SYNTHESIS OF A ROBUSTLY STABILIZING COMPENSATOR FOR AN UNCERTAIN JET ENGINE POWER PLANT

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Abstract

We propose a new algorithm based on interval analysis for design of a robust first order compensator for a jet engine. The proposed algorithm generates the entire set of stabilizing controller parameters. We demonstrate the algorithm on the acceleration control loop of a jet engine in the presence of parametric uncertainties.

Introduction

The performance requirements of modern, high technology aircraft have placed severe demands to engine control capability. Control requirements applied to gas turbine engines consist of ensuring safe, stable engine operation. Specific engine performance rating points are generally defined as basic steady state design goals for the control. A review of the basic theory of aero gas turbine engine operation and on control designs, which are currently in commercial and military use, is dealt by Spang and Brown [1].

The control logic of the modern Full Authority Digital Engine Control (FADEC) is comprised of many control loops, each of which has a specific purpose. Typical control loops include (but are not limited to) a high or low rotor speed governor, acceleration and deceleration loop, and various limiting loops for temperature, speed and fuel flow. With electronic controls, more accurate control of engine thrust can be obtained through control of compressor speed [2]. A block diagram of a typical compressor speed control is shown in Fig. 1. A compressor speed demand schedule establishes the desired compressor speed as a function of inlet temperature and throttle position. A variant of proportional control uses the derivative of rotor speed (Ndot or rotor acceleration) to control engine acceleration and deceleration as a function of inlet temperature. A block diagram of a typical Ndot controller is shown in Fig. 2. Direct control of acceleration, rather than speed, allows tighter control of engine acceleration thereby improving transient response and reducing mechanical stress.

All existing systems are subject to various disturbances and uncertainties. Mathematically, we can only approximate an existing system with a transfer function depending upon the information available about a system and the observations over a certain period of time. The difference between the performance of the actual system and model gives the estimate of uncertainties in the actual system. These uncertainties can be represented as variations in coefficients of transfer functions in frequency domain and form interval systems.

In the present work, an algorithm using interval analysis approach is proposed for the synthesis of a robustly stabilizing first order compensator of a jet engine interval plant having parametric uncertainties. Our aim is to develop algorithmic results which will enable us to determine if a robust first order stabilizer exists for a given interval plant. If it exists, then we obtain the set of stabilizing compensator parameters as a union of interval vectors (or boxes). Any value within this union represents a guaranteed robustly stabilizing compensator for the given interval plant. The algorithm is developed using a new powerful tool of interval analysis called sub-definite computation technique. The algorithm is generic and forms a very viable method for the design of individual control loops of both commercial and military jet engines. It is successfully applied to design an acceleration control loop of an experimental jet engine under development in India.

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** Professor, Systems and Control Engineering, Indian Institute of Technology Bomaby, Powai
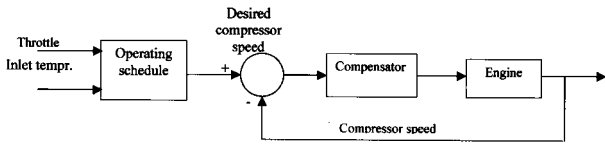   Mumbai - 400 076, India; Email: nataraj@ee.iitb.ac.in

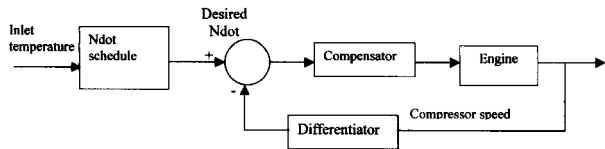*Fig. 1 Block diagram of the compressor speed control loop of jet engine*

Fig. 2 Block diagram of the Ndot controller of jet engine

The present paper is organized as follows: In section 2 we give a brief introduction to interval analysis. The basic concepts for robust stabilization are briefly given in section 3.

The algorithm for compensator synthesis is given in section 4. Section 5 deals with the interval analysis algorithm to accomplish the synthesis of first order robust controller. In section 6, the design of a robust first order compensator is carried out using the proposed technique for the acceleration control loop with the manipulated variable as main burner fuel flow and controlled variable as the compressor speed acceleration. Conclusions are drawn in section 7.

Interval Analysis

Interval analysis was introduced by Moore in 1966 [3]. It basically deals with the errors occurring in the computer implementation of numerical methods, basically because of finite word lengths of computers. It considers a closed interval to be a number of special type and works with that. It has tremendous potential for application in control systems as its natural property allows representation of uncertainty. It has not been used much in control applications and is yet to be explored in that direction. Our attempt is to utilize interval analysis methods in control applications and try to get an algorithm which is easy to use. This work is a first step in that direction.

An interval is a closed and bounded set of real numbers $\Re$: $X = [\underline{X}, \overline{X}] = \{x \in \Re : \underline{X} \le x \le \overline{X}\}$ where, $\underline{X}$ and $\overline{X}$ are the lower and upper endpoints of the interval X. The set of real intervals is denoted by I$\Re$. Set of all intervals in $X \subseteq \Re$ is denoted as I (X).

Definition 1: Intervals X and Y can be treated as numbers to define the elementary arithmetic operations $\{+, -, \cdot, /\}$ as follows.

$$X + Y = [\underline{X} + \underline{Y}, \overline{X} + \overline{Y}]$$
$$X - Y = [\underline{X} - \overline{Y}, \overline{X} - \underline{Y}]$$
$$X \cdot Y = [\min\{\underline{X}\underline{Y}, \underline{X}\overline{Y}, \overline{X}\underline{Y}, \overline{X}\overline{Y}\},$$
$$\max\{\underline{X}\underline{Y}, \underline{X}\overline{Y}, \overline{X}\underline{Y}, \overline{X}\overline{Y}\}]$$
$$\frac{X}{Y} = X \cdot \frac{1}{Y}, \qquad \frac{1}{Y} = \left[\frac{1}{\overline{Y}}, \frac{1}{\underline{Y}}\right], 0 \notin Y \qquad (1)$$

Here, $x \in X$ means, the real number x is in the interval X. Two intervals are equal if their corresponding endpoints are equal. The intersection of two intervals X and Y is empty, $X \cap Y = \Phi$, if either $\underline{X} > \overline{Y}$ or $\underline{Y} > \overline{X}$. Else, the intersection of X and Y is again an interval $X \cap Y = [\max(\underline{X}, \underline{Y}), \min(\overline{X}, \overline{Y})]$. Hull of two intervals is defined as: $X \cup Y = [\min(\underline{X}, \underline{Y}), \max(\overline{X}, \overline{Y})]$, and set inclusion: $X \subseteq Y$ if and only if $\underline{Y} \le \underline{X}$ and $\overline{X} \le \overline{Y}$. Further width of an interval $X = [\underline{X}, \overline{X}]$ is defined as $w(X) = \overline{X} - \underline{X}$, the mean as $m(X) = (\underline{X} + \overline{X})/2$, and the absolute value of an interval X as $|X| = \max(|\underline{X}|, |\overline{X}|)$.

Remark 1: It is very important fact that the elementary arithmetic operations are inclusion monotonic. That is, $X \subseteq X', Y \subseteq Y' \Rightarrow X \circ Y \subseteq X' \circ Y', \circ \in \{+, -, ., /\}$, provided the operations are well-defined as given in definition 1.

In addition to the elementary arithmetic operations, there are further common, mostly unary, operations on intervals. Basic definitions and important properties related to interval analysis are discussed in [3].

Basic Concepts of Robust Stabilization

The area of robust parametric stability analysis received a major impetus with the appearance of the seminal Kharitonov theorem regarding the Hurwitz stability of real interval polynomials. [4] contains a comprehensive account of developments in this field.

Consider a strictly proper interval plant family P comprising of plants of the form

$$P(s,q,r) = \frac{N_p(s,q)}{D_p(s,r)}$$

$$= \frac{q_0 + q_1 s + \ldots + q_l s^{l_l}}{r_0 + r_1 s + \ldots + s^k}; \quad k > l \tag{2}$$

where interval bounds are a priori given for each uncertain coefficient q_i and r_i. Let C(s) be a compensator in a feedback structure for this interval plant. If C(s) is such that it stabilizes the entire P, then C(s) is said to robustly stabilize P. Barmish et al. [5] have shown if the compensator for an interval plant is first order, the stability of only sixteen extreme plants is necessary and sufficient to stabilize the entire interval family. The present paper uses this extreme point results for controller synthesis.

Algorithm for Compensator Synthesis

Let $\underline{4}$ denote the set [1, 2, 3, 4]. Let $N_{i1}(s)$,

$i1 \in \underline{4}$, denote the Kharitonov polynomials associated with $N_P(s, g)$ in Eq. 2.

$$N_1(s) = \underline{q_0} + \underline{q_1} s + \overline{q_2} s^2 + \overline{q_3} s^3 + \underline{q_4} s^4 + \ldots..$$
$$N_2(s) = \underline{q_0} + \overline{q_1} s + \overline{q_2} s^2 + \underline{q_3} s^3 + \underline{q_4} s^4 + \ldots..$$
$$N_3(s) = \overline{q_0} + \underline{q_1} s + \underline{q_2} s^2 + \overline{q_3} s^3 + \overline{q_4} s^4 + \ldots..$$
$$N_4(s) = \overline{q_0} + \overline{q_1} s + \underline{q_2} s^2 + \underline{q_3} s^3 + \overline{q_4} s^4 + \ldots.. \tag{3}$$

Similarly, let $D_{i2}(s)$, $i2 \in \underline{4}$, denote the Kharitonov polynomials associated with $D_P(s, r)$ in Eq. (2) Consider that a robustly stabilizing PI controller

$$C(s) = K_1 + \frac{K_2}{s} \tag{4}$$

is to be synthesised for an interval plant family P with Kharitonov polynomials $N_1(s)$, $N_2(s)$, $N_3(s)$ and $N_4(s)$ and $D_1(s)$, $D_2(s)$, $D_3(s)$ and $D_4(s)$ for the numerator and denominator respectively [4].

The sixteen extreme plants are defined by

$$P_{i1,i2}(s) = \frac{N_{i1}(s)}{D_{i2}(s)} \tag{5}$$

with $i1, i2 \in \{1, 2, 3\ 4\}$.

The proposed compensator synthesis Algorithm using interval analysis is given below:

Algorithm: Synthesis of first order compensator using interval analysis.

Begin Algorithm

i) Set up sixteen Routh tables for closed loop polynomials associated with each extreme plant.

ii) Enforce positivity for each of the first column entries which are functions of K_1 and K_2. This leads to set of inequalities involving K_1 and K_2.
iii) To obtain the final controller, (K_1, K_2) should stabilize all sixteen extreme plants simultaneously. Let

$$K_{i1, i2,} \qquad i1, i2 \in \{1,2,3,4\} \tag{6}$$

denote the set of stabilizing gains corresponding to the $i1^{th}$ and $i2^{th}$ Kharitonov polynomial for the numerator and denominator, respectively. Then, the desired set of stabilizing gains is given by

$$K = K_1 K_2 \cap K_{i1,i2} \tag{7}$$

(iv) Solve the inequalities for each of the sixteen extreme plants using sub-definite computations technique (to solve the set of inequalities involving the sixteen extreme plants).

(v) Find all feasible solutions as interval boxes of specified accuracy in the initial bounds obtained at (iv) above.

End Algorithm

Remark 1: *The solution set obtained as a set of interval boxes contains all the feasible solutions. Any point within this box is a guaranteed stabilizing gain for the interval system.*

Remark 2: *A necessary and sufficient condition for the existence of a robust stabilizing controller is non - emptiness of the set of gains in (iii) above.*

Remark 3: *The interval plant can be stabilized by selecting any $(K_1, K_2) \in K$.*

A computer program has been developed for the above controller synthesis technique.

Solving the Constraint Set

To solve the set of constraints (obtained in the above synthesis procedures by enforcing positivity in the first column of the Routh tables), we use a new powerful general tool called constraint propagation. Constraint propagation has been gaining popularity in recent years, and has been successfully applied in fields such as robotics and computer-aided geometric design. In this technique, relationships among intermediate quantities in arithmetic expressions in constraints are used recursively to compute even narrower bounds on solution variables.

Several authors have applied the constraint propagation technique in nonlinear equation systems codes and software. The UniCalc solver [7] solves systems of nonlinear equations and inequalities with possibly inexact data. According to the algorithm used, the system to be solved can be overdetermined, underdetermined, and the system's parameter (coefficients, variables, initial conditions) can be imprecise and expressed as intervals. UniCalc uses the computations techniques based on the sub-definite computations method [8], which can be regarded as an analogue of constraint propagation with interval labels [9] To implement this method [3], algorithms of interval mathematics are used.

As a result of solving the algebraic system, we either find a parallelepiped that contains all solutions of the system, or a message about the system's incompatibility is issued. If the system has a unique solution, then the parallelepiped will be to reduced to a point. If the system has several solutions, then in order to locate all of them, it is necessary either to add the appropriate auxiliary relations, or to use the built-in tool for automatic root locating. The built-in root locating tool is used to separate roots and to narrow a box containing exactly one solution. This tool uses the following "bisection" process: it splits the resulting box in one of its dimensions, and repeats the basic algorithm for the sub-boxes. This process is continued recursively until either the width of the interval for the separated variable is smaller than the computational accuracy, or no more roots are found in this manner. After that, it splits the box in other dimensions as well until the width of the interval for each variable is smaller than the computation accuracy.

A detailed description of the algorithm of sub-definite calculations can be found in [8] and [10]. This solution technique is applied to solve the set of inequalities in the example below.

Jet Engine Application

Consider the SISO Jet engine interval plant with input as fuel flow and output as acceleration of compressor speed, Ndot (refer Fig. 2).

$$P(s, \alpha) = \frac{N_p(s)}{D_p(s, \alpha)} = \frac{q_o}{s^3 + r_2 s^2 + r_1 s} \qquad (8)$$

The uncertainty bounds are

$q_o \; \varepsilon \; [940, 980], \; r_1 \; \varepsilon [97, 107], \; r_2 \; \varepsilon [215, 230]$

Let us synthesize a compensator of the form

$$C(s) = \frac{N_c(s)}{D_c(s)} = K_1 + \frac{K_2}{s} \qquad (9)$$

to stabilize the interval plant.

The Kharitonov polynomials for the numerator and denominator of the effective plant of acceleration loop are:

$N_1(s) = 940; \; N_2(s) = 980;$

$D_1(s) = s^2 + 97s + 215;$

$D_2(s) = s^2 + 107s + 215;$

$D_3(s) = s^2 + 107s + 230;$

$D_4(s) = s^2 + 97s + 230; \qquad (10)$

Thus, there are 8 different extreme plants. Using these extreme plants, and PI compensator C(s), the associated closed loop polynomials are derived as follows.

$p_{1,1}(s) = s^3 + 97s^2 + (215 + 940K_1)s + 940K_2$

$p_{1,2}(s) = s^3 + 107s^2 + (215 + 940K_1)s + 940K_2$

$p_{1,3}(s) = s^3 + 107s^2 + (230 + 940K_1)s + 940K_2$

$p_{1,4}(s) = s^3 + 97s^2 + (230 + 940K_1)s + 940K_2$

$$p_{3,1}(s) = s^3 + 97s^2 + (215 + 980K_1)s + 980K_2$$

$$p_{3,2}(s) = s^3 + 107s^2 + (215 + 980K_1)s + 980K_2$$

$$p_{3,3}(s) = s^3 + 107s^2 + (230 + 980K_1)s + 980K_2$$

$$p_{3,4}(s) = s^3 + 97s^2 + (230 + 980K_1)s + 980K_2 \qquad (11)$$

In a similar manner, the other associated closed loop polynomial is obtained for all the extreme plants. Routh table is set up for all the closed loop polynomials. The stability of these polynomials can be obtained by enforcing positivity in the first column of the Routh table. There are 3 inequalities associated with each closed loop polynomial. Thus all the 24 inequalities are solved using the proposed interval analysis algorithm and initial bounds on both K_1 and K_2 are obtained as [0,2.19131e6]. Thus the given plant is stabilizable even with a very large value of K_1 and K_2. All feasible solutions of K_1 and K_2 can be obtained as intervals within a width of specified accuracy. As an illustration, Fig. 3 shows the set of robust PI stabilizers for $P(s)$ in the interval range [0,10] for both K_1 and K_2. Since this is non-empty, $P(s)$ is robustly stabilizable using any combination of K_1 and K_2 in the solution set obtained. Any $(K_1, K_2) \in K$ can be selected for stabilizing $P(s)$. All the roots of the 8 closed loop polynomials were found to lie in the left half of plane thereby confirming stability. Fig. 4 shows the closed loop step response for the interval plant with the designed compensator for $K_1 = 0.5$ and $K_2 = 0.25$. No overshoot in Ndot is observed; this is an important performance requirement in jet engines. Also, the settling time is small. Stable response is obtained for the impulse input at set point, which is shown in Fig. 5.
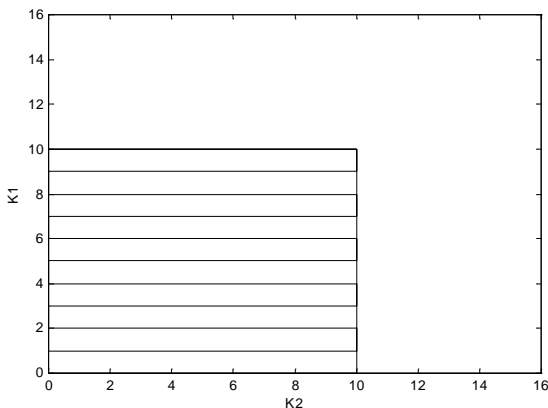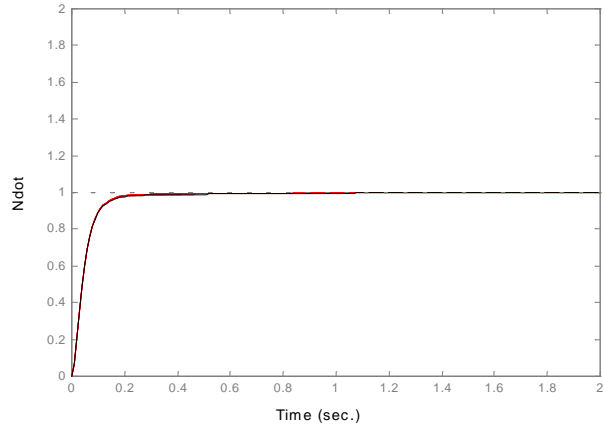

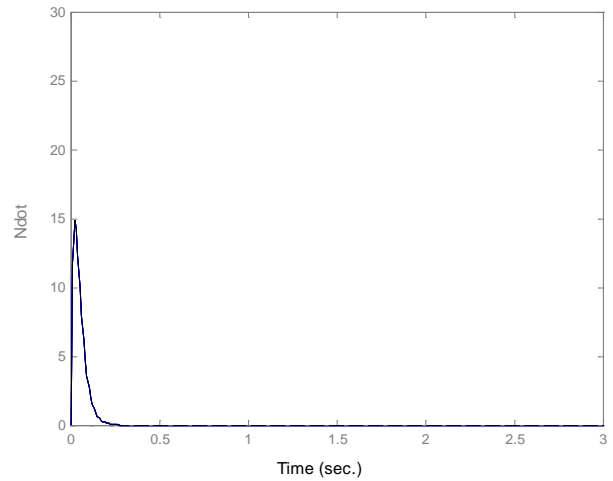Fig. 4 Closed loop system response to a
unit step command in Ndot

Fig. 5 Closed loop system response to a
unit impulse command in Ndot

Conclusion

We have established an interval analysis based algorithm for evaluation of set of robustly stabilizing first order compensator for an interval plant of jet engine. The algorithm guarantees that all feasible robust stabilizers lie within the bounds of computed interval boxes. The technique guarantees stability for the entire interval plant set. It also finds if a robust first order compensator for an individual loop is feasible or not. If feasible, the algorithm gives the entire solution set. Although synthesis procedure is for robustly stabilizing compensator, appropriate controller meeting desired performance specification can also be obtained using constraints on compensator coefficients and verifying the performance through simulations. Alternately, desired performance constraints can be derived as

Fig. 3 Stabilizable controller gains obtained by Interval
analysis for Ndot control loop

additional constraints and this synthesis technique can be extended to design jet engine robust compensators meeting both stability and performance requirements. Research is currently underway in this direction.

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