ISENTROPIC SWIRLING FLOW THROUGH SUPERSONIC NOZZLES - PART I (FREE VORTEX FLOW)

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Abstract

An analytical model has been developed and an approximate solution for the free vortex swirling supersonic flow through nozzles. Earlier work by Mager, is potential flow based. In the present model swirl number has been redefined to confine the flow properties within the valid range. Flow properties are always determined by the total conditions (P_0 and a_0), nozzle profile and swirl intensity as in the case of one-dimensional flow. Integral form of the continuity equation has been used in order to satisfy the boundary conditions, since radial velocity component is ignored. For free vortex case, it has been shown that there exists a minimum radius along the length of the nozzle at which the density is zero and flow seems to be void causing reasonable blockage to the flow at throat. In defining the non-dimensional swirl number (β), axial velocity at each section was employed for non-dimensionalization as compared to Mager's definition of using a global constant to find swirl number (α). The variations of performance parameters of the nozzle such as co-efficient of discharge (C_d) thrust efficiency (η_s) and impulse efficiency (η_I) and flow field parameters such as density, pressure, velocity etc. are presented as a function of swirl numbers (α, β) and compared with the available experimental data. The swirl numbers (α, β) are also correlated.

Nomenclature

a	= velocity of sound
f_1, f_s	= non-dimensional functions
k_1, k_2	= constants
$\dot{m}$	= mass flow rate
m	= mass flow rate (non-dimensionalised)
n	= integral index
r, θ, z	= cylindrical polar co-ordinates
v_r	= velocity component (radial)
v_θ	= velocity component (tangential)
v_z	= velocity component (axial)
A	= area
C_d	= coefficient of discharge
I	= integral function of void region
M	= Mach number of the specific (free vortex) flow
P	= pressure
R	= nozzle wall radius
T	= temperature
Th	= thrust on the exit plane
V	= velocity
V_θ	= velocity component at the nozzle wall (tangential)
V_z	= velocity component at the nozzle wall (axial)
α	= swirl number $\dfrac{V_\theta}{(V_\theta)_{max}}$
β	= swirl number $\dfrac{V_\theta}{V_z}$
λ	= swirl number $\dfrac{(V_\theta)_{r_\upsilon}}{V}$
ρ	= density
γ	= ratio of specific heats
Γ	= circulation
ϕ	= potential function
$\xi=r/R$	= non-dimensional radius
η_s	= thrust efficiency
η_I	= impulse efficiency

Subscript

0	= stagnation condition
ϑ	= edge of void

Superscript

*	= condition at the throat

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Introduction

Swirling flow in a nozzle occurs in a number of propulsion applications including the flow in turbofan and turbojet engines, spin stabilized rockets and integral rocket/ramjets. In certain cases the tangential velocity component is introduced implicitly by the flow itself due to the geometry as in turbofan and turbojets or due to spin stabilization. But in other cases such as integral rocket/ramjets the swirl is introduced explicitly in the conventional axial flow for the efficient utilization of propellant by mixing and stabilization. In general the gases are to be expanded from designated total conditions (P_0 and a_0) to the ambient conditions that prevail outside the nozzle. While for the rocket on its mission ambient conditions are inconsistent with the designed one and drastic changes in ambient pressure are observed which prevents the nozzle to expand the gases as desired. Due to improper and unsteady expansion, variations of thrust and impulse are reported. One of the strategies to address this problem is control of mass flow. The control of mass flow either by changing the total conditions or restructuring the profile is not so easy and leads to complicated mechanism. Hence the alternative is the mass blockage in the supersonic nozzle by introduction of swirl either by fixed vanes or rotating vanes at the inlet so that reasonable assumptions can be made. The conservation of angular momentum warrants intense swirl at the minimum area and weak swirl at the maximum area. Its axial velocity is impaired so that the addition of swirl can be thought as an effective means of restricting the throat area. No major losses of thrusts are expected as thrust occurs in the exit plane. The explicit introduction of swirl is also considered in other devices for different purposes as in the case of retaining heavy uranium atom inside the chamber in nuclear rockets, and in plasma rockets for arc stabilization and to reduce the heat transfer problem by uniform temperature distribution. Other areas in which swirl has considerable effect are noise suppression due to the reduction in the number of visible cells, cell structure variations and reduction of wavelength of primary shock cells. It is clear that the main objective of introducing the swirl in the otherwise conventional axial flow is for control of mass flow rate, noise suppression and enhancement of propellant mixing and stabilization. The swirl introduced at the upstream will persist to some extent throughout the flow.

The swirling nozzle flow is not so easy to analyze as the Mach Number varies both radially and axially which puts up lot of constraints to develop a simple analytical model. Of the infinite number of swirl profiles the two significant profiles are free vortex in which υ_θ is inversely proportional to radius and υ_z is constant over the section and solid body rotation or forced vortex in which both υ_θ and υ_z vary radially. Combination of the two profiles in any proportion are practically significant and those profiles are classified as inner biased and outer biased based on the centroid of the profile [1]. The fundamental problem is the establishment of choking criterion. By carrying over the concept of choking in the one-dimensional case either maximization of mass or sonic velocity criterion can be adopted. Even though the maximization of mass does not stipulate exact condition of choking but provides reasonable flow properties compared to the sonic velocity criterion. The earliest work was by Mager [2] in 1961, who presented the solution of isentropic swirling potential flow through a C-D nozzle without radial velocity component for effective mass blockage. The compressibility was missing in his work as "M" is defined as some non-dimensional velocity [3]. A free vortex profile without radial velocity component and specific heat ratio ($\gamma = 1.4$) and maximization of mass for choking criterion was used. His principle was extended by Swithenbank and Scotter [4] in 1964, to the case with $\gamma = 1.25$, whereas Glick and Kilgore [5] in 1967, presented results for the mass flux with γ in the range of 1.10 to 1.28 for the case of free vortex flow. Similar results were obtained by Bussi [6] in 1974, but he used a different technique based on variational principle. His theory was substantiated by Batson [7] in 1970 experimentally. The existence of vacuum core in the Mager's work [2] lead to speculation that reversed flow may occur along the axis, when a swirling flow exhausts to a finite exit pressure and it has been demonstrated by Donaldson et al. [8] in 1962 and So [9] in 1967 that the condition of reversed flow will occur only under certain conditions, when a strong vortex flow exhaust to atmospheric pressure. King [10] shows that reversed flow along the axis would not occur through the nozzle throat, when any real distribution of circulation is used. The case of choked forced vortex flow was attempted by Bastress [11] in 1965 with sonic velocity criterion, Manda [12] in 1966 who uses the constant enthalpy assumption to derive the variations of axial velocity and Carpenter and Johannesen [1, 13] in 1975 derived the choking criterion for a general swirling flow without making any additional assumptions beyond those usually made for quasi-cylindrical theory. The radial variations are presented by Lewellen et al. [14] in 1969, in which the numerical solution of choked forced vortex nozzle flow in the quasi-one dimensional form. The thrust characteristics, impulse efficiency and co-efficient of discharge for forced vortex flow are presented by

Gostintsev [15, 16] for a particular family of velocity profile assumed to exist at the throat known as Caldonazzos flows. The case of weak swirling flows with fairly large radii of curvature was covered by Moore [17] in 1964 and Smith [18] in 1971 who extended Halls analytical method for non-swirling flows. Hsu. C.T., [3] in 1971, shows that the swirling phenomena of an inviscid non-isoenergetic, non-isentropic flow are governed by Crocco's theorem and also demonstrated that the results obtained by Mager [2] for free vortex problem and the axial Mach Number distribution over a given section obtained by Lewellen [14] for forced vortex problem can be reduced from his results. The numerical simulation of swirling flow through nozzles were attempted by Armitage [19] in 1967 by explicit Lax-Wendroff scheme and Dutton [20] in 1987 with clines [21] code. The investigation of swirling viscous flow in supersonic propulsion nozzle was presented by Chang et al. [22] in 1989. Experimental studies of swirling flow in nozzles were attempted by Binnie et al. [23] in 1957, Iserland [24] in 1958 and with low swirl intensities by Massier [25, 26, 27] in 1965. Parkinson [28] in 1967, presents the effect on the mass flow of swirl through a supersonic nozzle. He made experiments with air to investigate this effect and enumerate the practical differences in the flow from the free vortex form of the theoretical model. He shows that the discharge is in general less than the ideal predictions.

In the present work an approximate solution for the free vortex (considered potential flow) through a nozzle is presented and compared with the available experimental results [28]. The flow field properties such as Non-dimensional velocity (Mach Number of the related flow M), pressure P, temperature t and the density ρ and the performance parameters such as co-efficient of discharge, thrust and impulse efficiency are presented as a function of swirl number (β). The swirl number (β) is defined as the ratio of tangential velocity at the wall (V_θ) to the axial velocity at the wall (V_z). But alternatively Mager has defined swirl number (α) as the ratio of tangential velocity at the wall to the stagnation enthalpy ($\sqrt{2C_pT_0}$ a global constant). When α^* at the throat tends to one the related mass flow tends to zero which is practically insignificant, since the swirl is considered as a secondary effect. The actual flow by which the swirl is imparted is not considered for discussion. An analytical model involving redefined (to confine the flow properties within the valid range) swirl numbers, use of integral form of continuity equation to satisfy the boundary conditions and employing axial velocity at each section for non-dimensionalization has been successfully attempted. The influence of swirl numbers (α, β) as the nozzle performance parameters have been evaluated and correlated with available experimental data.

Nozzle Profile

For this purpose a typical converging diverging (C-D) nozzle [20] with cylindrical inlet, conical convergent and divergent sections and circular arc transitions between these sections are considered (Fig.1). The area contraction ratio between the nozzle inlet and throat is $A/A^* = 0.668$, one used for the ramjet systems and corresponds to a Mach Number of 2.4682 at the exit. The nozzle inlet parameters are P_0 and a_0 and swirl number (α/β). Specific heat ratio of $\gamma = 1.4$ is used for the calculations.

Analytical Model

Consider a potential function ϕ in the cylindrical coordinate system r, θ, z with the corresponding velocity components υ_r ,υ_θ , and υ_z. The potential ϕ which assumed to be irrotational, not a function of r and the υ_z to be constant over the section is defined as

$$\phi = \Gamma\theta + \int v_z dz \tag{1}$$

where the vortex strength at the axis Γ (circulation per unit radians) is super imposed with axial velocity υ_z (Z). Let V_θ and V_z be the tangential and the axial velocity components at the wall respectively (V_z is constant over the section). Then

$$\phi = \frac{\partial}{\partial\theta}[RV_\theta] + \int v_z dz \tag{2}$$

and the components of velocity are given by

$$v_r = 0; \; v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\phi}; \; v_z = \frac{\partial}{\partial z}(RV_\theta) + V_z$$

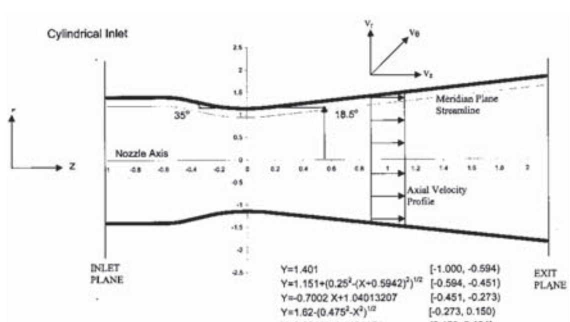

Fig. 1 C-D NOZZLE Geometry for Ramjet with M=2.46826

For the flow to be purely axial, it is implied from the Equation 2 that RV_θ has to equal k_1. Hence the total velocity $V^2 = \left(\frac{k_1}{r}\right)^2 + V_z^2$. The potential function must satisfy the continuity equation subject to boundary conditions. Since the radial velocity component is ignored, integral form of the continuity equation has been employed and the mass flow rate is given by $\dot{m} = \int_A \rho\, V_z\, dA$. By isentropic relation the density is defined as

$$\rho = \rho_0 \left[1 - \frac{\gamma - 1}{2a_0^2} \left(\left(\frac{k_1}{r}\right)^2 + V_z^2 \right) \right]^{\frac{1}{\gamma - 1}} \tag{3}$$

Since the profile is free vortex, there exists a minimum radius $r = r_\upsilon(z)$ at which the density (ρ) is zero and below which the void region exists

$$r_v^2 = \frac{\frac{\gamma - 1}{2} k_1^2}{a_0^2 - \frac{\gamma - 1}{2} v_z^2} \tag{4}$$

upon substitution of this limiting value the mass flow rate is given by the constant (k_2)

$$k_2 = 2\pi\rho_0 v_z \left[1 - \frac{\gamma - 1}{2} \left(\frac{V_z}{a_0}\right)^2 \right]^{\frac{1}{\gamma - 1}} \left(\int_{r_v}^{R} \left[1 - \left(\frac{r_v}{r}\right)^2 \right]^{\frac{1}{\gamma - 1}} r dr \right) \tag{5}$$

Defining the non-dimensional quantities

$$\xi = \frac{r}{R};\ M^2 = \frac{V_z^2}{(a_0^2 - \frac{\gamma - 1}{2} V_z^2)};\quad m = \frac{k_2}{\rho_0 A^* a_0};$$

$$\alpha = \frac{V_\theta}{(V_\theta)_{\max}} = \frac{k_1}{Ra_0} \sqrt{\frac{\gamma - 1}{2}}$$

for the definition of Mach number (M), even though υ_θ component was not involved its value is identical to what would exists for no swirl case. Hence the equations (5) and (4) can be written as functionals of (ξ_υ, M)

$$f_1(\xi_v, M) = \frac{MI}{\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma + 1}{2(\gamma - 1)}}} = \frac{A^*}{A} m \tag{6}$$

$$f_2(\xi_v, M) = \frac{\xi_v^2}{1 + \frac{\gamma - 1}{2} M^2} = \alpha^2 \tag{7}$$

where $I = 2 \int_{\xi_v}^{1} \left[1 - \left(\frac{\xi_v}{\xi}\right)^2 \right]^{\frac{1}{\gamma - 1}} \xi d\xi$

for $\gamma = 1.4$ i.e., $\gamma = \frac{n+1}{n-1}$ where $n = 6$ the integral is given by

$$I(\xi_v, 1.4) = \sqrt{(1 - \xi_v^2)}\,(1 + \frac{2}{3}\xi_v^2 (7 - \xi_v^2)) - \frac{5\xi_v^2}{2} \ln \left[\frac{1 + \sqrt{(1 - \xi_v^2)}}{1 - \sqrt{(1 - \xi_v^2)}} \right] \tag{8}$$

whose plot is shown in Fig.2, as $\xi_\upsilon^2 \to 1$; $I \to 0$.

It is observed that the functions f_1 and f_2 representing the mass and the swirl number should be a maximum at the throat. At the throat, equation (6) and (7) shows that the specific values of a^* and ξ_υ^* is a function of M^* only. Hence differentiating with respect to M^* and equating it to zero to satisfy the condition for maxima,

$$\left(\frac{df_1}{dM}\right)^* = \left(\frac{\partial f_1}{\partial M} + \frac{\partial f_1}{\partial \xi_v} \frac{\partial \xi_v}{\partial M}\right)^* = 0$$

$$\left(\frac{df_2}{dM}\right)^* = \left(\frac{\partial f_2}{\partial M} + \frac{\partial f_2}{\partial \xi_v} \frac{\partial \xi_v}{\partial M}\right)^* = 0$$

by elimination of Jacobian, the condition for maximum mass flow rate is given by

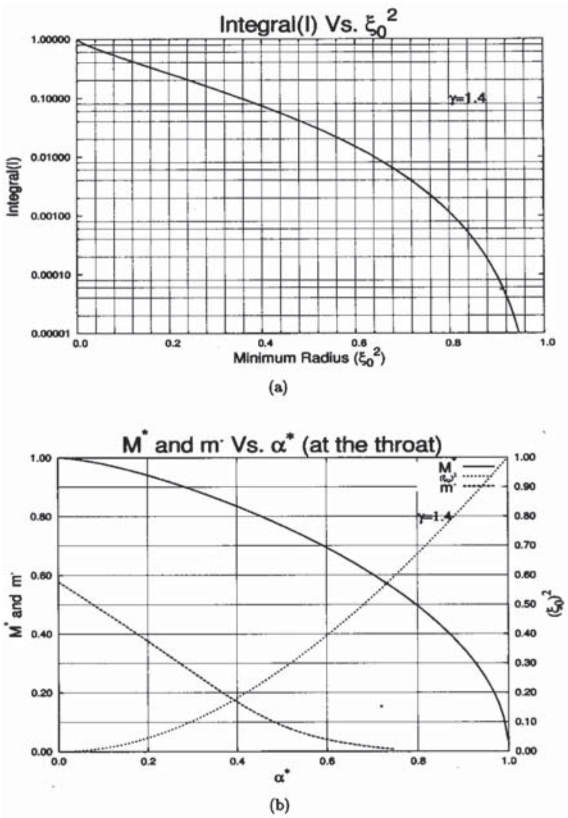

Fig. 2 (a) Variation of I Vs. Min. area (b) Variation of α^* Vs. M^*, m and ξ_v^2 at the throat

$$\left(\frac{\partial f_1}{\partial M}\frac{\partial f_2}{\partial \xi_v}\right)^* = \left(\frac{\partial f_1}{\partial \xi_v}\frac{\partial f_2}{\partial M}\right)^* = 0$$

which shows that for the fixed value of swirl a^* at the throat the minimum radius ξ_υ^2 is a function of non-dimensional velocity M^* only.

By substituting the partial derivatives in equation (7) and after some manipulations the derivative of $I(\xi_\upsilon)$,α^* (ξ_υ^*) and M^* $(\alpha^*, \xi_\upsilon^*)$ are given by

$$\alpha^* = \xi_v^* \left(1 - \frac{1}{\frac{3-\gamma}{\gamma-1} + \frac{2[1-(\xi_v^*)^2]^{\frac{1}{\gamma-1}}}{I^*}}\right)^{1/2} \tag{9}$$

$$\frac{\partial I}{\partial \xi_v} + \frac{2}{\xi_v}\left[I - \left(1-\xi_v^2\right)^{\frac{1}{\gamma-1}}\right] \tag{10}$$

$$(M^*)^2 = \frac{2}{\gamma-1}\left[\left(\frac{\xi_v^*}{\alpha^*}\right)^2 - 1\right] \tag{11}$$

Thus the equations determine the values of minimum radius (ξ_υ^*) and Mach Number (M^*) at the throat for different values of swirl value (α^*). Its plot for $\gamma = 1.4$ is shown in Fig.2. It is observed from the figure that as $\alpha^* \to 1$, $\xi_\upsilon^* \to$ and $M^* \to 0$ and for the no swirl case as $\alpha^* \to 0$, $M^* \to 1$, it approaches the conventional flow without swirl. Hence the non-dimensional mass flow is given by the equation (6)

$$m^{\cdot} = \frac{M^* I^*}{[1 + \frac{\gamma-1}{2}(M^*)^2]^{\frac{\gamma+1}{2(\gamma-1)}}} \tag{12}$$

The variation of m for various values of swirl intensity (α^*) at the throat is shown in Fig.2. In this way all the necessary quantities pertinent to the determination of the flow in the nozzle are established.

Defining the Swirl Number

A number of possibilities of non-dimensionalization and its correlation with nozzle performance parameters such as co-efficient of discharge, impulse efficiency were presented by Dutton [20]. The swirl number α is defined as the ratio of tangential component of velocity at the wall to its maximum value as if the entire energy is converted to tangential velocity component approaching the limit of zero flow conditions $(M^* \to 0)$. So when α at the throat goes to 1, virtually there will not be any flow and the entire throat is blocked. It does not have any significance as the swirl is considered to be a secondary effect. Hence an attempt has been made to redefine the swirl number. One such is defining it as the ratio of tangential velocity component at the minimum radius $(\upsilon_\theta)_{\tau\upsilon}$ to the total velocity.

$$\lambda = \frac{r_v (\upsilon_\theta)_{r_v}}{[R\sqrt{(\upsilon_\theta)_{r_v}^2 + (\upsilon_z)^2}]} \tag{13}$$

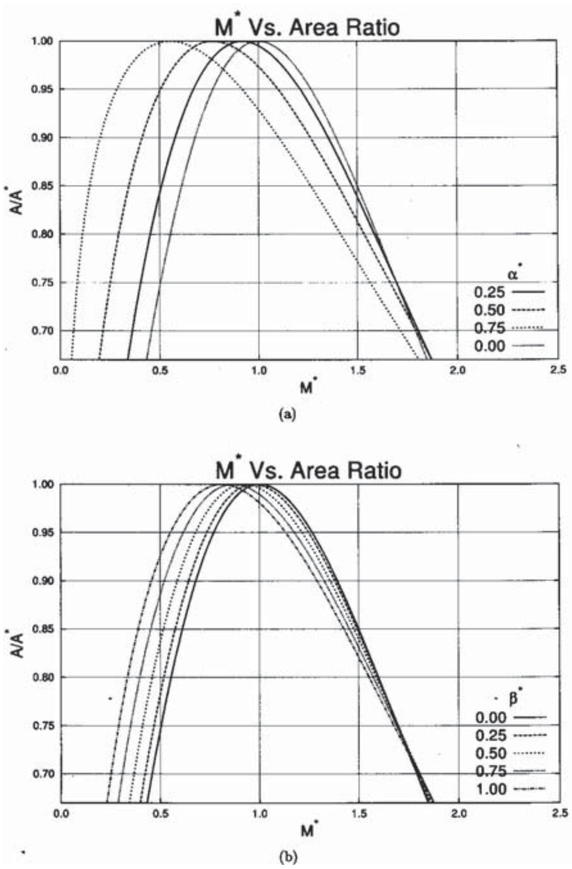

Fig. 3 Variation of M Vs. A/A for different values of alpha and beta*

defining it in terms of non-dimensional numbers

$$\lambda^2 = \frac{\xi_v^2}{\left[2 + \frac{\gamma - 1}{2}(M_*)^2\right]} \tag{14}$$

Based on the principle of maximization of the mass flow, the relationship between M^* and ξ_υ is given by

$$\frac{(M^*)^2 - 1}{(\gamma - 1)\,(M^*)^2} = \left[1 - \frac{(1 - \xi_v^2)^{\frac{1}{\gamma - 1}}}{I}\right]\left[\frac{1 + \frac{\gamma - 1}{2}}{2 + \frac{\frac{\gamma - 1}{2}}{(M^*)^2}}\right] \tag{15}$$

with the Mach Number

$$(M^*)^2 = \frac{2}{\gamma - 1}\left[\left(\frac{\xi_v^*}{\lambda^*}\right)^2 - 2\right] \tag{16}$$

which shows that as $\lambda \to 1$, $\xi_\upsilon \to 1$ for which the value of $(M^*)^2$ is negative and invalid that restricts the upper limit of λ^*. Similarly for the lower limit as $\lambda \to 0$ it makes indeterminate form and hence the flow will exists only for a shorter range of values for λ^*.

Another form of non-dimensionalization is presented as the ratio of tangential velocity component V_θ and axial velocity component V_z at the wall.

$$\beta = \frac{V_\theta(z)}{V_z(z)} \text{ where } V_z(z) = \frac{a_0 M}{\sqrt{1 + \frac{\gamma - 1}{2}(M^*)^2}} \quad \text{and}$$

$$a_0 = \sqrt{\gamma R T_0}$$

To proceed with the computation to any other section other than throat, the conservation of angular momentum $\alpha R = \alpha^* R^*$ and by equation (7),

$$\frac{\left(\frac{\xi_v}{\alpha^*}\right)^2}{1 + \frac{\gamma - 1}{2}M^2} = \frac{A^*}{A} \tag{17}$$

Hence the β after some manipulation is given by

$$\beta = \frac{\alpha}{M}\sqrt{\frac{2}{\gamma - 1}}\;\frac{\xi_v}{\alpha^*}\,\frac{R}{R^*} \tag{18}$$

and at the throat the equation reduces to

$$\beta^* = (\xi_v^*/M^*)\sqrt{\frac{2}{\gamma - 1}} \tag{19}$$

By restricting the magnitude of non-dimensionalization it is possible to achieve reasonable flow with β^* ranging from 0 to 1 at the throat. In order to correlate the new swirl number (β) with (α), a least square parabolic fit has been made,

$$\alpha^* = -1.69323734 \times 10^{-4} + 0.41829462\beta^* - 0.595541(\beta^*)^2 \tag{20}$$

which almost behaves linearly in the lower range as shown in Fig.5(a). The (β) at any other section is related to the (β^*) at the throat by

$$\beta = \beta^* \left(\frac{\alpha}{\alpha^*}\right)\left(\frac{x_v}{x_v^*}\right)\left(\frac{M^*}{M}\right)\left(\frac{R}{R^*}\right) \tag{21}$$

The effect of swirl on the mass flow rate, the Mach Number (M^*) and on the minimum radius (ξ_υ^2) are shown in Fig.2(b).

Estimation of Flow Properties at Other Sections

As one would expect for the given stagnation conditions and the swirl intensity, once the mass flow rate is established, the flow properties at any other section can be given by the equations (6) and (7) with conservation of angular momentum

$$\frac{A^*}{A} = \frac{1}{m}\left[\frac{MI}{1+\frac{\gamma-1}{2}M^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}} = \frac{\left(\frac{\xi_v}{\alpha^*}\right)^2}{1+\frac{\gamma-1}{2}M^2} \tag{22}$$

The computed variations of $\frac{A}{A^*}(M)$ for various values of α and β are shown in Fig.3. β shows a valid range of the flow, even though it is a process of scaling down of α. Once the mass flow rate is estimated, the non-dimensional velocity at the throat M^* and minimum radius (ξ_υ^2) as a function of area ratio from the equation (22) is evaluated as follows. The two simultaneous algebraic equations with two unknowns (ξ_υ) and (M) at any arbitrary section have been evaluated by the gradient method. Let $f_1(\xi_\upsilon, M) = 0$ and $f_2(\xi_\upsilon, M) = 0$ be the two simultaneous equations. By expanding it over the Taylor's series with two independent variables (ξ_υ, M) and ignoring the higher order terms, and that the assumed values are close to the root, the system of equations are given by

$$\frac{\partial f_1}{\partial \xi_v} h + \frac{\partial f_1}{\partial M} k = -f_1(\xi_v, M)$$

$$\frac{\partial f_2}{\partial \xi_v} h + \frac{\partial f_2}{\partial M} k = -f_2(\xi_v, M)$$

Now the problem is the evaluation of the step size h and k. By Cramer's rule it is given by

$$h = \frac{1}{J}\begin{vmatrix} -f_1(\xi_v, M) & \partial f_1/\partial M \\ -f_2(\xi_v, M) & \partial f_2/\partial M \end{vmatrix} \tag{23}$$

$$k = \frac{1}{J}\begin{vmatrix} -f_1(\xi_v, M) & \partial f_1/\partial \xi_v \\ -f_2(\xi_v, M) & \partial f_2/\partial \xi_v \end{vmatrix} \tag{24}$$

where the Jacobian J is given by

$$J = \frac{\partial(f_1, f_2)}{\partial(\xi_v, M)}$$

Estimation of Performance Parameters

The performance parameters such as coefficient of discharge (C_d) thrust efficiency (η_s) and impulse efficiency (η_I) are defined as follows to estimate the variations in the efficiency of the nozzle due to the introduction of swirl. The coefficient of discharge is defined as the ratio of mass flow rate with swirl to the ideal conditions.

$$C_d = \frac{\dot{m}}{\dot{m}_{(\beta=0)}} = \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{M^* I^*}{\left[1+\frac{\gamma-1}{2}(M^*)^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

The thrust (T) in its non-dimensionalised form is defined as $T = \frac{\text{thrust}}{P_0 A}$

$$= \frac{1}{P_0 A}\int_A (P + \rho V_z^2)\ ; \eta_S \text{ (at the exit plane)} = \frac{Th_{\beta^*}}{Th_{\beta=0}}$$

$$= \left(\frac{1+\frac{\gamma-1}{2}M^2_{(\beta=0)}}{1+\frac{\gamma-1}{2}M^2}\right)$$

$$\left(\frac{\gamma(1+M^2)I - (\gamma-1)(1-\xi_v^2)\left(\frac{\gamma}{\gamma-1}\right)}{1+\gamma M^2_{(\beta=0)}}\right) \tag{26}$$

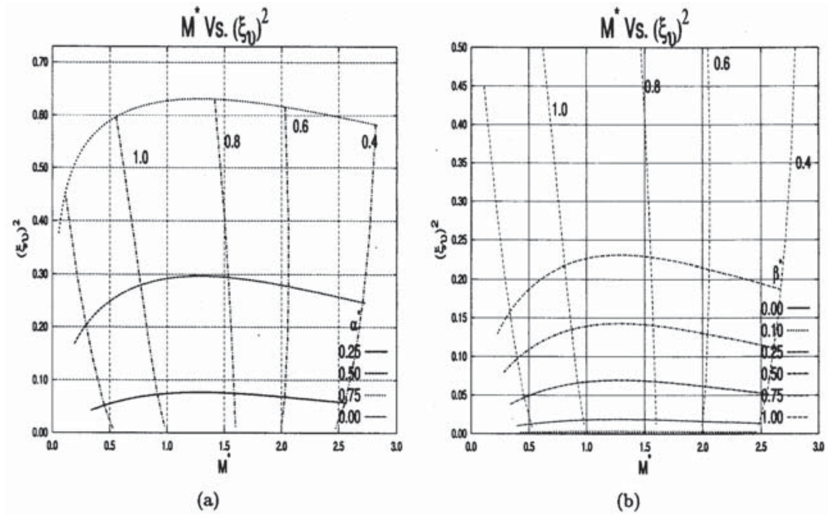

Fig. 4 Variations of Minimum Area (ξ_v^2) Vs. Mach Number (M^*) at the throat with constant area contours for α^* and β^*

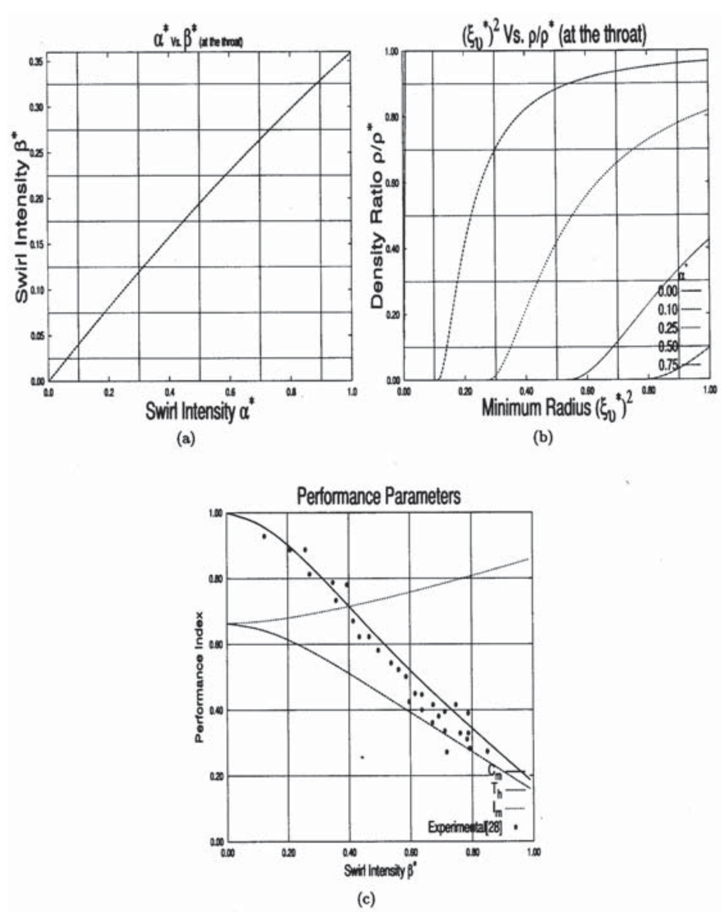

Fig. 5 (a) Varitaion of α^* Vs. β^* (b) Radial Variation of Density at the throat (c) Performance Parameters

The variation of these parameters with published experimental data [28] for different swirl intensities (β) is shown in Fig.5.

Results and Discussion

The computed variations of the constant I as a function of minimum radius (ξ_υ^2) is shown in Fig.2(a) and $\frac{A*}{A}(M)$ for various values of ($\alpha*/\beta*$) are shown in Fig.3. It has been shown that as $\xi_\upsilon^2 \rightarrow 0, I \rightarrow 1$ and $\xi_\upsilon^2 \rightarrow 1, I \rightarrow 0$ logarithmically.

As seen from the figure, all the curves are quite similar in shape and differ from the no swirl case by shifting towards lower Mach Number at the throat. For the maximum value of $\alpha*(0.75)$, the Mach Number at the throat is 0.6. But for same value of swirl with β it has been scaled down by achieving a Mach Number (0.8) which seems to be quite reasonable. By increasing the value of $\alpha*$ to 1, the Mach Number approaches the limit.

$$\lim_{\xi_v^2 \rightarrow 1} M^* = \sqrt{\frac{5}{7}(1-(\xi_v^*)^2}.$$

But for the case of $\beta^*= 1$ at the throat the Mach Number for choked conditions are 0.75. Convenient summary of present results is presented in Fig.4 which shows the variations of minimum radius as a function of Mach Number for various values of swirl intensity α^*/β^* with constant area lines. It is apparent that when located far downstream of the throat (neat exit) the Mach Number of the related flow tends to be more than that of the one-dimensional flow at same A*/A. Also it is clear that the minimum radius ξ_υ^2 became a maximum slightly down stream of the throat. Hence it shows that the swirl tends to have its greatest blockage near the throat. Further it has been shown by Mager [2] that maximum values of ξ_υ^2 do not necessarily imply the maximum values of ξ_υ^2.

The variations of pressure, density and temperature along the length of the nozzle for various values of swirl intensity are plotted in Figs. 6, 7, 8, 9, 10 and 11. The pressure variations shown that the expansion is not uniform for all values of swirl, so that it matches with the exhaust conditions. Hence the pressure ratio (P_0/P_e) is not constant for different values of swirl and over expanded to match the exhaust conditions. As the swirl intensity increase it is not possible for the gases to expand as desired and loss of throat and efficiency is expected. The radial variations of density at the throat is shown in Fig.5(b). It is quite reasonable at this juncture to remember that the real flow near the axis of rotation is dominated by viscous effects and hence the potential flow solutions invalid. Because of this viscous dominance the tangential velocity component will have a distribution of forced vortex. Hence it is expected that the density will not go to zero monotonically near the axis and this region will not be completely void of the flow. Due to sharp changes in υ_θ this region is easily identified and can be considered as core. There is a considerable variation of C_d with the swirl and in general it is always less than the ideal conditions.

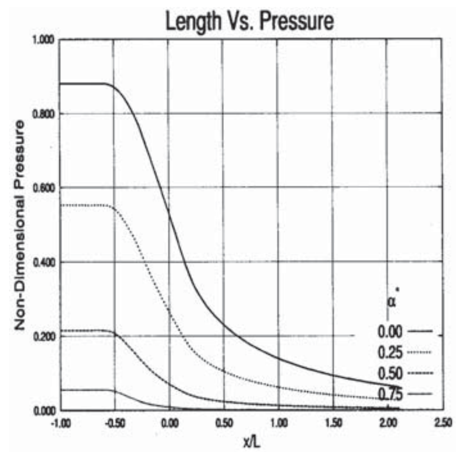

Fig. 6 Pressure (alpha)

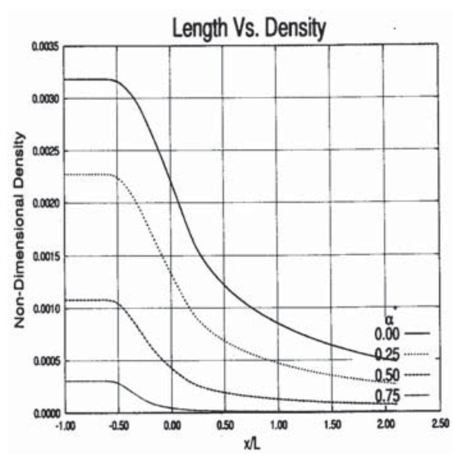

Fig. 7 Desity (alpha)

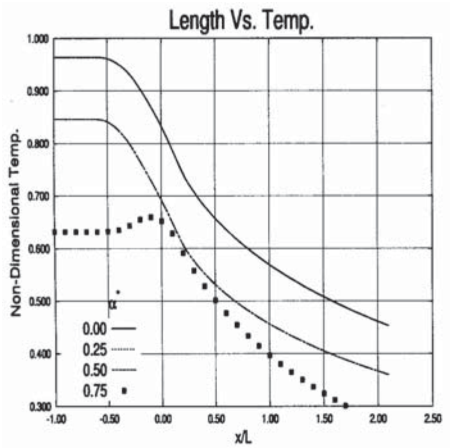

Fig. 8 Temperature (alpha)

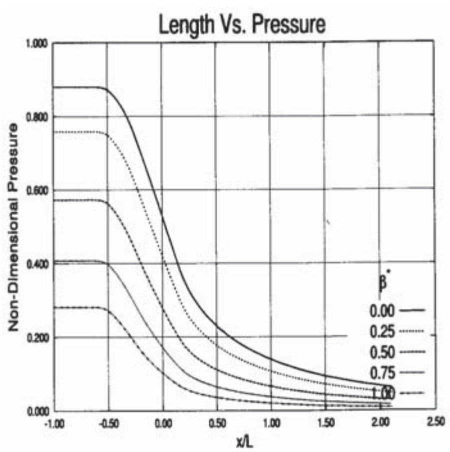

Fig. 9 Pressure (beta)

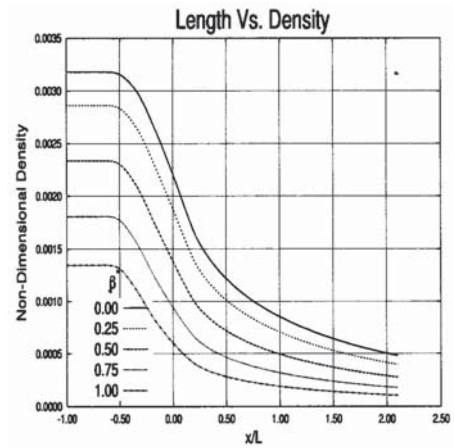

Fig. 10 Desity (beta)

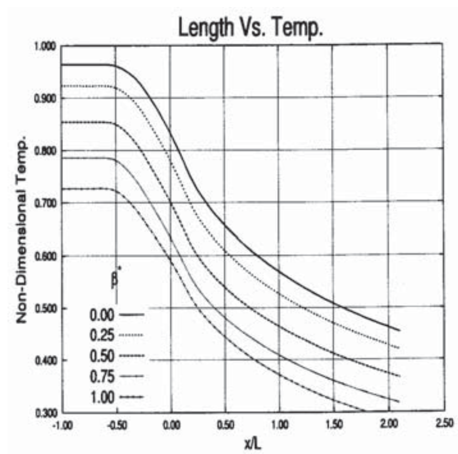

Fig. 11 Temperature (beta)

Conclusion

An analytical model has been developed to analyze the free vortex swirling supersonic flow through nozzle. It is observed through the analysis that there exists a minimum radius ξ_{υ}^2 which is a function of the swirl number (α/β) where the density approaches the limit of zero and flow seems to be void. The existence of ξ_{υ}^2 is due to the assumption that the inlet profile is free vortex. Different types of non-dimensionalization have been defined for swirl number oand one such swirl number (β) is defined. The drawback in swirl number α is that M^* tends to zero and α^* tends to 1. The proposed swirl number $\beta(0, 1)$ conveniently defines the swirl in entire range of 0 to 1 (as β tends to 1, M tends to 0.8), making the values comparable with experimental findings. The variation of mass flow as a function of swirl intensity shows a similar expression as that of Quasi-Cylindrical theory and for the no swirl case it approaches the conventional flow. It is concluded from the radial variations of density that the real flow near the axis is dominated by viscosity and there exists a forced vortex flow, hence the potential theory is invalid in the void region. The co-efficient of discharge, thrust co-efficient, and impulse function have been defined and plotted. The area average values of the flow field properties such as density, temperature and pressure are also plotted.

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