GENESIS OF VARIOUS OPTICAL ARRANGEMENTS OF CIRCULAR POLARISCOPE IN DIGITAL PHOTOELASTICITY

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Abstract

In the early developments for automation of photoelasticity several researchers have proposed phase shifting techniques (PST) with different optical arrangements. Several ways of calculating the isoclinic and isochromatic parameters were also reported. In many cases though the optical arrangements were different, the intensity equations remained the same. Although theoretically, each of the techniques provide the evaluation of isoclinics and isochromatics, when applied to experimental images only some of the techniques remain robust. This indicates that specific optical arrangements do play a very important role on the success of the phase shifting technique experimentally. One of the main sources of error is due to the mismatch of quarter wave plate. In this paper, the role of quarter wave plates in influencing the selection of appropriate optical arrangement for PST is studied systematically. Different possibilities of obtaining the photoelastic parameters with least error are explored.

Notations

I_i = Intensity of light transmitted for arbitrary positions of optical elements in a polariscope

I_a = Light intensity accounting for the amplitude of light vector and the proportionality for circular and plane polariscope arrangements respectively.

I_b = Background light intensity for circular and plane polariscope arrangements respectively.

$I'_{i-jL/R}$ = ith equation of Table 1, jth equation in the table of intensity equations including quarter wave plate error and L or R indicates the handedness of the input light.

K = Amplitude of incident light vector

$ke^{i\omega t}$ = Incident light vector

β = Orientation of analyzer axis w.r.t. x-axis

δ = Fractional retardation in radians introduced by the model

δ_c = Calculated value of fractional retardation in radians

δ'_c = Calculated value of fractional retardation in radians considering quarter wave plate error

ε = Quarter wave plate error

λ_{ref} = Wavelength of light for which the quarter wave plate introduces the retardation of $\pi/2$ radians

λ = Actual wavelength of light used

θ = Orientation of principal stress direction w.r.t. x-axis

θ_c = Calculated value of isoclinic parameter w.r.t. x-axis

θ'_c = Calculated value of isoclinic parameter w.r.t. x-axis considering quarter wave plate error

ξ = Orientation of the I quarter wave plate axis w.r.t. x-axis

η = Orientation of the II quarter wave plate axis w.r.t. x-axis

Introduction

Automation of the extraction of isoclinic and isochromatic data over the whole domain of the model was not possible till the advent of PC-based digital image processing hardware, which can record intensity data at video rates. Several techniques for automating this were proposed by various researchers [1]. Phase shifting technique (PST) is one of the widely used techniques for determining the isoclinic and isochromatic parameter at every point in the model. In phase shifting techniques, the phase-shifted images are recorded by changing the orientations of the various optical elements of the polariscope.

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Hecker and Morche [2] introduced the concept of phase shifting to photoelasticity for the determination of isochromatic parameter over the whole domain of the model. Extending the work of Hecker and Morche, Patterson and Wang [3] proposed a six step phase-shifting technique for the determination of both isoclinic and isochromatic parameters. Later, Ajovalasit et al. [4] proposed a six step phase-shifting algorithm which uses left and right circularly polarized lights to minimize the effect of quarter wave plate error on isoclinic and isochromatic parameter calculation. A limited study on the role of quarter wave plates on the performance of various algorithms proposed by Ajovalasit and his group has been reported [4]. However, a comprehensive study of role of quarter wave plate mismatch on the performance of algorithms proposed by various researchers is desirable.

One of the issues in digital photoelasticity is to reduce the number of images used for the calculation of the photoelastic parameters. Several four step methods have been reported in the literature [4-7]. Among the available four step methods, the one that performs better experimentally needs to be selected. A need thus exists to study the performance of the available four-step phase shifting techniques considering the quarter wave plate error. In this paper, a systematic study has been carried out on the role of quarter wave plate error on the performance of various six step and four step phase-shifting techniques.

Phase Shifting Techniques Based on Circular Polariscope

In phase shifting techniques, one attempts to evaluate the isoclinic and isochromatic parameters by processing the intensity information obtained from a generic circular polariscope as shown in Fig.1. The intensity of light transmitted through the generic arrangement of a circular polariscope with $\xi = 135°$ can be represented as

$$I_i = I_b + \frac{I_a}{2} + \frac{I_a}{2} \left[\sin 2(\beta_i - \eta_i)\cos\delta - \sin 2(\theta - \eta_i)\cos 2(\beta_i - \eta_i)\sin\delta\right] \quad (1)$$

where ξ, η and β represent the orientations of the first quarter wave plate, the second quarter wave plate and the analyzer respectively. The choice of appropriate set of intensity equations to evaluate the photoelastic parameters has been the study by several researchers and various researchers have come up with different optical combinations to get relevant intensity data. Table 1 gives the optical arrangements used in a six step algorithm [3]. From

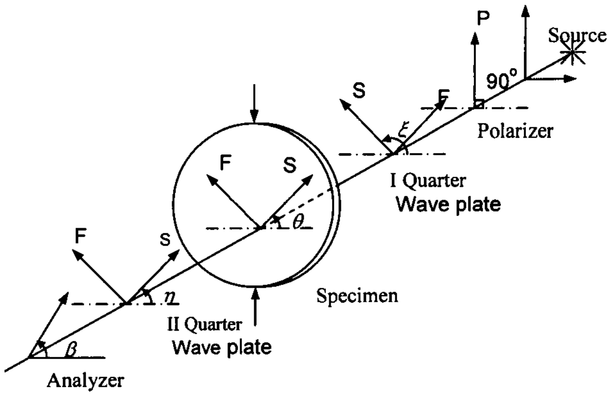

Fig. 1. Generic arrangement of a circular polariscope

the equations listed in Table 1, the isoclinic parameter is obtained as

$$\theta_c = \frac{1}{2}\tan^{-1}\left(\frac{I_5 - I_3}{I_4 - I_6}\right) = \frac{1}{2}\tan^{-1}\left[\frac{I_a \sin\delta \sin 2\theta}{I_a \sin\delta \cos 2\theta}\right]$$

for $\sin\delta \neq 0$ (2)

The isochromatic parameter is calculated as

$$\delta_c = \tan^{-1}\left(\frac{I_4 - I_6}{(I_1 - I_2)\cos 2\theta_c}\right) = \tan^{-1}\left(\frac{I_a \cos 2\theta \sin\delta}{I_a \cos 2\theta \cos\delta}\right)$$

for $\cos 2\theta_c \neq 0$ (3)

$$\delta_c = \tan^{-1}\left(\frac{I_5 - I_3}{(I_1 - I_2)\sin 2\theta_c}\right) = \tan^{-1}\left(\frac{I_a \sin 2\theta \sin\delta}{I_a \sin 2\theta \cos\delta}\right)$$

for $\sin 2\theta_c \neq 0$ (4)

However, the evaluation of isochromatic parameter using Eqs. (3) and (4) is inconvenient to use and a new equation to provide high modulation was proposed by Quiroga et. al [8] as

$$\delta_c = \tan^{-1}\left(\frac{(I_5 - I_3)\sin 2\theta_c + (I_4 - I_6)\cos 2\theta_c}{(I_1 - I_2)}\right) = \tan^{-1}\left(\frac{I_a \sin\delta}{I_a \cos\delta}\right) \quad (5)$$

The first two intensity equations of Table 1 correspond to bright and dark field arrangements used in conventional photoelasticity. However, the specific optical arrangements shown in Table 1 for these are not the ones popularly in use in conventional photoelasticity. Ajovalasit et. al [4] proposed a new set of optical arrangements in which the first two are the popular conventional polariscope arrangements and also modified the last two arrangements

Table 1: Optical arrangements for a six step phase shifting technique

Image No.	ξ	η	β	Intensity equation
1	$3\pi/4$	0	$\pi/4$	$I_1 = I_b + \frac{I_a}{2}(1+\cos\delta)$
2	$3\pi/4$	0	$3\pi/4$	$I_2 = I_b + \frac{I_a}{2}(1-\cos\delta)$
3	$3\pi/4$	0	0	$I_3 = I_b + \frac{I_a}{2}(1-\sin 2\theta \sin\delta)$
4	$3\pi/4$	$\pi/4$	$\pi/4$	$I_4 = I_b + \frac{I_a}{2}(1+\cos 2\theta \sin\delta)$
5	$3\pi/4$	$\pi/2$	$\pi/2$	$I_5 = I_b + \frac{I_a}{2}(1+\sin 2\theta \sin\delta)$
6	$3\pi/4$	$3\pi/4$	$3\pi/4$	$I_6 = I_b + \frac{I_a}{2}(1-\cos 2\theta \sin\delta)$

Table 2: Optical arrangements for the phase shifting technique proposed by Ajovalasit et. al

Image No.	ξ	η	β	Intensity equation
1	$3\pi/4$	$\pi/4$	$\pi/2$	$I_1 = I_b + \frac{I_a}{2}(1+\cos\delta)$
2	$3\pi/4$	$\pi/4$	0	$I_2 = I_b + \frac{I_a}{2}(1-\cos\delta)$
3	$3\pi/4$	0	0	$I_3 = I_b + \frac{I_a}{2}(1-\sin 2\theta \sin\delta)$
4	$3\pi/4$	$\pi/4$	$\pi/4$	$I_4 = I_b + \frac{I_a}{2}(1+\cos 2\theta \sin\delta)$
5	$\pi/4$	0	0	$I_5 = I_b + \frac{I_a}{2}(1+\sin 2\theta \sin\delta)$
6	$\pi/4$	$3\pi/4$	$\pi/4$	$I_6 = I_b + \frac{I_a}{2}(1-\cos 2\theta \sin\delta)$

Table 3: Multiple optical arrangements for obtaining intensity equations listed in Table 1 by left circu-

η \ β	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
0	I_3	I_1	I_5	I_2	I_3
$\pi/4$	I_2	I_4	I_1	I_6	I_2
$\pi/2$	I_3	I_2	I_5	I_1	I_3
$3\pi/4$	I_1	I_4	I_2	I_6	I_1
π	I_3	I_1	I_5	I_2	I_3

of Table 1. These are shown in Table 2. In the last two arrangements, the input quarter wave plate orientation is changed from $3\pi/4$ to $\pi/4$ which made the input light to change from left circular to right circular. The isoclinic and isochromatic parameters can be obtained from Eqs. (2) and (5) respectively.

In the early developments of phase shifting techniques, the researchers have proposed several optical arrangements. Though they looked different at first sight, they were yielding one of the equations reported in Table 1. Ramesh [1] reported that there could be multiple optical arrangements from which the intensity equations given in Table 1 can be obtained. The multiple optical arrangements with left circularly polarised light are given in Table 3. It is instructive to note that with left circularly polarized light ($\xi = 3\pi/4$), I_1 can be obtained by 6 arrangements, I_2 can be obtained by 6 arrangements, I_3 can be obtained by 6 arrangements, I_4 can be obtained by 2 arrangements, I_5 can be obtained by 3 arrangements and I_6 can be obtained by 2 arrangements. If the incident light is right circularly polarised, the various multiple optical arrangements are summarized in Table 4.

Intensity Equations Including Quarter Wave Plate Error

One of the commonest problems in photoelastic analysis is the mismatch of quarter wave plate. Its influence on the experimental performance of phase shifting algorithms can be understood if the intensity equations listed in Tables 1 and 2 are re-derived considering the quarter wave plate error.

Quarter wave plates used for generating circularly polarized light introduce a retardation of $\pi/2$ only when used with a light of reference wavelength λ_{ref}. When used with light of any other wavelength (λ) than the reference wavelength (λ_{ref}), quarter wave plates introduce a retardation of $\pi/2 + \varepsilon$ instead of $\pi/2$, where quarter wave plate error (ε) is defined as

$$\varepsilon = \frac{\pi}{2}\left(\frac{\lambda_{ref}}{\lambda} - 1\right) \tag{6}$$

The Jones matrix representation of a quarter wave plate with a quarter wave plate error of ε oriented at an arbitrary angle θ is given by [9]

$$\frac{1}{\sqrt{2}}\begin{bmatrix} \left(\cos\frac{\varepsilon}{2}-\sin\frac{\varepsilon}{2}\right)-i\left(\cos\frac{\varepsilon}{2}+\sin\frac{\varepsilon}{2}\right)\cos 2\theta & -i\left(\cos\frac{\varepsilon}{2}+\sin\frac{\varepsilon}{2}\right)\sin 2\theta \\ -i\left(\cos\frac{\varepsilon}{2}+\sin\frac{\varepsilon}{2}\right)\sin 2\theta & \left(\cos\frac{\varepsilon}{2}-\sin\frac{\varepsilon}{2}\right)+i\left(\cos\frac{\varepsilon}{2}+\sin\frac{\varepsilon}{2}\right)\cos 2\theta \end{bmatrix} \tag{7}$$

The components of the light vector along the analyzer axis (E_β) and perpendicular to the analyzer axis ($E_{\beta+\pi/2}$) of the generic circular polariscope can be obtained as

$$\begin{Bmatrix} E_\beta \\ E_{\beta+\pi/2} \end{Bmatrix} = \frac{1}{2}\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

$$\begin{bmatrix} \left(\cos\frac{\varepsilon}{2}-\sin\frac{\varepsilon}{2}\right)-i\left(\cos\frac{\varepsilon}{2}+\sin\frac{\varepsilon}{2}\right)\cos 2\eta & -i\left(\cos\frac{\varepsilon}{2}+\sin\frac{\varepsilon}{2}\right)\sin 2\eta \\ -i\left(\cos\frac{\varepsilon}{2}+\sin\frac{\varepsilon}{2}\right)\sin 2\eta & \left(\cos\frac{\varepsilon}{2}-\sin\frac{\varepsilon}{2}\right)+i\left(\cos\frac{\varepsilon}{2}+\sin\frac{\varepsilon}{2}\right)\cos 2\eta \end{bmatrix}$$

$$\begin{bmatrix} \cos\frac{\delta}{2}-i\sin\frac{\delta}{2}\cos 2\theta & -i\sin\frac{\delta}{2}\sin 2\theta \\ -i\sin\frac{\delta}{2}\sin 2\theta & \cos\frac{\delta}{2}+i\sin\frac{\delta}{2}\cos 2\theta \end{bmatrix}$$

$$\begin{bmatrix} \left(\cos\frac{\varepsilon}{2}-\sin\frac{\varepsilon}{2}\right)-i\left(\cos\frac{\varepsilon}{2}+\sin\frac{\varepsilon}{2}\right)\cos 2\xi & -i\left(\cos\frac{\varepsilon}{2}+\sin\frac{\varepsilon}{2}\right)\sin 2\xi \\ -i\left(\cos\frac{\varepsilon}{2}+\sin\frac{\varepsilon}{2}\right)\sin 2\xi & \left(\cos\frac{\varepsilon}{2}-\sin\frac{\varepsilon}{2}\right)+i\left(\cos\frac{\varepsilon}{2}+\sin\frac{\varepsilon}{2}\right)\cos 2\xi \end{bmatrix}$$

$$\times \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} k e^{i\omega t} \tag{8}$$

The intensity of the light transmitted by the optical system can be obtained as

$$I = E_\beta E_\beta^* \tag{9}$$

where E_β^* is the complex conjugate of E_β^*.

Tables 5 and 6 give the intensity equations including quarter wave plate error for the multiple optical arrangements corresponding to intensity equation I_1 listed in Tables 3 and 4 respectively. The intensity of light transmitted is obtained by the symbolic computational software

Table 4: Multiple optical arrangements for obtaining intensity equations listed in Table 1 by right circularly polarized ($\xi = 3\pi/4$)

η \ β	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
0	I_5	I_2	I_3	I_1	I_5
$\pi/4$	I_1	I_6	I_2	I_4	I_1
$\pi/2$	I_5	I_1	I_3	I_2	I_5
$3\pi/4$	I_2	I_6	I_1	I_4	I_2
π	I_5	I_2	I_3	I_1	I_5

Maple. The intensity equation $I'_{i-jL/R}$ corresponds to i^{th} image of Table 1, j^{th} optical arrangement to get this intensity equation (Table 3) with L or R indicating the handedness of the input light.

The intensity equation I_1 is the bright field arrangement in conventional photoelasticity. In the presence of quarter wave plate mismatch not all the arrangements yield the same value of the intensity of light transmitted. The multiple optical arrangements mentioned in Tables 5 and 6 can be segregated into three categories namely the optical arrangements with quarter wave plates crossed, quarter wave plates parallel and others in which the quarter wave plates are neither crossed nor parallel. The individual relative orientations of the elements do play a role and this

Table 5: Intensity equations including quarter wave plate error corresponding to multiple optical arrangements for obtaining I1 using left circularly polarized light listed in Table 3

ξ	η	β	Intensity Equations
Quarter wave plates Crossed			
$3\pi/4$	$\pi/4$	$\pi/2$	$I'_{1-1L} = I_b + I_a\left\{\left(1-\cos^2 2\theta\sin^2\varepsilon\right)\cos^2\left(\frac{\delta}{2}\right)+\cos^2 2\theta\sin^2\varepsilon\right\}$
Quarter wave plates Parallel			
$3\pi/4$	$3\pi/4$	0	$I'_{1-2L} = I_b + I_a\left\{\sin^2\left(\frac{\delta}{2}\right)\sin^2 2\theta+\sin 2\theta\sin\delta\sin\varepsilon\cos\varepsilon +\left(1-2\sin^2\left(\frac{\delta}{2}\right)+\sin^2\left(\frac{\delta}{2}\right)\cos^2 2\theta\right)\cos^2\varepsilon\right\}$
$3\pi/4$	$3\pi/4$	π	$I'_{1-3L} = I_b + I_a\left\{\sin^2\left(\frac{\delta}{2}\right)\sin^2 2\theta+\sin 2\theta\sin\delta\sin\varepsilon\cos\varepsilon +\left(1-2\sin^2\left(\frac{\delta}{2}\right)+\sin^2\left(\frac{\delta}{2}\right)\cos^2 2\theta\right)\cos^2\varepsilon\right\}$
Other arrangements			
$3\pi/4$	π	$\pi/4$	$I'_{1-4L} = I_b + \frac{I_a}{2}\{1-\sin 2\theta\cos 2\theta(1-\cos\delta)- \sin\varepsilon\cos\varepsilon\sin\delta(\cos 2\theta-\sin 2\theta) +(\cos\delta+\sin 2\theta\cos 2\theta(1-\cos\delta))\cos^2\varepsilon\}$
$3\pi/4$	0	$\pi/4$	$I'_{1-5L} = I_b + \frac{I_a}{2}\{1-\sin 2\theta\cos 2\theta(1-\cos\delta)- \sin\varepsilon\cos\varepsilon\sin\delta(\cos 2\theta-\sin 2\theta) +(\cos\delta+\sin 2\theta\cos 2\theta(1-\cos\delta))\cos^2\varepsilon\}$
$3\pi/4$	$\pi/2$	$3\pi/4$	$I'_{1-6L} = I_b + \frac{I_a}{2}\{1+\sin 2\theta\cos 2\theta(1-\cos\delta)+ \sin\varepsilon\cos\varepsilon\sin\delta(\sin 2\theta+\cos 2\theta) +(\cos\delta-\sin 2\theta\cos 2\theta(1-\cos\delta))\cos^2\varepsilon\}$

Intensity equation without including quarter wave plate error $I_1 = I_b + \frac{I_a}{2}(1+\cos\delta)$

is brought out in Tables 5 and 6. Thus from the experimental standpoint, one needs to select those optical combinations that yield the least error in the presence of quarter wave plate mismatch.

Inspection of Tables 5 and 6 shows that in the presence of quarter wave plate mismatch, intensity of light transmitted is also a function of ε. Intensity equations corresponding to optical arrangements with quarter wave plates crossed are identical for both right circularly ($\xi = 3\pi/4$) and left circularly ($\xi = \pi/4$) polarized lights. However, the intensity equations for parallel and other optical arrangements differ when the input light handedness is changed. The sinε term in these equations gets changed to -sinε when the input light is changed.

Table 6: Intensity equations including quarter wave plate error corresponding to multiple optical arrangements for obtaining I_1 using right circularly polarized light listed in Table 4

ξ	η	β	Intensity Equations
Quarter wave plates crossed			
$\pi/4$	$3\pi/4$	$\pi/2$	$I'_{1-1R} = I_b + I_a\left\{\left(1-\cos^2 2\theta\sin^2\varepsilon\right)\cos^2\left(\frac{\delta}{2}\right)+\cos^2 2\theta\sin^2\varepsilon\right\}$
Quarter wave plates Parallel			
$\pi/4$	$\pi/4$	0	$I'_{1-2R} = I_b + I_a\left\{\sin^2\left(\frac{\delta}{2}\right)\sin^2 2\theta - \sin 2\theta\sin\delta\sin\varepsilon\cos\varepsilon + \left(1-2\sin^2\left(\frac{\delta}{2}\right)+\sin^2\left(\frac{\delta}{2}\right)\cos^2 2\theta\right)\cos^2\varepsilon\right\}$
$\pi/4$	$\pi/4$	π	$I'_{1-3R} = I_b + I_a\left\{\sin^2\left(\frac{\delta}{2}\right)\sin^2 2\theta - \sin 2\theta\sin\delta\sin\varepsilon\cos\varepsilon + \left(1-2\sin^2\left(\frac{\delta}{2}\right)+\sin^2\left(\frac{\delta}{2}\right)\cos^2 2\theta\right)\cos^2\varepsilon\right\}$
Other arrangements			
$\pi/4$	0	$3\pi/4$	$I'_{1-4R} = I_b + \frac{I_a}{2}\{1+\sin 2\theta\cos 2\theta(1-\cos\delta) - \sin\varepsilon\cos\varepsilon\sin\delta(\sin 2\theta+\cos 2\theta) + (\cos\delta - \sin 2\theta\cos 2\theta(1-\cos\delta))\cos^2\varepsilon\}$
$\pi/4$	$\pi/2$	$\pi/4$	$I'_{1-5R} = I_b + \frac{I_a}{2}\{1-\sin 2\theta\cos 2\theta(1-\cos\delta) - \sin\delta\sin\varepsilon\cos\varepsilon\ (\sin 2\theta-\cos 2\theta) - (\cos\delta + \sin 2\theta\cos 2\theta(1-\cos\delta))\cos^2\varepsilon\}$
$\pi/4$	π	$3\pi/4$	$I'_{1-6R} = I_b + \frac{I_a}{2}\{1+\sin 2\theta\cos 2\theta(1-\cos\delta) - \sin\varepsilon\cos\varepsilon\sin\delta(\sin 2\theta+\cos 2\theta) + (\cos\delta - \sin 2\theta\cos 2\theta(1-\cos\delta))\cos^2\varepsilon\}$

Intensity equation without including quarter wave plate error $I_1 = I_b + \frac{I_a}{2}(1+\cos\delta)$

Since the equations are quite complex, the relative behaviour of the equations could be understood by plotting a graph of % error in intensity equation defined by

$$\% \text{ Error in Intensity} = \frac{I_Q - I_i}{I_i} \times 100 \qquad (10)$$

where I_i is the intensity of light transmitted in the absence of quarter wave plate error and I_Q is considering the quarter wave plate error. These plots are obtained for $\varepsilon = -9°$ [4] and δ varying from 0 to 2π with θ as parameter for $\theta = 0°$, 22.5°and 45°. Figs. 2a to 2h give the plot of error in intensity for the case of bright field.

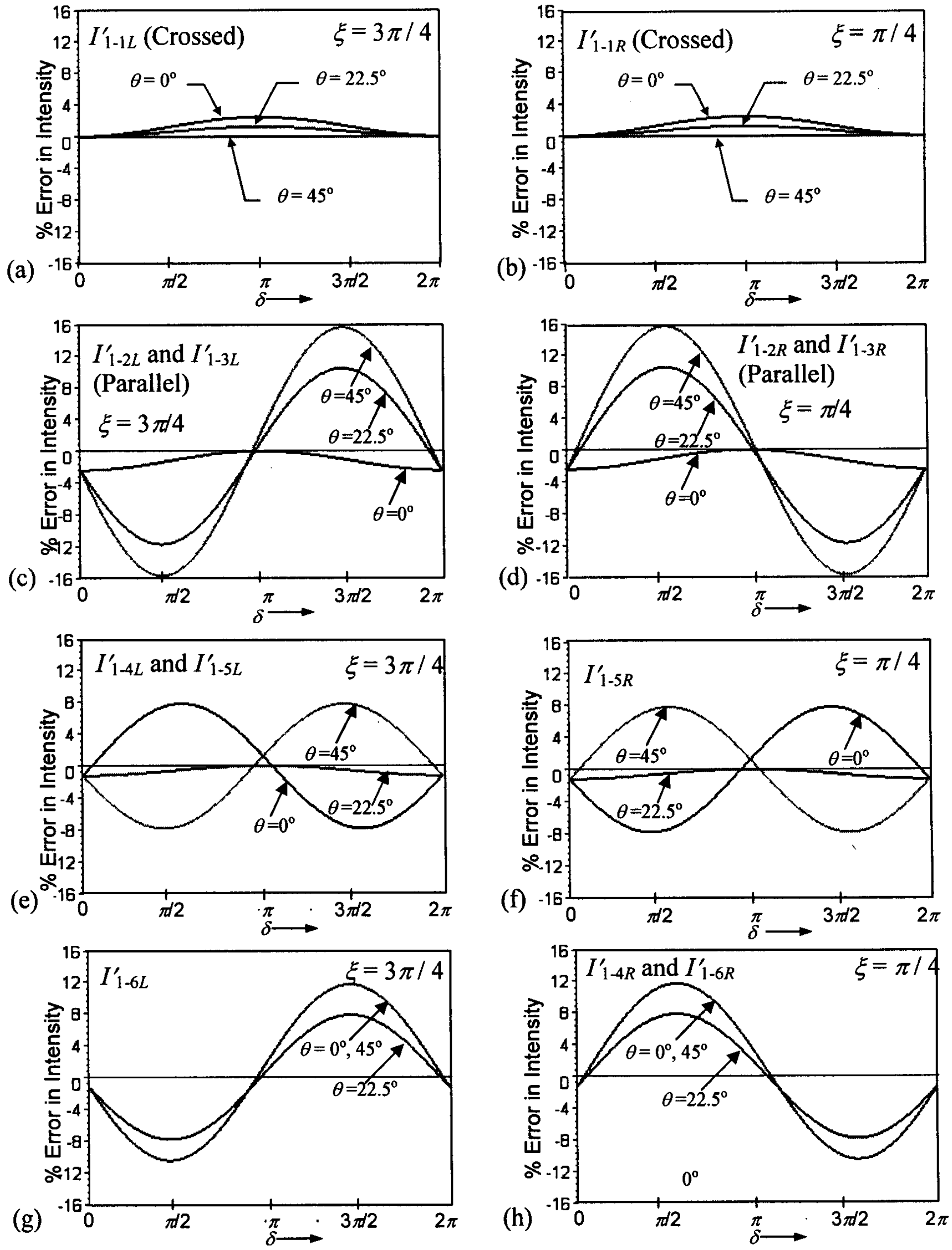

Fig. 2. Influence of quarter wave plate mismatch ($\varepsilon = -9°$) on the intensity of light transmitted for various optical arrangements for a left circularly polarized light ($\xi = 3\pi/4$) in Table 5 and right circularly polarized light ($\xi = \pi/4$) in Table 6 as a function of δ with θ as parameter, which would have given the same intensity equations as I_1 in Table 1

The intensity equation I_2 is the conventional dark field arrangement. Here again in the presence of quarter wave plate mismatch, the intensity equations could be grouped into three groups. Figures 3a to 3h give the plots of error in intensity for the case of dark field for the two incident light conditions. From Figs. 2 and 3, it is clear that the error in intensity is minimum for crossed quarter wave plates and maximum for the parallel arrangements for both bright field (I_1) and dark field (I_2) and reconfirms the wisdom of using crossed quarter wave plates in conventional photoelasticity.

The same study is extended to the rest of the optical arrangements corresponding to the intensity equations I_3, I_4, I_5 and I_6 of Table 1. Surprisingly, for intensity equations I_3 to I_6 all the multiple optical arrangements of

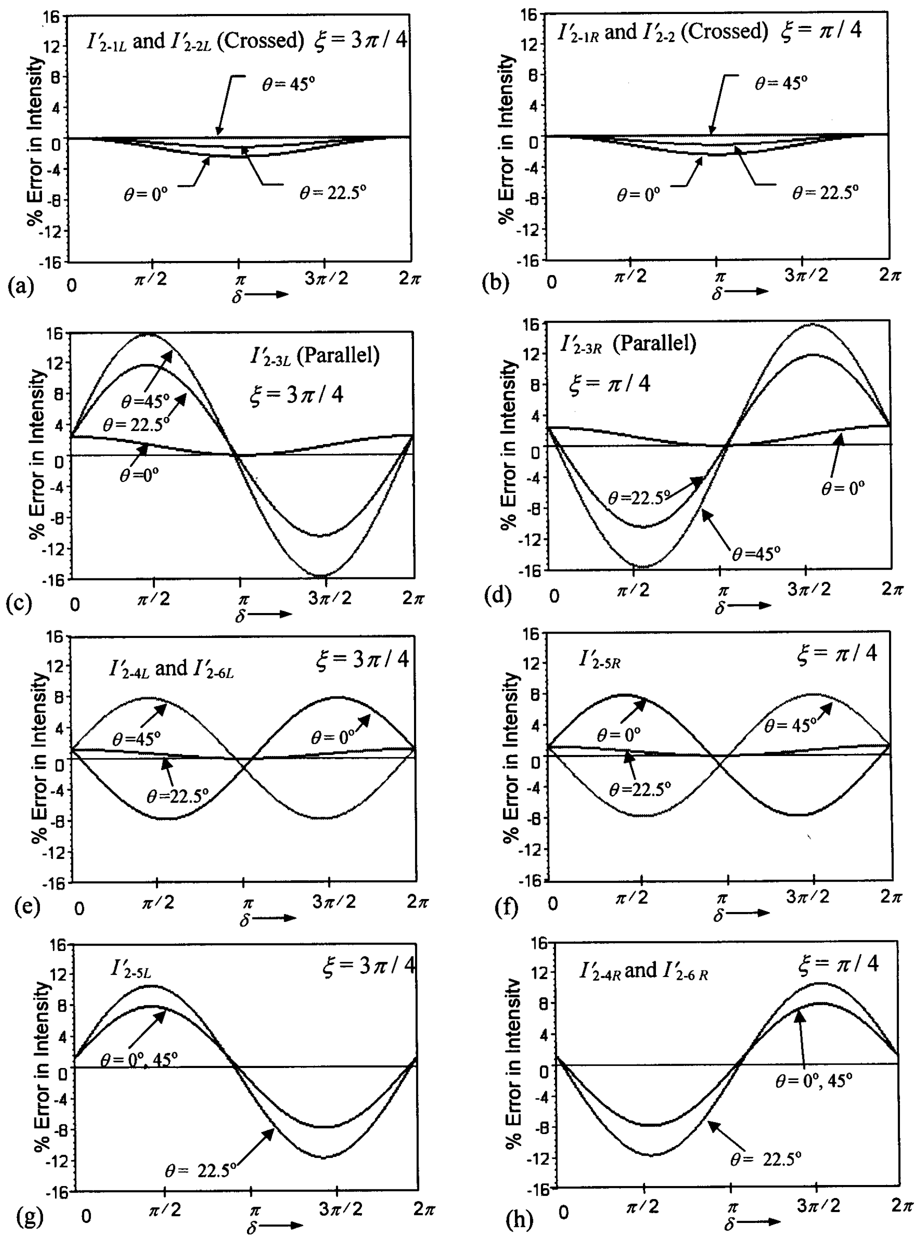

Fig. 3. Influence of quarter wave plate mismatch ($\varepsilon = -9°$) on the intensity of light transmitted for various optical arrangements for a left circularly polarized light ($\xi = 3\pi/4$) in Table 3 and right circularly polarised light ($\xi = \pi/4$) in Table 4 as a function of δ with θ as parameter, which would have given the same intensity equations as I_2 in Table 1

Tables 3 and 4 yield the same intensity equation in the presence of quarter wave plate mismatch. However, when the input handedness is changed its behaviour is similar to I_1 and I_2 viz., the term $\sin\varepsilon$ gets changed to $-\sin\varepsilon$. These are summarised in Table 7.

Selection of Intensity Equations to Reduce the Influence of Quarter Wave Plate Error on Photoelastic Parameters Evaluation

Table 8 gives the intensity equations including quarter wave plate error corresponding to the optical arrangements given in Table 1. The equations for θ_c and δ_c can be recast to get the values of θ'_c and δ'_c in the presence of quarter wave plate error.

$$\theta'_c = \frac{1}{2}\tan^{-1}\left(\frac{I'_5 - I'_3}{I'_4 - I'_6}\right) \tag{11}$$

$$\delta_c = \tan^{-1}\left(\frac{(I'_5 - I'_3)\sin 2\theta'_c + (I'_4 - I'_6)\cos 2\theta'_c}{(I'_1 - I'_2)}\right) \tag{12}$$

By substituting the intensity equations of Table 8 in Eq. (11) and Eq. (12), they get simplified to

$$\theta'_c = \frac{1}{2}\tan^{-1}\left(\frac{\sin 2\theta \sin\delta \cos\varepsilon - (\cos^2 2\theta + \sin^2 2\theta \cos\delta)\sin\varepsilon}{\cos 2\theta \sin\delta \cos\varepsilon + (1 - \cos\delta)\sin 2\theta \cos 2\theta \sin\varepsilon}\right) \tag{13}$$

Table 7: Intensity equations including quarter wave plate error corresponding to multiple optical arrangements given in Tables 3 and 4 for obtaining intensity equations I_3 to I_6 in Table 1

Handedness of the input light	Intensity Equations
Left Circular	$I'_{3L} = I_b + \frac{I_a}{2}\{1 + (\cos^2 2\theta + \cos\delta\sin^2 2\theta)\sin\varepsilon - \sin 2\theta\sin\delta\cos\varepsilon\}$
Right Circular	$I'_{3R} = I_b + \frac{I_a}{2}\{1 - (\cos^2 2\theta + \cos\delta\sin^2 2\theta)\sin\varepsilon - \sin 2\theta\sin\delta\cos\varepsilon\}$
Left Circular	$I'_{4L} = I_b + \frac{I_a}{2}\{1 + \cos 2\theta\sin\delta\cos\varepsilon + (1 - \cos\delta)\sin 2\theta\cos 2\theta\sin\varepsilon\}$
Right Circular	$I'_{4R} = I_b + \frac{I_a}{2}\{1 + \cos 2\theta\sin\delta\cos\varepsilon - (1 - \cos\delta)\sin 2\theta\cos 2\theta\sin\varepsilon\}$
Left Circular	$I'_{5L} = I_b + \frac{I_a}{2}\{1 - (\cos^2 2\theta + \sin^2 2\theta\cos\delta)\sin\varepsilon + \sin 2\theta\cos\varepsilon\sin\delta\}$
Right Circular	$I'_{5R} = I_b + \frac{I_a}{2}\{1 + (\cos^2 2\theta + \sin^2 2\theta\cos\delta)\sin\varepsilon + \sin 2\theta\cos\varepsilon\sin\delta\}$
Left Circular	$I'_{6L} = I_b + \frac{I_a}{2}\{1 - (1 - \cos\delta)\sin 2\theta\cos 2\theta\sin\varepsilon - \cos 2\theta\cos\varepsilon\sin\delta\}$
Right Circular	$I'_{6R} = I_b + \frac{I_a}{2}\{1 + (1 - \cos\delta)\sin 2\theta\cos 2\theta\sin\varepsilon - \cos 2\theta\cos\varepsilon\sin\delta\}$

Table 8: Intensity equations including quarter wave plate error corresponding to the optical arrangements listed in Table 1

$$I'_1 = I_b + \frac{I_a}{2}\{1 - \sin 2\theta\cos 2\theta(1 - \cos\delta) - \sin\varepsilon\cos\varepsilon\sin\delta(\cos 2\theta - \sin 2\theta) + (\cos\delta + \sin 2\theta\cos 2\theta(1 - \cos\delta))\cos^2\varepsilon\}$$

$$I'_2 = I_b + \frac{I_a}{2}\{1 + \sin 2\theta\cos 2\theta(1 - \cos\delta) + \sin\varepsilon\cos\varepsilon\sin\delta(\cos 2\theta - \sin 2\theta) - ((1 - \cos\delta)\sin 2\theta\cos 2\theta + \cos\delta)\cos^2\varepsilon\}$$

$$I'_3 = I_b + \frac{I_a}{2}\{1 + (\cos^2 2\theta + \cos\delta\sin^2 2\theta)\sin\varepsilon - \sin 2\theta\sin\delta\cos\varepsilon\}$$

$$I'_4 = I_b + \frac{I_a}{2}\{1 + \cos 2\theta\sin\delta\cos\varepsilon + (1 - \cos\delta)\sin 2\theta\cos 2\theta\sin\varepsilon\}$$

$$I'_5 = I_b + \frac{I_a}{2}\{1 - (\cos^2 2\theta + \cos\delta\sin^2 2\theta)\sin\varepsilon + \sin 2\theta\sin\delta\cos\varepsilon\}$$

$$I'_6 = I_b + \frac{I_a}{2}\{1 - \cos 2\theta\sin\delta\cos\varepsilon - (1 - \cos\delta)\sin 2\theta\cos 2\theta\sin\varepsilon\}$$

$$\delta'_c = \tan^{-1}\left(\frac{\begin{array}{l}(\sin 2\theta \sin\delta \cos\varepsilon - (cos^2 2\theta + \sin^2 2\theta\cos\delta)\sin\varepsilon)\sin 2\theta'_c \\ + (\cos 2\theta \sin\delta\cos\varepsilon \\ + (1-\cos\delta)\sin 2\theta\cos 2\theta\sin\varepsilon)\cos 2\theta'_c\end{array}}{\begin{array}{l}\cos\delta\cos^2\varepsilon + (sin 2\theta - \cos 2\theta)\sin\varepsilon\cos\varepsilon\sin\delta \\ -(1-\cos\delta)\sin 2\theta\cos 2\theta\sin^2\varepsilon\end{array}}\right) \quad (14)$$

For θ'_c to represent isoclinic parameter, its expression should be a function of only θ at $\delta \neq 0, \pi, 2\pi\ldots$ as in Eq. (2). For δ'_c to represent isochromatic parameter, its expression should be a function of only δ as in Eq. (5). But Eq. (13) is a function of θ, δ and ε and Eq. (14) is a function of θ'_c, θ, δ and ε. Thus, the θ'_c and δ'_c obtained from Eq. (11) and Eq. (12) are not representing the isoclinic or isochromatic parameter correctly. In other words if the values of θ'_c or δ'_c are used for further analysis they represent these parameters with error.

It is seen in the previous section that the error in intensities in the case of bright field and dark field is minimum only when the quarter wave plates are crossed. When the light is changed from right circularly polarized to left circularly polarized, the error in the intensity of light transmitted gets negated for the equations I_3 to I_6. To eliminate or minimize the effect of quarter wave plate error in the evaluation of isoclinic and isochromatic parameters, the intensity equations should be selected such that the intensity equations corresponding to both left and right circularly polarized lights are judiciously used.

The possible combinations of intensity equations to get θ'_c and δ'_c with minimum error are given in Table 9. Table 10 gives the equations for isoclinic and isochromatic parameters including quarter wave plate error for the combinations listed in Table 9. It is interesting to note that the fifth combination of intensity equations given in Table 9 corresponds to the six-step algorithm proposed by Ajovalasit et. al [4]. The expressions for θ'_c and δ'_c for the six cases mentioned in Table 10 are

$$\theta'_c = \tan^{-1}\left(\frac{I_a \sin\delta \sin 2\theta \cos\varepsilon}{I_a \sin\delta \cos 2\theta \cos\varepsilon}\right) \quad (15)$$

$$\delta'_c = \tan^{-1}\left(\frac{I_a \sin\delta\cos\varepsilon}{I_a\left[\cos\delta(1-\cos^2\theta\sin^2\varepsilon) + \cos^2 2\theta\sin^2\varepsilon\right]}\right) \quad (16)$$

Unlike Eq. (13) Eq. (15) is a function of only θ when $\sin\delta \neq 0$. Hence it represents the isoclinic parameter (θ_c). But Eq. (16) is a function of θ, δ and ε. Hence, similar to δ'_c obtained by Eq. (12), δ'_c obtained from Eq. (14) represents the isochromatic parameter with error. The plot of error $(\delta'_c - \delta)/2\pi$ due to quarter wave plate mismatch for the cases of Eq. (14) and Eq. (16) is shown in Fig. 4.

Table 9: Combinations of intensity equations including quarter wave plate error that gives θ'_c and δ'_c with minimum error as in Eqs. (15) and Eq. (16)

Combination 1	Combination 2	Combination 3	Combination 4	Combination 5	Combination 6
I'_1 Crossed	I'_1 Crossed	I'_1 Crossed	I'_1 Crossed	I'_1 Crossed	I'_1 Crossed
I'_2 Crossed	I'_2 Crossed	I'_2 Crossed	I'_2 Crossed	I'_2 Crossed	I'_2 Crossed
I'_{3L}	I'_{5L}	I'_{3L}	I'_{5L}	I'_{3L}	I'_{3R}
I'_{3R}	I'_{5R}	I'_{3R}	I'_{5R}	I'_{4L}	I'_{4R}
I'_{4L}	I'_{6L}	I'_{6L}	I'_{4L}	I'_{5R}	I'_{5L}
I'_{4R}	I'_{6R}	I'_{6R}	I'_{4R}	I'_{6R}	I'_{6L}

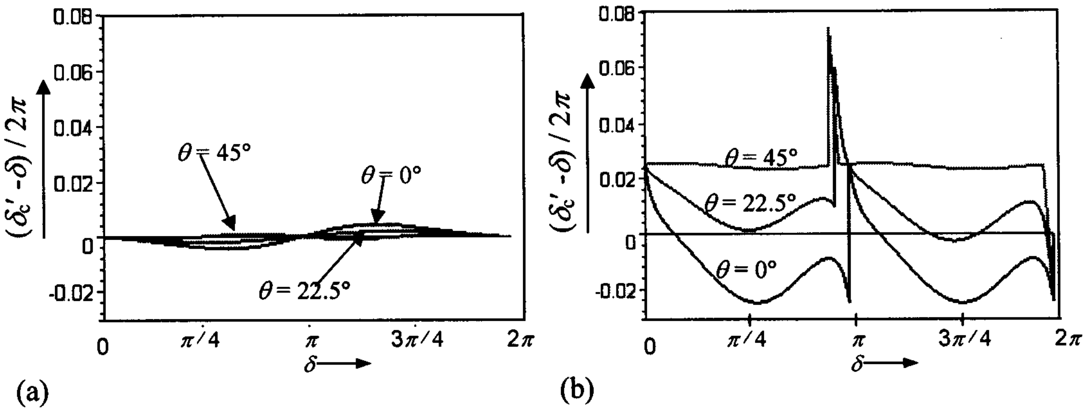

Fig. 4. Error in isochromatic parameter $(\delta'_c/\delta)/2\pi$ for six step phase shifting techniques (a) Ajovalasit six step algorithm (b) Patterson six step algorithm

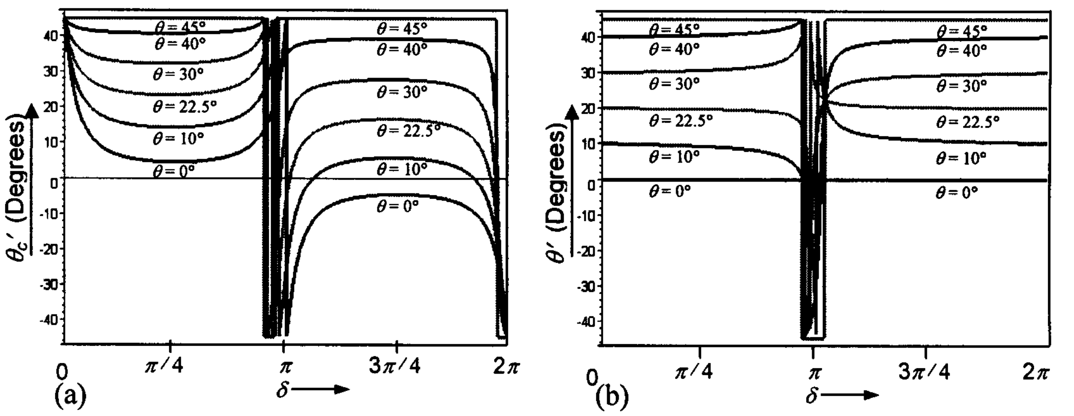

Fig. 5. Plot of variation of θ'_c with δ for various Four Step Algorithms with θ as parameter. (a) variation of θ'_c for Ajovalasit, Barone and Patterson four step algorithm. (b) Variation of θ'_c for Asundi four step algorithm

Four Step Phase-Shifting Techniques

One of the issues in phase shifting techniques is to minimize the number of images required for the evaluation of photoelastic parameters. Since there are four unknowns (I_b, I_a, θ and δ), a minimum of four images are required. Several four step methods have been reported in the literature [4-7]. Table 11 gives the optical arrangements along with the corresponding intensity equations including quarter wave plate error for Ajovalasit et. al 4-step, Asundi et. al 4-step, Barone et. al 4-step and Patterson et. al 4-step phase shifting techniques. Since the available four step methods use different combination of optical arrangements, the one that performs better in the presence of quarter wave plate mismatch needs to be selected. It is also to be kept in mind that the selected method is amenable for simultaneous recording of four phase shifted images.

Although the methodology of evaluation of isoclinic data is straight forward, in isochromatic parameter evaluation the proponents of four step methods did not take care of the issue of modulation as it was done in six step methods. Extending the methodology adopted for six step methods to four step methods, the equations are recast in this study to have high modulation over the field and is summarized in Table 12. The expressions for isoclinic parameter in the presence of quarter wave plate mismatch are given in Table 13. The Table shows that the algorithms of Ajovalasit et. al, Barone et. al and Patterson and Wang give same expression for isoclinic evaluation. The expression for evaluation of isoclinic parameter by Asundi's algorithm is different. However, by looking at the graphs in Fig. 5, it is seen that these set of algorithms give isoclinic values that are quite different from the actual values. In summary, the algorithms cannot be classified on the basis of isoclinic evaluation. Table 14 gives the simplified ex-

Table 10: Equations for isoclinic parameter θ'_c and isochromatic parameter δ'_c for combinations of intensity equations mentioned in Table 9

	Isoclinic Parameter (θ'_c)	Isochromatic parameter (δ'_c)
1	$\theta'_c = \frac{1}{2}\tan^{-1}\left(\frac{I'_m - (I'_{3L} + I'_{3R})}{(I'_{4L} + I'_{4R}) - I'_m}\right)$	$\delta'_c = \tan^{-1}\left(\frac{(I'_m - (I'_{3L} + I'_{3R}))\sin 2\theta'_c + ((I'_{4L} + I'_{4R}) - I'_m)\cos 2\theta'_c}{(I'_1 - I'_2)}\right)$
2	$\theta'_c = \frac{1}{2}\tan^{-1}\left(\frac{(I'_{5L} + I'_{5R}) - I'_m}{I'_m - (I'_{6L} + I'_{6R})}\right)$	$\delta'_c = \tan^{-1}\left(\frac{((I'_{5L} + I'_{5R}) - I'_m)\sin 2\theta'_c + (I'_m - (I'_{6L} + I'_{6R}))\cos 2\theta'_c}{(I'_1 - I'_2)}\right)$
3	$\theta'_c = \frac{1}{2}\tan^{-1}\left(\frac{I'_m - (I'_{3L} + I'_{3R})}{I'_m - (I'_{6L} + I'_{6R})}\right)$	$\delta'_c = \tan^{-1}\left(\frac{(I'_m - (I'_{3L} + I'_{3R}))\sin 2\theta'_c + (I'_m - (I'_{6L} + I'_{6R}))\cos 2\theta'_c}{(I'_1 - I'_2)}\right)$
4	$\theta'_c = \frac{1}{2}\tan^{-1}\left(\frac{(I'_{5L} + I'_{5R}) - I'_m}{(I'_{4L} + I'_{4R}) - I'_m}\right)$	$\delta'_c = \tan^{-1}\left(\frac{((I'_{5L} + I'_{5R}) - I'_m)\sin 2\theta'_c + ((I'_{4L} + I'_{4R}) - I'_m)\cos 2\theta'_c}{(I'_1 - I'_2)}\right)$
5	$\theta'_c = \frac{1}{2}\tan^{-1}\left(\frac{(I'_{5R} - I'_{3L})}{(I'_{4L} - I'_{6R})}\right)$	$\delta'_c = \tan^{-1}\left(\frac{((I'_{5R} - I'_{3L}))\sin 2\theta'_c + ((I'_{4L} - I'_{6R}))\cos 2\theta'_c}{(I'_1 - I'_2)}\right)$
6	$\theta'_c = \frac{1}{2}\tan^{-1}\left(\frac{(I'_{5L} - I'_{3R})}{(I'_{4R} - I'_{6L})}\right)$	$\delta'_c = \tan^{-1}\left(\frac{((I'_{5L} - I'_{3R}))\sin 2\theta'_c + ((I'_{4R} - I'_{6L}))\cos 2\theta'_c}{(I'_1 - I'_2)}\right)$
$I'_m = I'_1 + I'_2$		

pression for isochromatic parameter including quarter wave plate error (δ') in terms of ε, δ and θ. Figure 6 shows the variation of error $[(\delta' - \delta)/2\pi]$ in isochromatic parameter as a function of retardation (δ) introduced by the model with θ as parameter. The error $[(\delta' - \delta)/2\pi]$ varies as a function of δ and θ. To compare the four step algorithms in an overall sense, the RMS of the error $[(\delta' - \delta)/2\pi]$ for $\delta = 0$ to 2π for different values of θ is plotted as shown in Fig. 7. From Fig. 7 it is clear that RMS error of isochromatic parameter by Barone et. al and Patterson and Wang four step methods are higher than the four step algorithm of Ajovalasit et. al. On the other hand, the RMS error by Asundi's algorithm in the region $\theta = 0°$ to $7.5°$ and $\theta = 30°$ to $45°$ is lower than that of Ajovalasit et. al's algorithm, but is much higher in the region $\theta = 7.5°$ to $30°$. Since input quarter wave plate has to be kept at different orientation to record four phase shifted images in the algorithm of Asundi, the methodology is not quite convenient to device a hardware for recording four phase shifted images simultaneously. On the other hand, in the algorithm of Ajovalasit et. al the orientation of input quarter wave plate is not altered and hence it is amenable for devising a hardware to record four images simultaneously.

Conclusion

Equations including quarter wave plate error for the intensity of light transmitted in a generic circular polariscope for all the multiple optical arrangements are derived. The influence of quarter wave plate error on the selection of appropriate optical arrangements for phase shifting techniques is studied. Various possibilities of obtaining the isoclinic and isochromatic parameter by six step algorithm with least error in the presence of quarter wave plate mismatch are explored. The available four step methods are analyzed based on their performance in the presence

Table 11: Intensity equations for various four step phase shifting algorithms considering quarter wave plate

Optical Arrangement			Intensity Equations
Ajovalasit[4]			
$3\pi/4$	$\pi/4$	$\pi/2$	$I_1' = I_b + I_a\left[\left(1-\cos^2 2\theta\sin^2\varepsilon\right)\cos^2\frac{\delta}{2} + \cos^2 2\theta\sin^2\varepsilon\right]$
$3\pi/4$	$\pi/4$	0	$I_2' = I_b + I_a\left[\left(1-\cos^2 2\theta\sin^2\varepsilon\right)\sin^2\frac{\delta}{2}\right]$
$3\pi/4$	0	0	$I_3' = I_b + \frac{I_a}{2}\left[1+\left(\cos^2 2\theta + \sin^2 2\theta\cos\delta\right)\sin\varepsilon - \sin 2\theta\cos\varepsilon\sin\delta\right]$
$3\pi/4$	$\pi/4$	$\pi/4$	$I_4' = I_b + \frac{I_a}{2}\left[1+\left(1-\cos\delta\right)\sin 2\theta\cos 2\theta\sin\varepsilon + \cos 2\theta\cos\varepsilon\sin\delta\right]$
Asundi [5]			
$3\pi/4$	$\pi/4$	$\pi/2$	$I_1' = I_b + I_a\left[\left(1-\cos^2 2\theta\sin^2\varepsilon\right)\cos^2\frac{\delta}{2} + \cos^2 2\theta\sin^2\varepsilon\right]$
$3\pi/4$	$\pi/4$	0	$I_2' = I_b + I_a\left[\left(1-\cos^2 2\theta\sin^2\varepsilon\right)\sin^2\frac{\delta}{2}\right]$
$3\pi/4$	$\pi/4$	$\pi/4$	$I_3' = I_b + \frac{I_a}{2}\left[1+\left(1-\cos\delta\right)\sin 2\theta\cos 2\theta\sin\varepsilon + \cos 2\theta\cos\varepsilon\sin\delta\right]$
π	$\pi/2$	$\pi/4$	$I_4' = I_b + \frac{I_a}{2}\left[1-\left(1-\cos\delta\right)\sin 2\theta\cos 2\theta\sin\varepsilon + \cos 2\theta\cos\varepsilon\sin\delta\right]$
Barone [7]			
$3\pi/4$	0	$\pi/4$	$I_1' = I_b + \frac{I_a}{2}\left(1-\left(1-\cos\delta\right)\sin 2\theta\cos 2\theta + \left(\cos\delta + \left(1-\cos\delta\right)\sin 2\theta\cos 2\theta\right)\cos^2\varepsilon + \left(\sin 2\theta - \cos 2\theta\right)\sin\varepsilon\cos\varepsilon\sin\delta\right)$
$3\pi/4$	0	$3\pi/4$	$I_2' = I_b + \frac{I_a}{2}\left(1+\left(1-\cos\delta\right)\sin 2\theta\cos 2\theta - \left(\cos\delta + \left(1-\cos\delta\right)\sin 2\theta\cos 2\theta\right)\cos^2\varepsilon - \left(\sin 2\theta - \cos 2\theta\right)\sin\varepsilon\cos\varepsilon\sin\delta\right)$
$3\pi/4$	0	0	$I_3' = I_b + \frac{I_a}{2}\left[1+\left(\cos^2 2\theta + \sin^2 2\theta\cos\delta\right)\sin\varepsilon - \sin 2\theta\cos\varepsilon\sin\delta\right]$
$3\pi/4$	$\pi/4$	$\pi/4$	$I_4' = I_b + \frac{I_a}{2}\left[1+\left(1-\cos\delta\right)\sin 2\theta\cos 2\theta\sin\varepsilon + \cos 2\theta\cos\varepsilon\sin\delta\right]$
Patterson [6]			
$3\pi/4$	0	$\pi/4$	$I_1' = I_b + \frac{I_a}{2}\left(1-\left(1-\cos\delta\right)\sin 2\theta\cos 2\theta + \left(\cos\delta + \sin 2\theta\cos 2\theta\left(1-\cos\delta\right)\right)\cos^2\varepsilon + \left(\sin 2\theta - \cos 2\theta\right)\sin\varepsilon\cos\varepsilon\sin\delta\right)$
$3\pi/4$	0	0	$I_2' = I_b + \frac{I_a}{2}\left(1+\cos\delta\sin\varepsilon + \left(1-\cos\delta\right)\cos^2 2\theta\sin\varepsilon - \sin 2\theta\sin\delta\cos\varepsilon\right)$
$3\pi/4$	$\pi/4$	$\pi/4$	$I_3' = I_b + \frac{I_a}{2}\left(1+\left(1-\cos\delta\right)\sin 2\theta\cos 2\theta\sin\varepsilon + \cos 2\theta\sin\delta\cos\varepsilon\right)$
$3\pi/4$	$\pi/2$	$\pi/2$	$I_4' = I_b + \frac{I_a}{2}\left(1-\cos\delta\sin\varepsilon - \left(1-\cos\delta\right)\cos^2 2\theta\sin\varepsilon + \sin 2\theta\sin\delta\cos\varepsilon\right)$

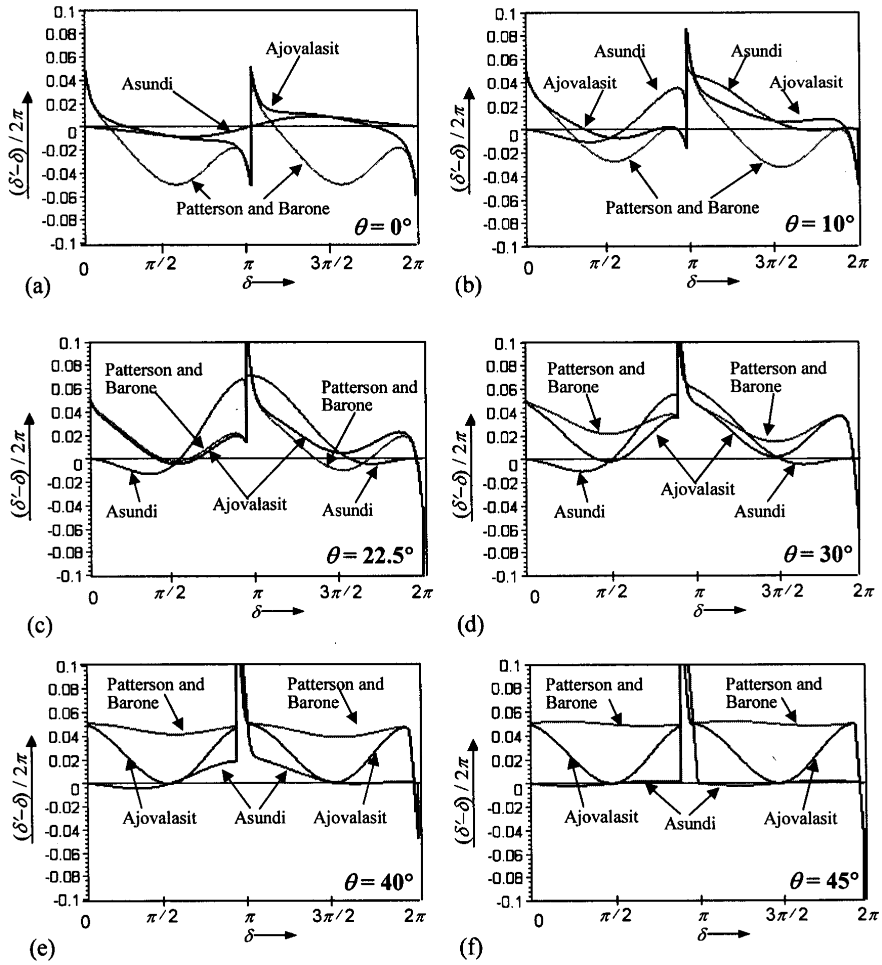

Fig. 6. Plot of error in isochromatic parameter $(\delta'_C/\delta)/2\pi$ due to quarter wave plate error ($\varepsilon = -9°$) for various Four Step Algorithm with θ as parameter. (a) for θ = 0°, (b) θ = 10°, (c) θ = 22.5°, (d) θ = 30°, (e) θ = 40°, (f) θ = 45°

Table 12: Equations for isochromatic parameter in Table 13 recast to get high modulation over the whole field

	Isochromatic parameter
Ajovalasit et. al(4 step)	$\delta_c = \tan^{-1}\left(\frac{((I_1+I_2)-2I_3)\sin 2\theta_c + (2I_4-(I_1+I_2))\cos 2\theta_c}{I_1-I_2}\right) = \tan^{-1}\left(\frac{I_a \sin\delta}{I_a \cos\delta}\right)$
Asundi et. al (4 step)	$\delta_c = \tan^{-1}\left(\frac{(2I_4-(I_1+I_2))\sin 2\theta_c + (2I_3-(I_1+I_2))\cos 2\theta_c}{I_1-I_2}\right) = \tan^{-1}\left(\frac{I_a \sin\delta}{I_a \cos\delta}\right)$
Barone et. al (4 step)	$\delta_c = \tan^{-1}\left(\frac{((I_1+I_2)-2I_3)\sin 2\theta_c + (2I_4-(I_1+I_2))\cos 2\theta_c}{I_1-I_2}\right) = \tan^{-1}\left(\frac{I_a \sin\delta}{I_a \cos\delta}\right)$
Patterson et. al (4 step)	$\delta_c = \tan^{-1}\left(\frac{(2I_4-(I_4+I_2))\sin 2\theta_c + (2I_3-(I_4+I_2))\cos 2\theta_c}{2I_1-(I_4+I_2)}\right) = \tan^{-1}\left(\frac{I_a \sin\delta}{I_a \cos\delta}\right)$

Table 13: Simplified forms of isoclinic parameter for various four step algorithms

	Isoclinic parameter	Isoclinic parameter including quarter wave plate error
Ajovalasit	$\theta_c = \frac{1}{2}\tan^{-1}\left(\frac{(I_1+I_2)-2I_3}{2I_4-(I_1+I_2)}\right)$	
Barone	$\theta_c = \frac{1}{2}\tan^{-1}\left(\frac{(I_1+I_2)-2I_3}{2I_4-(I_1+I_2)}\right)$	$\theta'_c = \frac{1}{2}\tan^{-1}\left(\frac{(\sin 2\theta\cos\varepsilon\sin\delta-(\cos^2 2\theta+\sin^2 2\theta\cos\delta)\sin\varepsilon)I_a}{(\cos 2\theta\cos\varepsilon\sin\delta+(1-\cos\delta)\sin 2\theta\cos 2\theta\sin\varepsilon)I_a}\right)$
Patterson	$\theta_c = \frac{1}{2}\tan^{-1}\left(\frac{2I_4-(I_4+I_2)}{2I_3-(I_4+I_2)}\right)$	
Asundi et al. (4 step)	$\theta_c = \frac{1}{2}\tan^{-1}\left(\frac{2I_4-(I_1+I_2)}{2I_3-(I_1+I_2)}\right)$	$\theta'_c = \frac{1}{2}\tan^{-1}\left(\frac{(\sin 2\theta\cos\varepsilon\sin\delta+(1-\cos\delta)\sin 2\theta\cos 2\theta\sin\varepsilon)I_a}{(\cos 2\theta\cos\varepsilon\sin\delta+(1-\cos\delta)\sin 2\theta\cos 2\theta\sin\varepsilon)I_a}\right)$

Table 14: Simplified forms of isochromatic parameter for various four step algorithms

Ajovalasit 4-step	$\delta'_c = \tan^{-1}\left(\frac{((\cos^2 2\theta+\sin^2 2\theta\cos\delta)\sin\varepsilon-\sin 2\theta\cos\varepsilon\sin\delta)\sin 2\theta'_c+((1-\cos\delta)\sin 2\theta\cos 2\theta\sin\varepsilon+\cos 2\theta\cos\varepsilon\sin\delta)\cos 2\theta'_c}{(1-\cos\delta)\sin 2\theta\cos 2\theta\sin\varepsilon+\cos 2\theta\cos\varepsilon\sin\delta}\right)$
Asundi 4-step	$\delta_c = \tan^{-1}\left(\frac{((1-\cos\delta)\sin 2\theta\cos 2\theta\sin\varepsilon+\sin 2\theta\cos\varepsilon\sin\delta)\sin 2\theta'_c+((1-\cos\delta)\sin 2\theta\cos 2\theta\sin\varepsilon+\cos 2\theta\cos\varepsilon\sin\delta)\cos 2\theta'_c}{(1-\cos^2 2\theta\sin^2\varepsilon)\cos\delta+\cos^2 2\theta\sin^2\varepsilon}\right)$
Barone and Patterson 4-step	$\delta_c = \tan^{-1}\left(\frac{((\cos^2 2\theta+\sin^2 2\theta\cos\delta)\sin\varepsilon-\sin 2\theta\cos\varepsilon\sin\delta)\sin 2\theta_c+((1-\cos\delta)\sin 2\theta\cos 2\theta\sin\varepsilon+\cos 2\theta\cos\varepsilon\sin\delta)\cos 2\theta_c}{(\sin 2\theta-\cos 2\theta)\sin\varepsilon\cos\varepsilon\sin\delta-(1-\cos\delta)\sin 2\theta\cos 2\theta+(\cos\delta+(1-\cos\delta)\sin 2\theta\cos 2\theta)\cos^2\varepsilon}\right)$

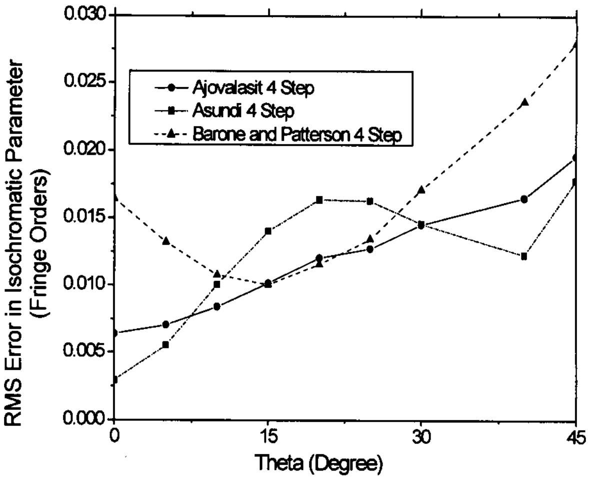

Fig. 7. Plot of RMS error in isochromatic parameter for various four step algorithms as a function of θ

of quarter wave plate error. The four step methodology of Ajovalasit et. al is amenable for devising a hardware to record four phase shifted images simultaneously. The error in isochromatic parameter evaluation by this four step method is smaller than the methods by Patterson and Wang and Barone et. al and is comparable to the Asundi four step method.

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