# LATERAL GUIDANCE FOR THE ENTRY PHASE OF WINGED VEHICLES

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#### Abstract

A lateral guidance algorithm developed for the high Mach entry phase of winged vehicles is presented in this paper. The vehicle is guided to meet the cross range and velocity azimuth at the end of entry phase. During entry phase, the vehicle has to be guided to meet the desired downrange and cross range requirements under dispersions in initial conditions and aerodynamic uncertainties. Guidance algorithms developed to meet the down range (in-plane) requirements uses bank angle (rotation of the vehicle about velocity vector) modulation. Bank angle modulation results in lateral drift of the vehicle leading to cross range dispersions and velocity azimuth deviations. A numerical predictor which integrates the equations of motion to predict cross range error and azimuth error at the end of entry phase is developed. A dual bank reversal algorithm is developed which determines the time of bank reversals using a predictor-corrector algorithm onboard. Simulation studies carried out show that the guidance algorithm meets the requirements even for large dispersions in initial conditions and aerodynamic uncertainties.

#### Introduction

Atmospheric entry phase is critical for winged vehicles. High Mach entry phase is characterized by high deceleration levels and active constraints. This phase typically starts at an altitude of 120 km and ends at about 20km (Mach 2 approx). During this phase, the vehicle has to be guided to meet the desired downrange and cross range requirements under dispersions in initial conditions and aerodynamic uncertainties. Guidance algorithms developed to meet the down range (in-plane) dispersions use bank angle (rotation of the vehicle about velocity vector) modulation. Bank angle modulation results in lateral drift of the vehicle leading to cross range dispersions and velocity azimuth deviations.

Many approaches are considered in literature to reduce the cross range dispersions and velocity azimuth errors. In space shuttle entry guidance design by Harpold et al. (1979) [1], the bank reversal manoeuvres which are also called as roll reversals are commanded as a function of azimuth error from the runway. In Evolved Acceleration Guidance Logic for Entry (EAGLE) which was proposed by A. Saraf et al. (2004) [2] a reduced order numerical predictor is used to predict the cross range error. A single bank reversal strategy is planned to reduce the cross range dispersion. An automated lateral guidance which determines the bank reversals by evaluating information from the reference cross range profile, current cross range and estimated actual lift to drag ratio is given in [3]. In Commanded drag guidance scheme provided in [4], bank reversal is based on azimuth error. The azimuth error is decided as the function of initial range dispersions. This does not cater to uncertainties occurring during flight.

This paper presents the lateral guidance scheme with dual bank reversal strategy developed to meet the velocity azimuth and cross range requirements. The new lateral guidance scheme is a continuation of [4].

# **Problem Formulation**

During entry phase, the vehicle has to be guided to meet the desired downrange and cross range requirements under dispersions in initial conditions and with aerodynamic uncertainties.

Let r (radial distance in m), v (velocity in m/s),  $\gamma$ (flight path angle in rad),  $\eta$  (Velocity azimuth in rad),  $\delta$  (latitude in rad),  $\lambda$  ( longitude in rad) be the states of the vehicle.

Let  $\omega_e$  (rad/s) be the earth rotation rate, m(kg) the mass of the vehicle,  $r_e$ (m) radius of earth,  $\rho$ (kg/m<sup>3</sup>) be the atmospheric density, s(m<sup>2</sup>) be the reference area, c<sub>L</sub> be the coefficient of lift, c<sub>D</sub> be the coefficient of drag, h(m) be the altitude and h<sub>s</sub>(m) the atmospheric scale height, D(m/s<sup>2</sup>) the drag deceleration, L(m/s<sup>2</sup>) the lift acceleration. The equations of motion of a winged vehicle over a rotating earth are given in (1) to (9).

$$r = v \sin(\gamma) \tag{1}$$

$$\dot{\lambda} = \frac{v\cos\left(\gamma\right)\sin\left(\eta\right)}{r\cos\left(\delta\right)}$$
(2)

$$\dot{\delta} = \frac{v\cos\left(\gamma\right)\cos\left(\eta\right)}{r} \tag{3}$$

$$\dot{v} = -D - \frac{\mu}{r^2} \sin(\gamma) + \omega_e^2 r \cos(\delta)$$
  
(sin (\gamma) cos (\delta) - cos (\gamma) cos (\gamma) sin (\delta)) (4)

$$\dot{\gamma} = (L\cos(\sigma) - \left(\frac{\mu}{r^2} - \frac{\nu^2}{r}\right)\cos(\gamma) + 2\omega_e \nu \sin(\eta)\cos(\delta) + \omega_e^2 r\cos(\delta)(\cos(\gamma)\cos(\delta) + \sin(\gamma)\cos(\eta)\sin(\delta)))/\nu$$
(5)

$$\dot{\eta} = -\left(\frac{L\sin(-\sigma)}{\cos(\gamma)} - \frac{v^2}{r}\cos(\gamma)\sin(\eta)\tan(\delta) + 2\omega_e v (\tan(\gamma)\cos(\eta)\cos(\delta) - \sin(\delta)) - \frac{\omega_e^2 r\sin(\eta)\sin(\delta)\cos(\delta)}{\cos(\gamma)}\right)/v$$
(6)

where

$$D = \frac{1}{2m} \rho v^2 c_D S r_e \tag{7}$$

$$L = \frac{1}{2m} \rho v^2 c_L S r_e \tag{8}$$

$$\rho = \rho_0 e^{-\frac{h}{s}} \tag{9}$$

Forces acting on the entry vehicle are given in Fig.1. Trajectory control can be achieved by varying bank angle and angle of attack. In this paper, angle of attack is scheduled as a function of Mach number. Trajectory control is done by bank angle modulation.

# Lateral Guidance Scheme

Bank angle modulation is used to meet downrange requirements. This results in lateral drift of the vehicle leading to cross range dispersions and velocity azimuth deviations. The schematic of the lateral guidance scheme is given Fig.2.

In lateral guidance scheme, a numerical predictor is used to predict the cross range (CR) and velocity azimuth at the end of entry phase. The predictor uses the bank angle history required to meet the downrange requirements (computed by the in-plane guidance). The dual bank reversal strategy uses the predicted cross range and velocity azimuth error to compute the required time of bank reversals onboard.

## **Numerical Predictor**

A numerical predictor which integrates the equations of motion to predict cross range error and azimuth error at the end of entry phase is developed. The current states of the vehicle are propagated using Euler's method of integration. Predictor uses the bank angle history ( $\sigma$ ) that is computed by the in-plane guidance scheme to meet the downrange requirements. Since the numerical integration of the original equations of motion ((1) to (9)) calls for long execution time in on-board computer, an approximate mathematical model is derived which has all essential aspects of the mission reducing the computational complexity.

The following are the assumptions made.

- i) Motion over a non rotating earth is considered ( $\omega_{e} = 0$ )
- ii) Variation in longitude is neglected ( $\dot{\lambda} = 0$ ).
- iii) The term  $v^2/r$  is small and is neglected.

iv) As angle of attack is scheduled as a function of Mach number, lift and drag coefficients are also represented as a function of Mach number. Thus the two variable dependency of aero coefficients is simplified.

It is estimated through simulations that the error due to above assumptions in prediction is within 1 km in cross range and 1 deg in velocity azimuth. Applying these assumptions in (1) to (6), the simplified equations used in predictor are given from (10) to (14).

$$\dot{r} = v \sin\left(\gamma\right) \tag{10}$$

$$\dot{\delta} = \frac{v\cos\left(\gamma\right)\cos\left(\eta\right)}{r} \tag{11}$$

$$\dot{v} = -D - \frac{\mu}{r^2} \sin\left(\gamma\right) \tag{12}$$

$$\dot{\gamma} = \left[ L\cos\left(\sigma\right) - \frac{\mu}{r^2}\cos\left(\gamma\right) \right] / v \tag{13}$$

$$\dot{\eta} = -\left[\frac{L\sin\left(-\sigma\right)}{\cos\left(\gamma\right)}\right] / v \tag{14}$$

The numerical predictor propagates the current states of the vehicle till the end of entry phase (Mach 2) using (10) to (14) using the bank angle history required to meet downrange requirements. The expected latitude and velocity azimuth at the end of entry phase is determined.

The predicted cross range (CRp) is computed from latitude as in (15). Let  $\delta p$  be the predicted latitude at the end entry phase and  $\delta l$  is the latitude of runway, then cross range predicted is

$$CRp = (\delta p - \delta l)*111 \text{ km}$$
(15)

## **Dual Bank Reversal Strategy**

A dual bank reversal which determines time of two bank reversals onboard is developed. The nominal trajectory is planned with two bank reversals such that the error in velocity azimuth and cross range is zero at the end of entry phase.

Through simulations it is found out that the effectiveness of first bank reversal to correct cross range (CR) dispersions is more when compared to second bank reversal.

The sensitivity of different first and second bank reversal times on final cross range is given in Fig.3. The errors in predicted cross range and velocity azimuth at the end of entry phase are computed onboard. The errors are modelled as a function of bank reversal time. For a particular performance during flight, time for two bank reversal is computed by the algorithm in three steps.

- Keeping second bank reversal time fixed (as nominal), algorithm computes the first bank reversal time using Newton Raphson method such that the cross range requirement at entry end is met.
- Using the first bank reversal time as obtained in the above step, the second bank reversal time is found such that the desired azimuth is met at end of entry phase.
- Both the bank reversal times are fine tuned simultaneously using two variable Newton - Raphson's method such that the predicted errors in crossrange and velocity azimuth is zero.

The bank angle history for nominal performance is shown in Fig.4 which shows the two bank reversals. Error in velocity azimuth and cross range throughout entry phase along with the bank angle command for nominal performance is given in Fig.5. It is inferred that errors are zero at the end of entry phase.

## Validation of Dual Bank Reversal Strategy

Guidance scheme is validated for initial condition dispersions at start of entry phase. Studies are carried out with dispersions of  $\pm 100$  m/s in entry velocity,  $\pm 1$  deg in flight path angle,  $\pm 0.15$  deg in longitude,  $\pm 10\%$  variations in aerodynamic lift and drag coefficients. Cases with combinations of the above dispersions are also studied.

The ground trace and velocity azimuth error during entry phase for typical cases is given in Fig.6 and Fig.7. The ground trace is given in terms of downrange and cross range which is computed from the latitude and longitude.

From Fig.6 and Fig.7, it is inferred that, at the end of entry phase, the dispersion in cross range is within 2 km for all the cases. The velocity azimuth error is within 7 degrees. The corresponding errors in velocity azimuth and cross range are 30 deg and 14 km respectively in earlier version [4]. The bank angle commands are given in Fig.8. The commands are limited between  $\pm$  80 deg. From Fig.8, it is seen that the timings of the two bank reversals vary depending on the performance.

# Special Case: Lateral Guidance Logic with Single Bank Reversal

In missions where high precision in velocity azimuth at entry end is not required, single bank reversal strategy is sufficient to meet the cross range requirements. The numerical predictor is used predict the cross range at entry end. The error in predicted cross range is used to determine the bank reversal time using Newton Raphson's technique on board. The bank angle command along with the cross range and velocity azimuth error is given in Fig.9.

From Fig.9, it is inferred the cross range error is zero at the end of entry phase. The azimuth error is about 30 deg.

## Validation

Validation of the single bank reversal strategy is carried out for dispersions in initial cases and aerodynamic uncertainties as mentioned in the previous section. The ground trace is given in Fig.10.

From Fig.10, it is inferred that the variation in cross range at the end of entry phase, is within 2 km for all the cases. From the simulations it is also observed that the maximum error in velocity azimuth is within 45 degrees.

#### Conclusion

A lateral guidance scheme using a dual bank reversal strategy which guides the vehicle to meet desired cross range and velocity azimuth is developed. A numerical predictor is used to predict the cross range and velocity azimuth at the end of entry phase for the desired bank angle profile The algorithm uses Newton Raphson's method to determine the time of the two bank reversals onboard such that the crossrange and velocity azimuth errors at the end of entry phase is zero. Guidance algorithm is validated for large dispersions in initial conditions and the algorithm meets the requirements at end of entry. The cross range

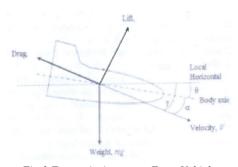


Fig.1 Forces Acting on an Entry Vehicle

error is within 2km at the end of entry phase. The azimuth error is about 7 deg.

As a special case, guidance strategy employing a single bank reversal strategy is also developed. The cross range error is within 2km at the end of entry phase. The azimuth error is about 45 deg.

This is applicable to missions in which high precision in velocity azimuth at entry end is not required.

#### References

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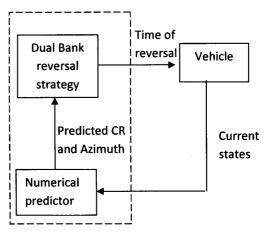


Fig.2 Lateral Guidance Scheme

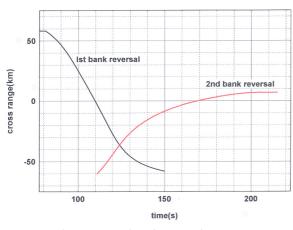


Fig.3 Sensitivity of Bank Reversal Time on CR

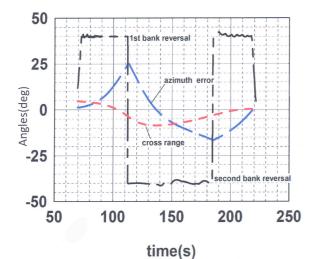


Fig.5 Error in Azimuth and Cross Range

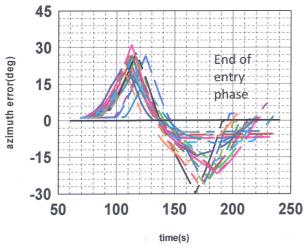
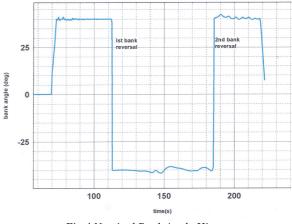
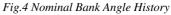


Fig.7 Velocity Azimuth Error History





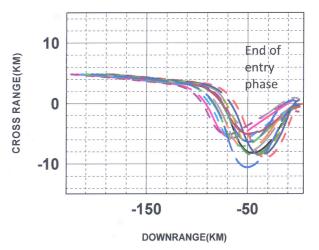


Fig.6 Ground Trace (Down range vs crossrange) with Two Bank Reversals

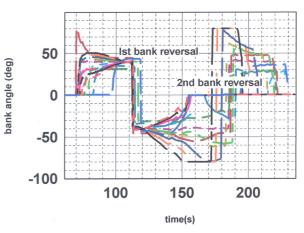


Fig.8 Bank Angle Command

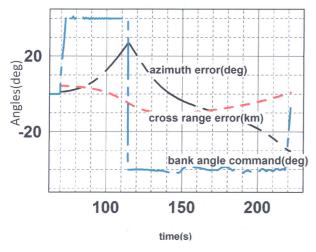


Fig.9 Nominal Bank Angle Command

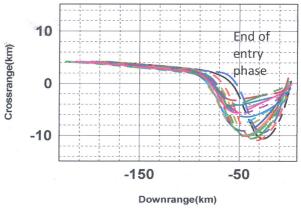


Fig.10 Ground Trace with Single Bank Reversal