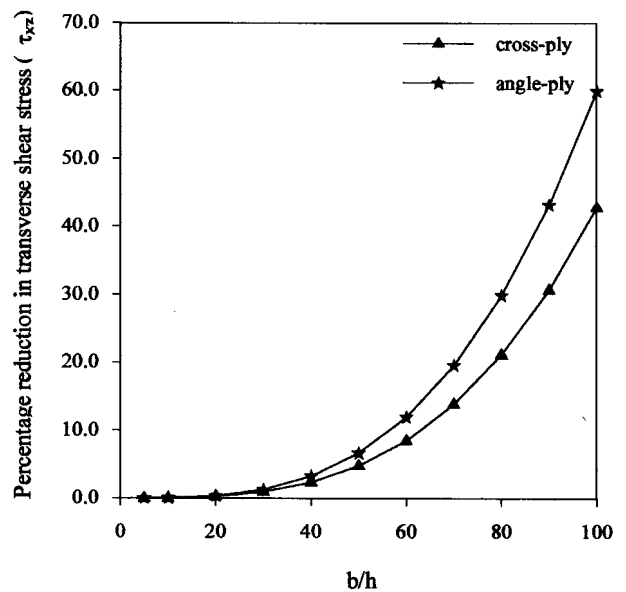


Fig. 6 Variation of percentage reduction in transverse shear stress with  $b/h$  ratio

these figures, it is seen that the percentage reduction in shear stresses is more compared to normal stress. Also the difference in percentage reduction of stresses between cross-ply and angle-ply laminates is more than the difference in percentage reduction in displacement, with lower values exhibited by angle-ply laminates except for transverse shear stress.

A square simply supported 4-layer cross-ply laminate (PZT/0/90/90/0/PZT) subjected to a uniform load is considered to study the effect of Q/V ratio (Q being the non-dimensional load expressed as  $Q = qb^4/E_2h^4 = 10$  and V, the voltage applied to the actuator). The variation of non-dimensional central deflection [ $\bar{w} = 100E_2wh^3/(qb^4)$ ] with various Q/V ratios, keeping Q the same, for different thickness ratios (b/h = 10, 50 and 100), is shown in Fig. 7. From the figure it is clear that thick plates require very high voltage to attain appreciable reduction in deflection compared to thin plates.

It is apparent from the foregoing analyses that for a given value of width-to-thickness ratio and mechanical load acting on the smart plate, there exists a particular voltage of actuator for which the mid-plane deflection is zero. To study this aspect, a square simply supported 4-layer substrate (PZT/0/90/90/0/PZT), subjected to a sinusoidal load,  $q = q_0 \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}$ , where  $q_0 = 500 \text{ N/m}^2$  and a sinusoidal voltage of the form,

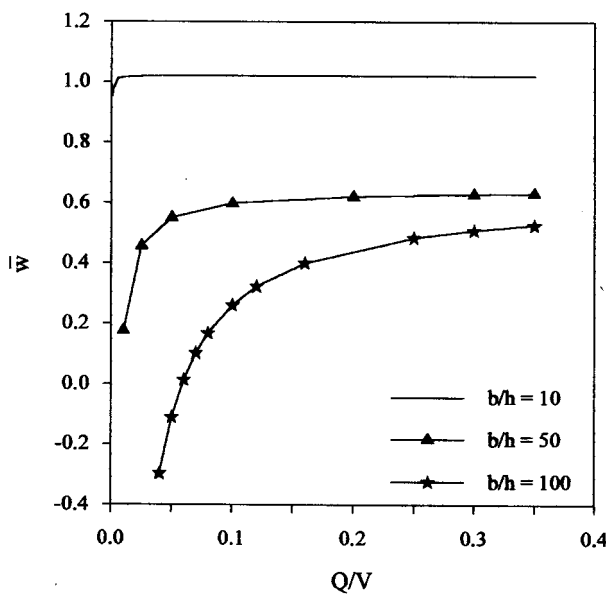


Fig. 7 Variation of non-dimensional central deflection with Q/V ratio

$V = V_o \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}$ , with equal and opposite sign to the actuators, is analysed. Fig.8 illustrates such a value of voltage necessary to make the deflection zero for a particular value of b/h. From the figure, it is clear that the effectiveness of the actuator increases linearly as the value of b/h decreases. i.e., higher voltage is required to be applied to the actuator surface as b/h increases. This is not in contradiction to the observation in the previous example because the load was non-dimensional in that example.

To study the effect of piezoelectric elements bonded in patches, a square simply supported 4-layer cross-ply laminate (PZT/0/90/90/0/PZT) is analysed for a uniform non-dimensional load of  $Q = 100$ . The aim of the study is to determine the size of the piezoelectric patch required to reduce the deflection of substrate to around 50%. The sides of the patch placed at the centre are increased from 25% to 100% of the sides of the plate with a constant voltage of 1000 volts applied to the actuators. The corresponding deflections are presented in non-dimensional form in Fig.9. The non-dimensional central deflection of the 4-layer substrate without piezoelectric layers under the uniform loading is found to be 0.6850. From the figure, it is seen that the deflection is reduced to around 51% with a piezoelectric patch of size 0.625a and 0.625b. It is also seen that the deflection decreases at a fast rate upto a patch size of 0.75a and 0.75b, beyond which increase in patch size does not cause much additional decrease in deflection.

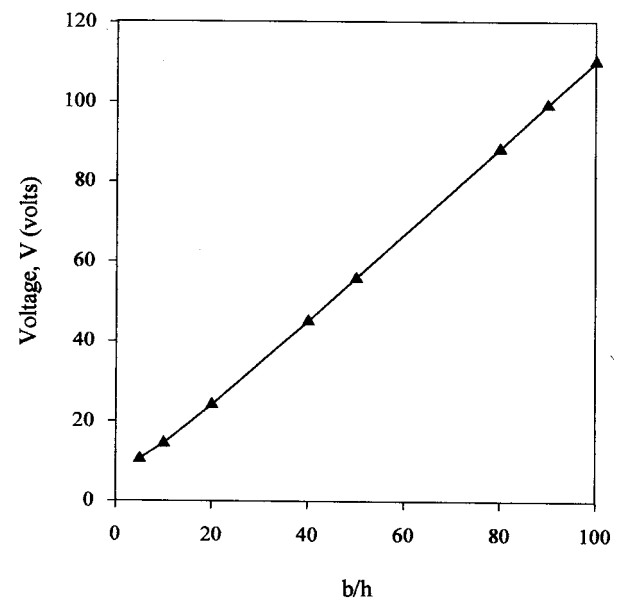


Fig. 8 Values of voltage for zero mid-plane deflection

**Dynamic Analysis**

Dynamic response of smart composite plates, when the piezoelectric elements at top and bottom surfaces are used as actuators, is studied in this section. Element mass matrix is developed by HRZ lumping scheme [24] and the effect of damping is taken into account in terms of damping ratios. The electrical load vector developed as a result of the applied voltage is given in Eq.(10). Time-wise history of displacement response is calculated using Wilson- $\theta$  method [25]. The displacement at any time under consideration is calculated by considering the effect of mechanical load as well as the electrical load.

To have an idea regarding the percentage reduction in amplitude of vibration, simply supported 4-layer cross-ply (0/90/90/0) and angle-ply (45/-45/-45/45) laminates with surface-bonded piezoelectric elements are considered. A constant voltage of 50 volts with equal and opposite sign is applied to the actuators. In both cases, a uniform load of non-dimensional value,  $Q = 50$ , is applied. The thickness of each actuator layer is considered as 1% of the total thickness of substrate. Geometry and material properties of the plate are given below.

$$a = b = 0.25 \text{ m}, h = 0.0025 \text{ m}.$$

Actuators :

$$E = 63.0 \text{ GPa}, \nu = 0.3, d_{31} = d_{32} = 254.0 \times 10^{-12} \text{ m/V}, \rho = 7600 \text{ kg/m}^3$$

Substrate :

$$E_1 = 175.0 \text{ GPa}, E_2 = 7.0 \text{ GPa}, \nu_{12} = 0.25, G_{12} = G_{13} = 3.5 \text{ GPa}, G_{23} = 1.4 \text{ GPa}, \rho = 2700 \text{ kg/m}^3$$

Peak values of displacements and stresses obtained for different width-to-thickness ratios, with and without voltage applied to the actuators, are expressed in non-dimensional form as given below and are presented in Figs.10-12.

$$\bar{w} = 100wE_2h^3/qb^4, \bar{\sigma}_x = \sigma_x h^2/qb^2, \bar{\tau}_{xz} = \tau_{xz} h/qb, \bar{\tau}_{yz} = \tau_{yz} h/qb$$

All response quantities are found to decrease with the application of voltage, the reduction increasing with  $b/h$  ratio. A reduction of 27% for cross-ply laminates and 26% for angle-ply laminates is observed in the case of plates with  $b/h = 100$ .

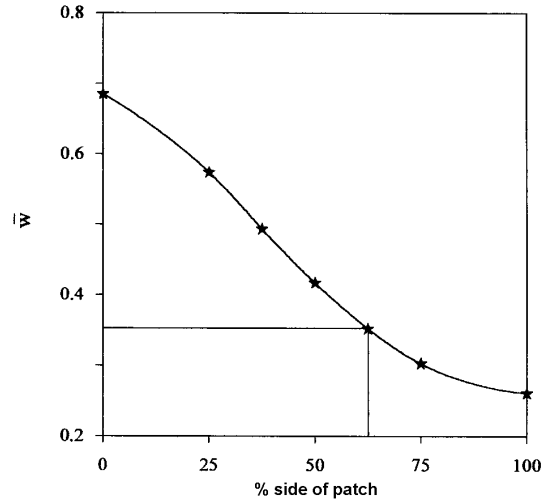


Fig. 9 Reduction in central deflection with size of patch

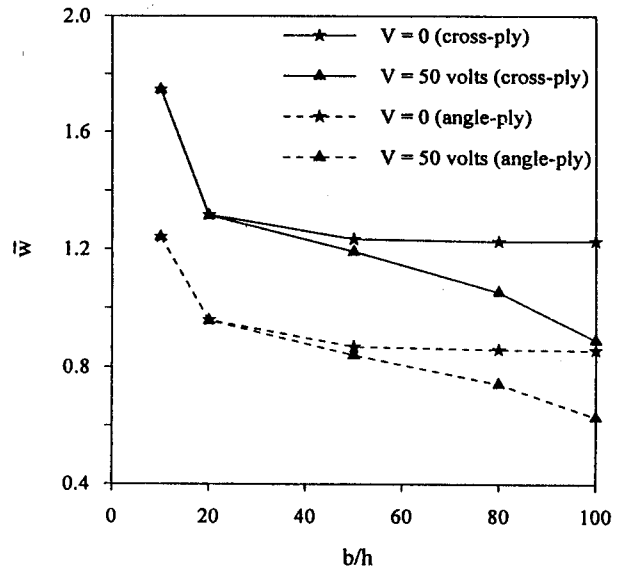


Fig. 10 Variation of peak displacement with b/h ratio

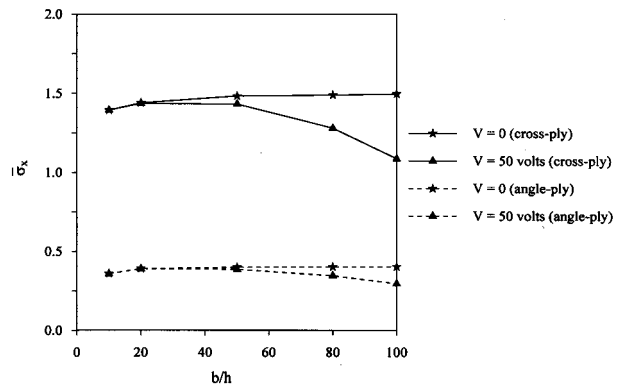


Fig. 11 Variation of peak normal stress with b/h ratio



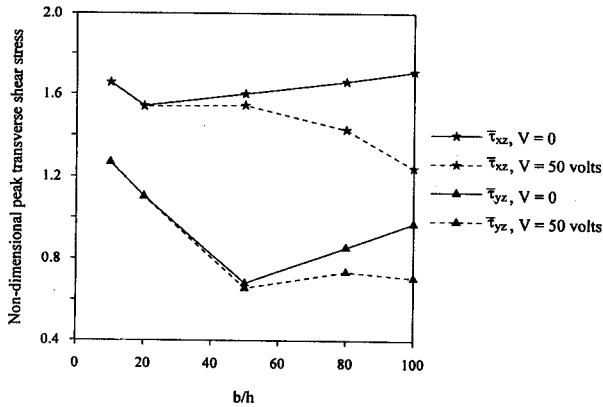


Fig. 12 Variation of peak transverse shear stress with  $b/h$  ratio (cross-ply)

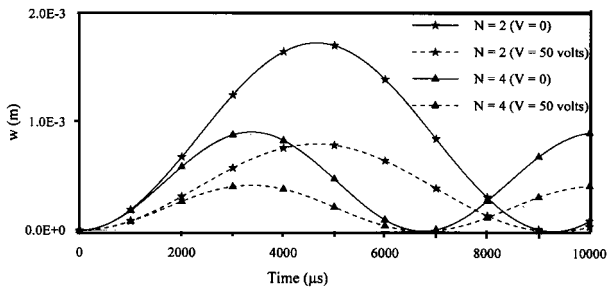


Fig. 13 Effect of actuator voltage with number of layers

To study the effect of actuator voltage with number of layers, simply supported thin ( $b/h = 100$ ) smart cross-ply laminates with anti-symmetric arrangements are considered having two (PZT/0/90/PZT) and four (PZT/0/90/0/90/PZT) layers. In both cases, a suddenly applied uniform load of non-dimensional value,  $Q = 25$  is considered. The variation of central deflection without voltage and with equal and opposite voltage of 50 volts, for the two stacking sequences is presented in Fig.13. From the figure, it is seen that there is a reduction in deflection of around 50% from 2-layers to 4-layers, both with and without actuator voltage. This implies that the number of layers does not have any additional influence on the reduction in deflection.

**Vibration Control**

The generic idea of electrically actuating piezoelectric elements in order to develop a desired mechanical deformation of the elements can be extended to the synthesis of smart structures with controllable dynamic response characteristics. This is accomplished by simultaneously considering the mechanical properties of the load-bearing

structure, electro-mechanical properties of the piezoelectric materials employed as sensors and actuators and their spatial distribution in the smart structure, the mechanical properties of the bond at the interface between the piezoelectric elements and the substrate, the control algorithm for the system and the desired response characteristics.

The charge generated across the thickness of each individual piezoelectric lamina can be obtained using Gauss law as

$$Q_s^k(t) = \frac{1}{2} \left[ \int_{S_l(z=z_k)} D \, dx \, dy + \int_{S_l(z=z_{k+1})} D \, dx \, dy \right] \quad (12)$$

where  $S_l = S_b \cap S_p$  is the intersection of electrode surfaces on both sides of the lamina. As the electrodes are assumed to be placed on the transverse surfaces with the poling direction  $z$ , only the component  $D_z$  of the electric displacement vector is nonzero. Since no external electric field is applied, the sensed charge can be calculated from Eq. (12) using Eq. (2), where

$$D = D_z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ e_{31} & e_{32} & e_{33} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = e_{31} \epsilon_x + e_{32} \epsilon_y + e_{33} \gamma_{xy} \quad (13)$$

In third-order shear deformation theory, the charge developed is evaluated as follows.

$$Q_s(t) + \frac{1}{2} \int_s \left[ 2 \left( e_{31} \epsilon_x^o + e_{32} \epsilon_y^o + e_{33} \gamma_{xy}^o \right) + \left( z_k + z_{k+1} \right) \left( e_{31} \kappa_x + e_{32} \kappa_y + e_{33} \kappa_{xy} \right) + \left( z_k^3 + z_{k+1}^3 \right) \left( e_{31}^* \kappa_x^* + e_{32}^* \kappa_y^* + e_{33}^* \kappa_{xy}^* \right) \right] dx \, dy \quad (14)$$

The current,  $I(t)$ , on the surface of the sensor is given by

$$I(t) = \frac{dQ_s(t)}{dt} \quad (15)$$

In order to numerically simulate Eq. (15), the simple backward difference numerical differentiation is used.

To study the influence of piezoelectric effect on the response of cross-ply and angle-ply laminates in thin and thick plate range, simply supported 4-layer symmetric and anti-symmetric cross-ply and angle-ply laminates

with  $b/h = 10$  and  $100$  and having the geometry and properties same as those in dynamic analysis are analysed. A damping ratio of  $0.02$  is assumed and a uniformly distributed load having a non-dimensional value of  $50$  is suddenly applied to the plate surface. Velocity type controller with gains of  $2000$  volts/ampere and  $4000$  volts/ampere is used. The charge developed in each element under the action of external load is evaluated using Eq. (14) and the corresponding electrical load vector due to amplified voltage is evaluated using Eq. (10). The central displacements obtained for thick ( $b/h = 10$ ) and

thin ( $b/h = 100$ ) symmetric cross-ply laminates with and without gain are presented in Figs.14 and 15. From the figures it is clear that the central displacement decays faster for the actuator with proper gain, as compared to actuator without gain. Moreover, vibration dies out much faster for thick plates. Also it has been observed that the gain cannot be increased beyond a certain value from where the effect of actuator reduces to that of negative damping. Other results are not presented for the sake of brevity.

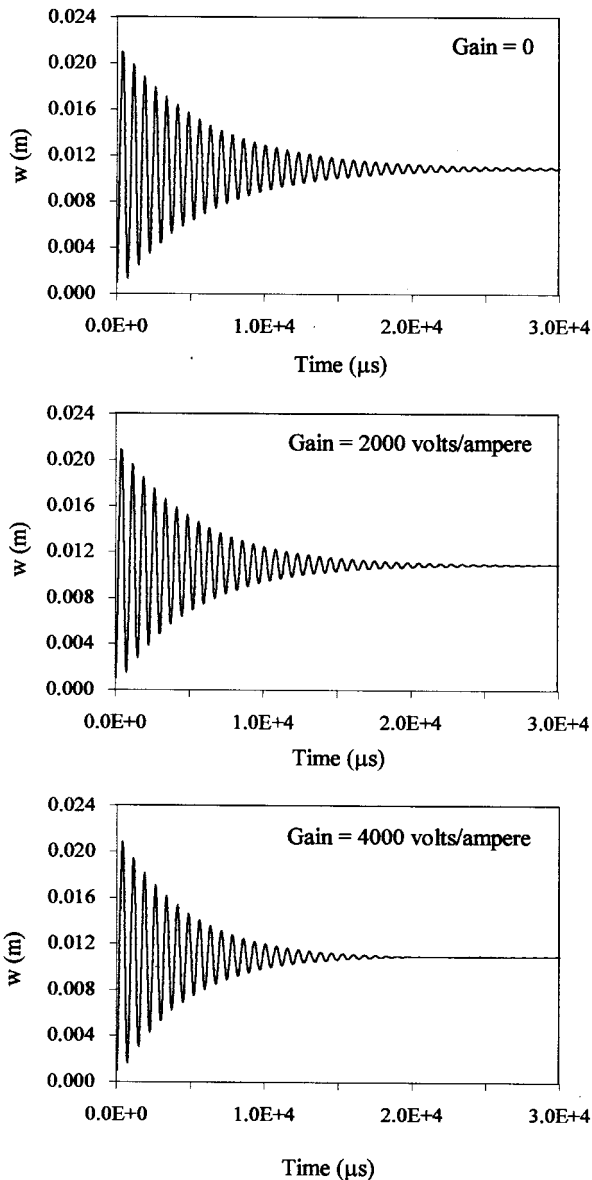


Fig. 14 Displacement response of 4-layer symmetric cross-ply laminate with different gains ( $b/h = 10$ )

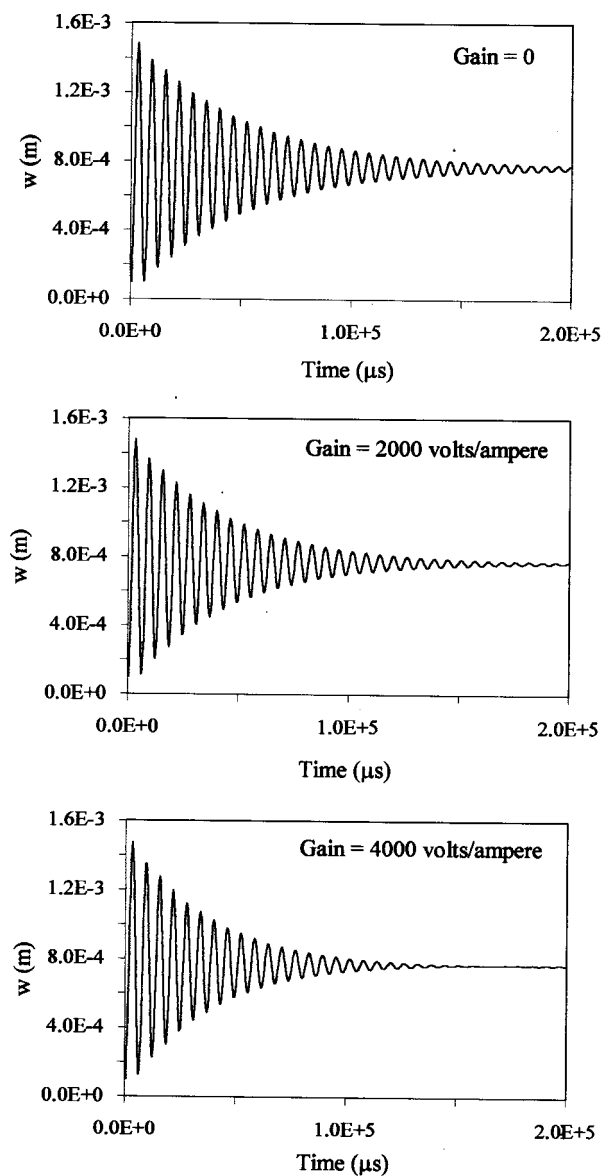


Fig. 15 Displacement response of 4-layer symmetric cross-ply laminates with different gains ( $b/h = 100$ )

### Conclusions

A finite element model based on third-order shear deformation theory with displacement degrees of freedom alone is presented for the analysis of smart composite plates. Numerical studies conducted to understand the behaviour of such plates under static and dynamic loadings reveal the following facts. Reduction in deflection is directly proportional to the actuator voltage, in the case of both thin and thick plates. The rate of reduction in deflection corresponding to a given voltage is more in thin plates than in thick plates for a given non-dimensional load. In other words, thick plates require very high voltage to attain considerable reduction in transverse deflection compared to thin plates. In the thin plate range, the percentage reduction in deflection and stresses increases at a very fast rate as  $b/h$  ratio increases. A smaller patch at a proper location depending on the required reduction in deflection is better than a piezoelectric layer of the same length and width as that of the substrate. The voltage applied to the actuator layer of a smart composite plate does not have significant influence in the frequencies of vibration. The amplitude of vibration reduces with increase in voltage without changing the frequency of vibration. An active control of vibration is achieved by suitably amplifying the voltage sensed by the sensor and using a control algorithm. The simple finite element model used in the study is capable of predicting the static and dynamic behaviour of laminated composite plates with piezoelectric elements.

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