# EFFECTIVENESS OF DISCRETE PIEZO-PATCH LOCATIONS IN CLAMPED-SIMPLY SUPPORTED PLATES

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**Abstract**

*Present study investigates the problem of determining the control effectiveness of piezoelectric patches for controlling vibration in higher modes of clamped-simply supported plates. A modal analysis approach is used to identify high average strain (or curvature) locations, as desired patch locations and represented in the form of geometrical parameters. The 9-patch configuration is used and modal analysis of integrated structure is carried out, which clearly shows that the initial patch configuration yields better effectiveness for square plates. In case of rectangular plates, the above strategy provides mixed results for the patch distribution effectiveness, for the modes considered here. Next, a concept of 'stretching' the patch distribution is applied for improving the effectiveness of the patch distribution and a Fig. of Merit (FOM), based on modal strains, is defined for quantifying the effectiveness. The results show that particular patch distribution geometry is suitable only for a specific group of modes. In view of this, a combined Fig. of Merit, based on the concept of modal weights, is also introduced, which has the potential to help in determining a suitable patch distribution through a formal optimization methodology. The control effectiveness comparison study of the initial and stretched patch configuration of plates also supports the quantification of control effectiveness of patch configuration through the FOM.*

## Introduction

Thin plates are common structural elements employed in many engineering applications and are subject to a wide variety of excitations, including acoustic excitations. When plates are very thin, as in aerospace applications, large number of vibration modes lie within the bandwidth of the acoustic excitation. It is well known that significant vibration of plates in their higher modes can have adverse consequences for the overall fatigue life of such plates and, therefore, it is necessary to address the problem of control of vibration in higher modes for plates that are exposed to high frequency acoustic excitation. In recent times, discrete piezo patches have emerged as preferred elements that can sense and cancel elastic vibration, by generating control strains. However, the effectiveness of such structural vibration control depends on many factors e.g. number of patches, their placement and size, as well as the plate boundary condition and its aspect ratio. One of the major challenges in controlling higher modes of vibration is to either maximize the vibration reduction for a given number of patches or to minimize number of patches for a specified control objective. In such a case, it becomes important to locate the discrete patches in order to maximize their collective effectiveness. However, it is known that such locations not only differ significantly from mode to mode but also have non-uniqueness for higher vibration modes. Present study uses a modal analysis based strategy to identify and characterize the patch locations that may be the best for a given control objective, in plates having mixed homogeneous boundary conditions.

In literature, researchers have proposed different methods for placement of piezo patches and notable among these are works of Tzou et al [2], Yaman et al [3], Young-Hun Lim [4] and D. Halim et al [5]. The approaches include; the piezo patches in X- configuration for controlling bending and twisting modes, placement of 12-patches on rectangular plate to study the response by changing the position of patches as a group and a patch location strategy, based on the identification of regions of high curvature, using a 5-patch configuration for clamped square plate. Authors [6] in a recent study have proposed a new 9-patch configuration for clamped square plate, which is shown to provide improvements in the overall effectiveness, in comparison to the 5-patch configuration of [4].

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The present study proposes to extend the above concept also to clamped-simply supported plates of different aspect ratios and also uses a modal strain based Figure Of Merit for quantifying the patch effectiveness. The objective of the present study is to provide a simple and effective mechanism of placing patches on plate type structures for controlling multimode vibration of plates. The high curvature zones are natural locations for piezo patches in effective control of vibration. The aim of the present study is to arrive at a simple strategy for identifying these high curvature points, based only on the 1$^{st}$ mode analysis and the geometric parameters of plate. Further, to generalize this approach, commercial software ANSYS© multi-physics is used.

### Modelling and Validation

The application of the finite element modeling techniques in the smart structures has grown significantly during the last decade, resulting in availability of piezo-electric elements in commercial finite element codes such as ANSYS©. In the present study, ANSYS© is used for generating the modal information of clamped-simply supported plates, without and with discrete piezo patches. In the reference [4], twenty-node solid elements have been used for piezo patch portion as well as for the part of the plate beneath the piezo patches. However, around the patches, thirteen-node transition elements have been proposed for connecting the solid elements to the remaining part of the plate, which is modelled by nine noded flat shell elements. Yaman et al [3] have demonstrated that modeling of the passive portion (plate without patches) using consistent solid elements with the actuator elements yields better results.

Further, the analysis carried out by authors in a earlier study [6] has shown that, results obtained by using solid 45 and solid 5 elements available in ANSYS© for the passive portion and for the actuator portions respectively, are in close agreement with the results given in reference [4]. Therefore, the same modelling strategy is adopted in the present study. Material models are defined separately for passive structure and piezoelectric materials and assigned as volume attributes. The key points at the interfaces of passive plate and piezoelectric patches are merged. The mesh sizes of piezoelectric discrete patches are taken as 4x4x1. The electrical degrees of freedom, in volts, are coupled for the nodes at the top and bottom surfaces of the piezo patches. Lastly, in case of simply supported boundary condition, it is necessary to apply appropriate rotational constraints also and, as the Solid 45 elements available in ANSYS© have only three translational degrees of freedom, three rotational degrees of freedoms are added externally, for correctly representing the boundary condition. In order to verify the adequacy of the above model, shell63 elements available in ANSYS© are used for generating the host plate frequencies and it is found that differences between frequencies of all the eight modes are less than 0.2%. Therefore, the finite element model adopted in the present study is considered to be adequate.

### Modal Analysis of the Host Plate

The present study is based on the premise that modal order and the corresponding mode shape function provide a significant insight into the best actuator locations for vibration control. It is well known that, for enhancing the effectiveness of the strain-based actuators it is necessary to place all of them in the regions of maximum strain/curvature. Therefore, mode shapes of the clamped-simply supported square and rectangular plate structures are obtained and analyzed for maximum strain/curvature points and as well as nodal lines (zero strain). In case of rectangular plates, mode shapes are also obtained by interchanging the boundary conditions along parallel edges from clamped to simply supported. In the present study, the plate geometry of references [4] and [6] is adopted for the sake of validation of the modelling philosophy.

The square plate of size 305x305x0.8mm$^3$ is modeled by using solid45 elements. The mesh size of 60x60x1 is arrived at through a frequency and modal deformation convergence study and the corresponding first eight mode shapes are shown in Figs. 1(a)-(h). It can be seen that high modal deflection (and consequently modal strain) regions lie either on a diagonal, or on a perpendicular bisector, for all the eight modes considered in the present analysis. This fact is used to initially place patches along the two diagonals and the two perpendicular bisectors. In order to bring out the influence of plate aspect ratio on the patch effectiveness, rectangular plates with aspect ratio of 1.5 and 2 are chosen, by suitably increasing the length of the two of the opposite edges. The mesh size of 90x60x1 and 120x60x1 are selected for plate aspect ratio of 1.5 and 2.0 respectively, in order to ensure convergence for larger aspect ratio plates by keeping the element aspect ratio as 1.0. Mode shape plots of rectangular plates with aspect ratio 1.5 are given in Figs. 2(a)-(h) and Figs. 3(a)-(h), wherein the clamped and the simply supported boundary conditions are flipped. It is found that these mode shapes are similar to those reported in ref. [7]. These plots also

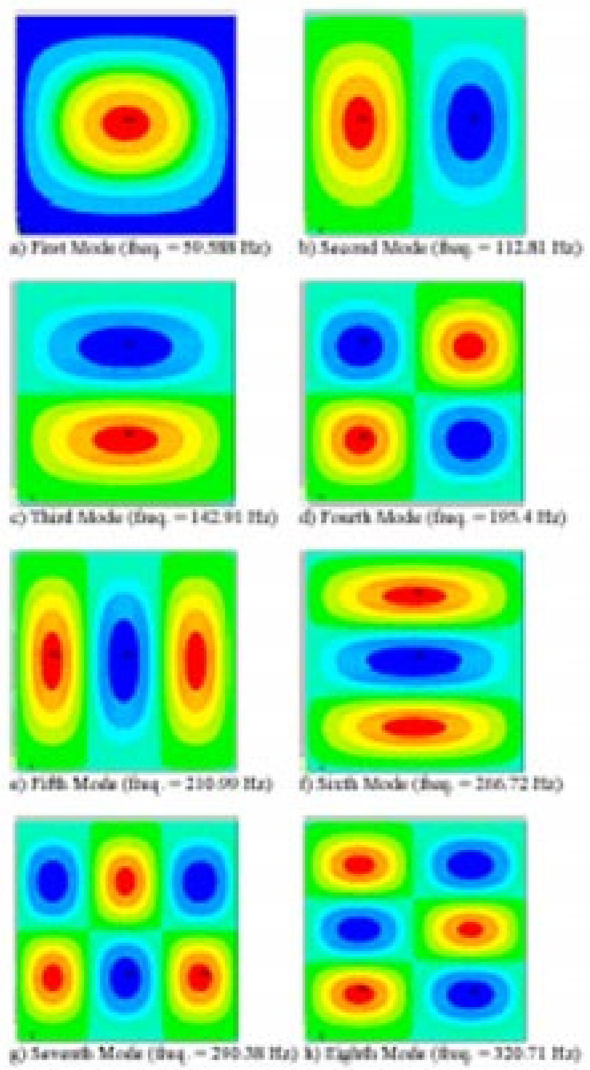


*Fig. 1 First eight mode shapes of clamped-simply supported square plate*

show that, the boundary condition along the longer parallel edges determines the overall mode pattern. It is further seen from these plots that high curvature regions largely lie either on a perpendicular bisector or on a diagonal, as noticed in the case of square plate. Therefore, the strategy of placing the patches along these four lines in the present case is found to be consistent with the modal analysis.

### Modal Analysis of Integrated Structure with 9-Patch Configuration

The schematic of the proposed configuration of host square plate with nine patches is shown in Fig. 4. Nine piezoelectric patches made of PZT-5H are selected for use as collocated sensors and actuators in bimorph configuration for control of multimode vibration. All the patches are of equal size (20 x 20 x 0.1 mm$^3$) and are initially placed along the two diagonals and along the two perpendicular bi-sectors of the plate geometry. The central patch is placed at the maximum strain point of first mode, while remaining eight patches are located on the basis of a simple arithmetic mean of maximum strain locations of all the remaining seven modes on the four sides of central patch, as an initial placement guess. These locations of patches are shown in Fig. 4 in terms of geometrical parameters and treated as initial patch configuration.

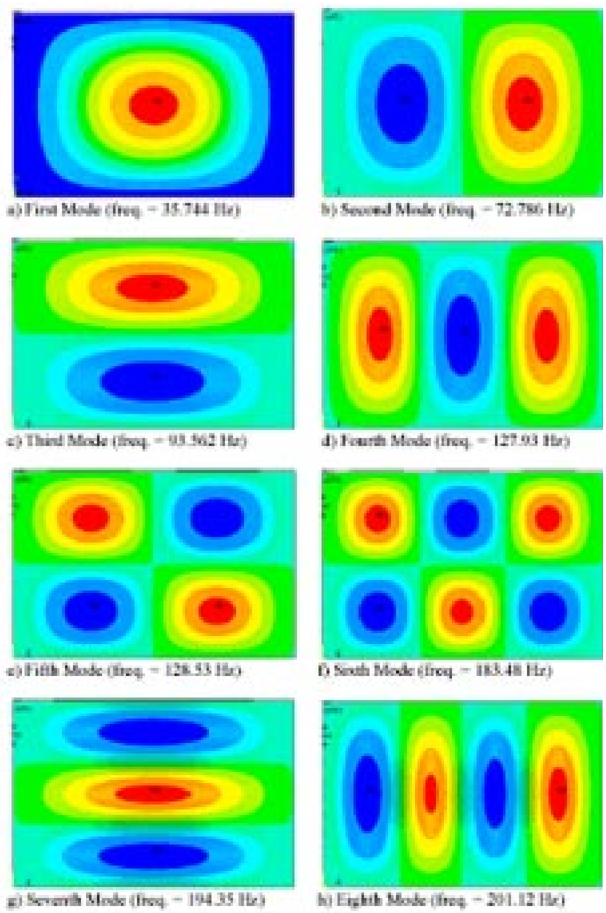


*Fig. 2 First eight mode shapes of clamped-simply supported (longer edges) rectangular*

It is to be noted here that, while initial patch placement guess is ad hoc and empirical in nature, the fact that it takes into account the maximum curvature points of the host plate modes of interest, provides a consistency and physics based backing to this guess. The modal analysis of integrated structure with passive piezoelectric patches (actua-

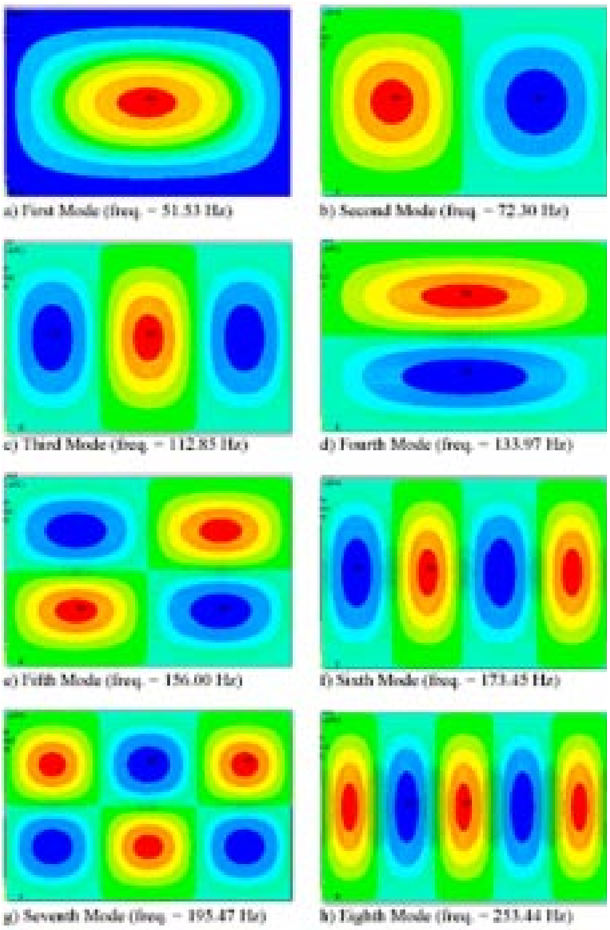


*Fig. 3 First eight mode shapes of clamped (longer edges)-simply supported rectangular plate of aspect ratio 1.5*

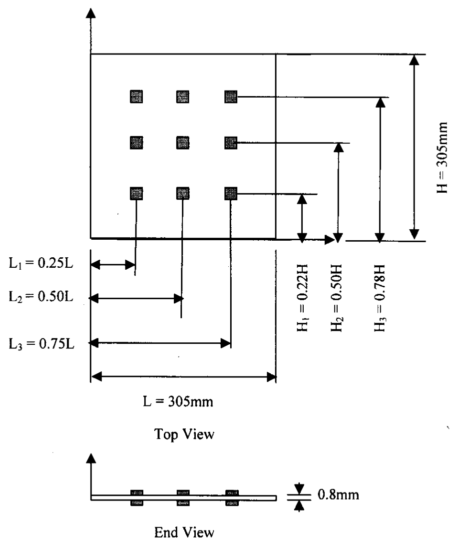


*Fig. 4 Schematic of simply supported (along length) and clamped (along height) host square plate*

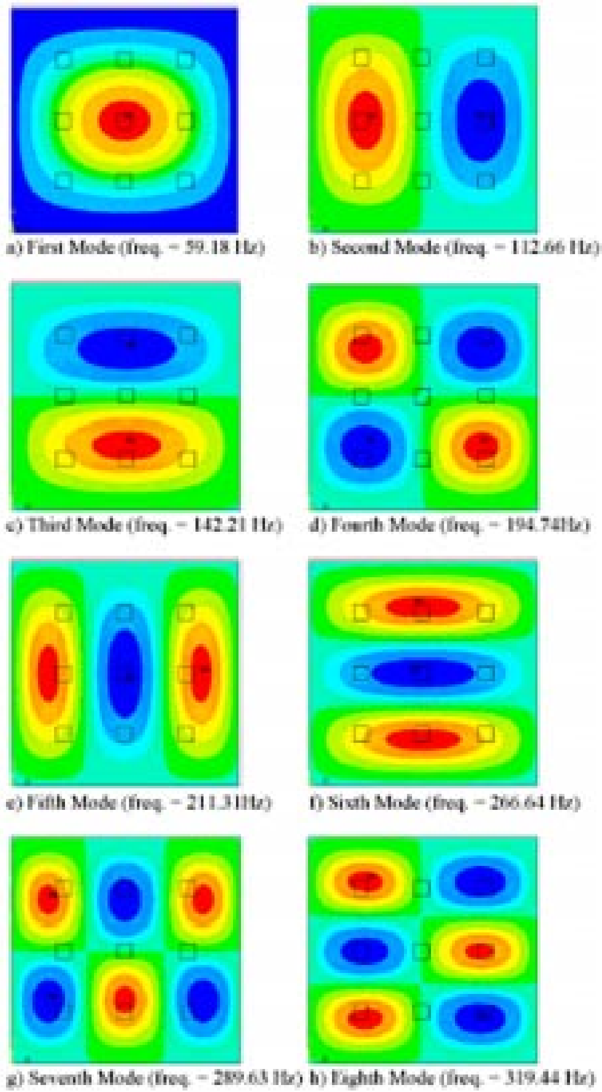


*Fig. 5 First eight mode shapes of square plate integrated with 9-patch configuration*

tors) is performed and mode shape diagrams are studied. Figs. 5(a)-(h) show the patch locations, along with the square plate mode shapes and it can be seen that at least 2-3 patches are at, or very close to, the regions of high curvature for all the eight modes considered here. This result supports the adequacy of the procedure for arriving at the initial patch configuration and indicates that if such a patch configuration is used for vibration control, it is likely to be reasonably effective for the higher as well as lower modes. Next, rectangular plates with aspect ratio of 1.5 and 2 respectively, are analyzed with initial patch configuration arrived at in the same manner and the patch distribution is kept doubly symmetric. The schematic of

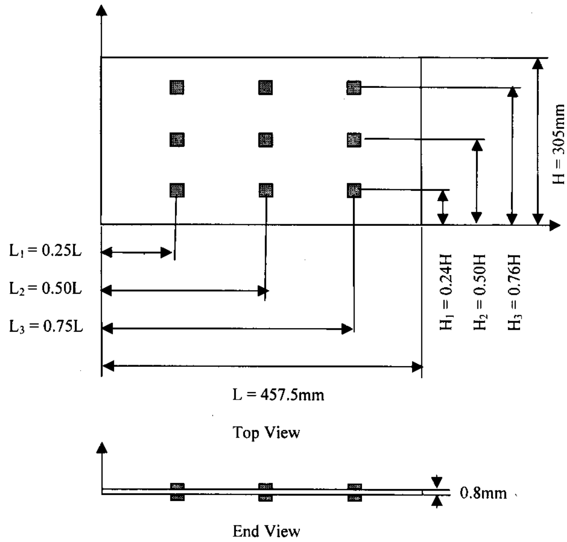


*Fig. 6 Schematic of simply supported (longer edges) and clamped (shorter edges) host plate of aspect ratio 1.5 with 9- patch configuration*

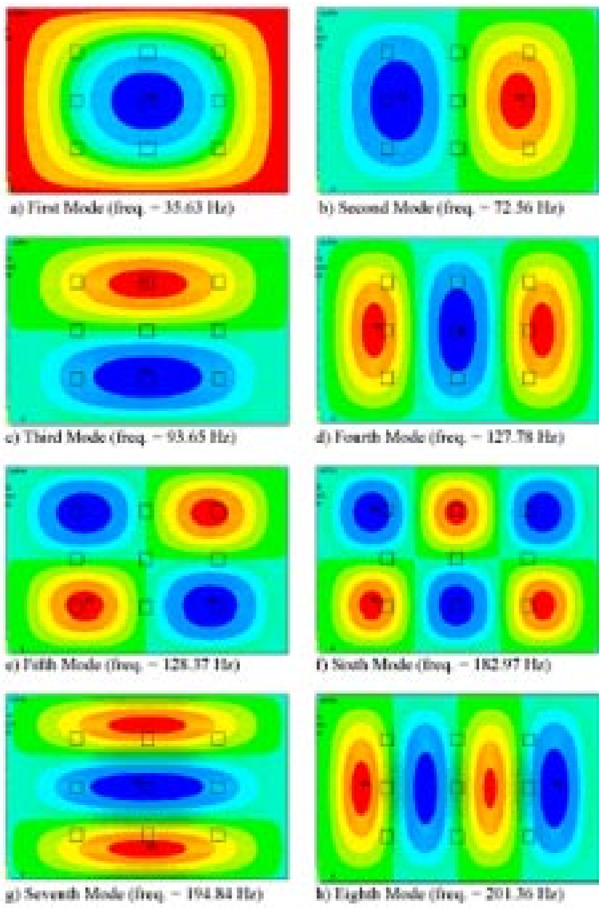

*Fig. 7 First eight mode shapes of clamped-simply supported (longer edges) rectangular plate of aspect ratio 1.5 integrated with initial patch configuration*

the initial patch configuration for the plate aspect ratio 1.5 is shown in Fig. 6, with patch locations in terms of geometrical parameters. Figs. 7(a)-(h) give mode shape plots for simply supported (longer edges) and clamped (shorter edges) rectangular plate of aspect ratio 1.5 and it can be seen that, excepting for the eighth mode shape, for all the other mode shapes, there are at least 2-3 patches that lie on the regions of high curvature. In the case of eighth mode shape, it is found that all the patches are fairly close to inflection points (or points having zero or small modal curvature), resulting in poor control effectiveness. After interchanging the boundary conditions along parallel edges, mode shape plots of same rectangular plate are shown in Figs. 8(a)-(h). In this case, it is found that for the sixth mode, all the patches are closer to nodal lines. The same trend is also observed in the case of plate with aspect ratio 2.0, thus bringing out a deficiency in the initial patch configuration guess.

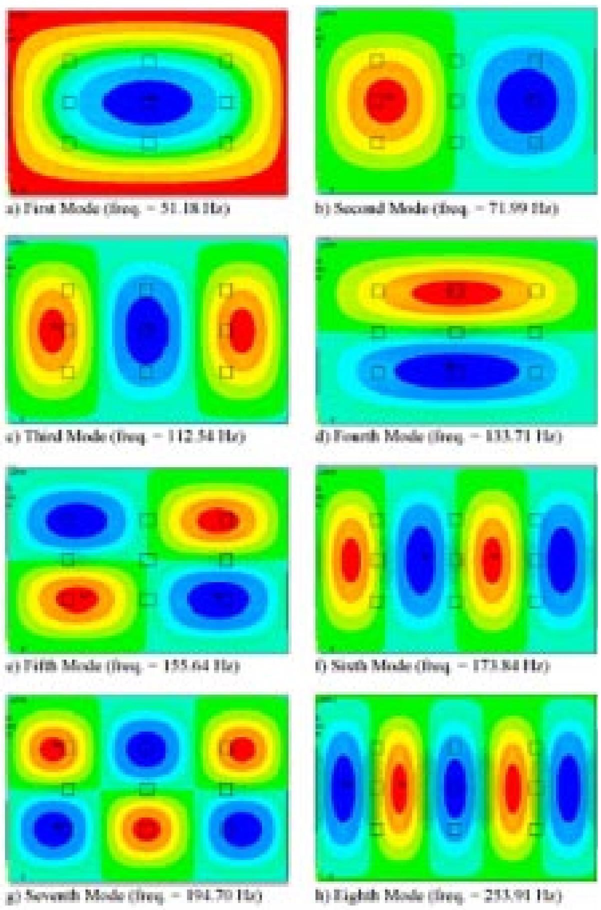

*Fig. 8 First eight mode shapes of clamped (longer edges)-simply supported rectangular plate of aspect ratio 1.5 integrated with initial patch configuration*

### Modal Analysis of 'Stretched' Integrated Structure

It is interesting to note from the eighth mode of Fig.7 and sixth mode of Fig. 8 that the patches are away from the maximum strain region only along the x-axes, while being placed correctly along the y-axis. Therefore, with the objective of controlling first eight modes by same number of patches, the patch distribution for plate aspect ratio 1.5 is 'stretched' along x-axes by 25% from the initial patch configuration and a study is conducted for this 'stretched' patch distribution (with passive patches), for the same plate aspect ratio. Mode shapes of the 'stretched' patch distribution for the plate of aspect ratio of 1.5 are shown in Figs. 9(a)-(h). It is seen that while patch distribution for all other modes remains practically unchanged, in the case of the eighth mode, patches move closer to the high curvature regions, indicating a possible improvement in the effectiveness of the patch distribution for eighth mode.

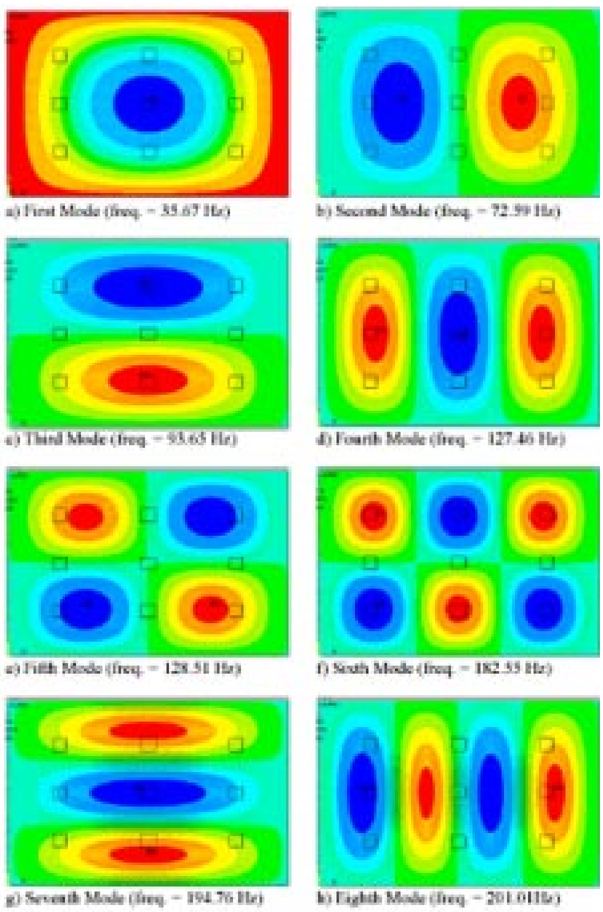

*Fig. 9 First eight mode shapes of clamped-simply supported (longer edges) rectangular plate of aspect ratio1.5 with 'stretched' patch configuration*

### Quantification of the Patch Distribution Effectiveness

The preceding analyses have shown that the 9-patch configuration is capable of providing control in all the eight modes, but with varying degree of effectiveness. This variation arises due to the fact that each patch senses a different strain, depending upon its proximity to the region of high modal curvature. In order to compare various candidate patch configurations for their control effectiveness, it is necessary to define a quantitative measure of the effectiveness of the patch distribution. In view of the fact that a patch responds to strain, a Figure Of Merit (FOM), based on the elastic bending strain magnitude, is considered to be the most appropriate. The strain values are taken at the centre of all patches, along with maximum strain value for each mode from the modal strain plot and a root mean square modal Figure of Merit is defined as,

$$FOM_j = \sqrt{(\sum \varepsilon_{ij}^2)/\varepsilon_{maxj}^2} \quad (1)$$

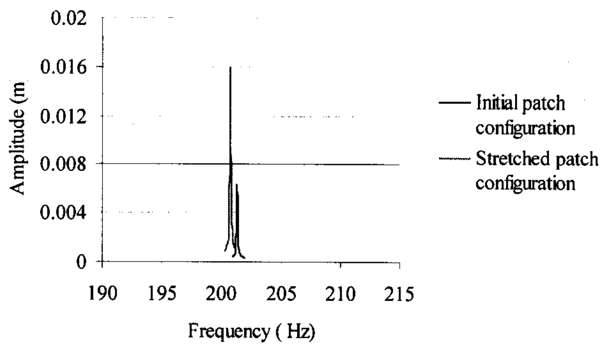


*Fig. 10 Frequency response plots of eighth mode of simply supported (longer edges) and clamped plate of aspect ratio 1.5 with initial and stretched patch configuration*

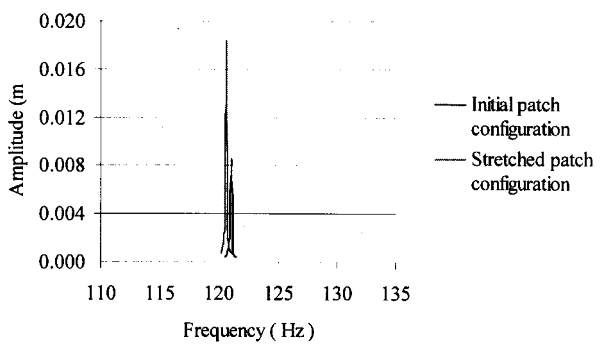


*Fig. 11 Frequency response plots of sixth mode of simply supported (longer edges) and clamped plate of aspect ratio 2.0 with initial and stretched patch configuration*

where, $FOM_j$ is the Figure of Merit of the $j^{th}$ elastic mode, $\varepsilon_{ij}$ is elastic strain component for $i^{th}$ patch and $j^{th}$ mode and $\varepsilon_{maxj}$ is maximum strain value for $j^{th}$ mode. The FOMs, calculated for an integrated structure of clamped-simply supported (longer edges) rectangular plates with 9-patch configuration having various patch distributions, are given in Table 1. Table 2 gives the same result after interchanging the boundary conditions along parallel edges. It is seen from Table 1 that while the effectiveness of the initial patch configuration for the plate of aspect ratio 1.5 in eighth mode is very small, it increases to more than double its value for the 'stretched' patch configuration. However, there is a marginal adjustment in the effectiveness for other modes as the patch distribution is 'stretched'. From Table 2 it is seen that the effectiveness of the initial patch configuration in sixth mode is very small which increases to about twice its earlier value for 'stretched' patch configuration. The same behavior is also seen for the plate of aspect ratio 2.0 and it can be concluded that a 'stretching' of the distribution in longer direction, improves the effectiveness of patch distribution in controlling a particular mode of interest. It is further noted that the FOM values in Table 1 are closer to the values for plates with all edges simply supported and similarly, the values in Table 2 approach those estimated for plates with all edges clamped. This shows that the nature of constraint on longer parallel edges dominates the overall plate modal behavior. It should be mentioned here that, in the definition of the FOM given in equation (1), all the eight modes are considered to be equally important and in reality, vibration control objectives can focus, either on a specific mode, or on a group of modes, that may be of interest from the point of view of excitation spectrum. In such a case it is more beneficial to assign a weightage to a particular mode and define a combined figure of merit as,

$$FOM_j = (1/No.\ of\ modes)\sqrt{\sum FOM_j^2\ w_j^2} \qquad (2)$$

where, $w_j$ is weightage for the $j^{th}$ mode.

These weightages can vary as per the expected excitation spectrum as well as with the importance assigned to a particular mode and make the process of deciding patch configuration, suitable for a formal optimization exercise. Total FOMs are calculated for the various patch distribution aspect ratios for the same plate based on a set of weights, representing various situations and are given in Table 3 and 4. It can be seen that depending on the control objective, a different patch distribution becomes more effective.

## Control Effectiveness Comparison of Stretched Patch Configuration

The piezoelectric patch bonded to plate surface, which generates maximum vibration after activating, can effectively control the same magnitude of vibration by changing the polarity of voltage supply. Therefore, for the comparison of control effectiveness of initial patch configuration and stretched patch configuration, the study is conducted for estimating the vibration generated in each mode by activating the piezoelectric patches (actuators) integrated with plates. For the comparison purpose, total voltage supplied to the 9-patches (actuators) is kept same

**Table 1 Figure of Merit for different patch distribution aspect ratios & for two values of plate aspect ratios of clamped-simply supported (longer edges) plate Patch Distribution Plate Aspect Ratio 1.5 Plate Aspect Ratio 2**

| Mode Shapes | Patch Distribution | | | |
|---|---|---|---|---|
| | Plate Aspect Ratio 1.5 | | Plate Aspect Ratio 2 | |
| | Initial Patch configuration | Stretched Patch Configuration | Initial Patch Configuration | Stretched Patch Configuration |
| 1 | 1.106 | 0.954 | 1.138 | 0.980 |
| 2 | 1.085 | 0.740 | 1.203 | 0.799 |
| 3 | 1.221 | 1.063 | 1.395 | 1.367 |
| 4 | 1.302 | 1.330 | 1.224 | 1.070 |
| 5 | 1.340 | 1.091 | 1.354 | 1.067 |
| 6 | 1.615 | 1.725 | 0.535 | 1.114 |
| 7 | 1.370 | 1.208 | 1.644 | 1.690 |
| 8 | 0.400 | 1.056 | 0.874 | 1.196 |

**Table 2: Figure of Merit for different patch distribution aspect ratios and for two values of plate aspect ratios of clamped (longer edges)-simply supported plate**

| Mode Shapes | Patch Distribution | | | |
|---|---|---|---|---|
| | Plate Aspect Ratio 1.5 | | Plate Aspect Ratio 2 | |
| | Initial Patch Configuration | Stretched Patch Configuration | Initial Patch configuration | Stretched Patch Configuration |
| 1 | 0.810 | 0.714 | 0.744 | 0.657 |
| 2 | 1.194 | 1.010 | 1.023 | 0.846 |
| 3 | 1.471 | 1.561 | 1.422 | 1.503 |
| 4 | 0.985 | 0.870 | 0.487 | 1.222 |
| 5 | 1.177 | 0.977 | 0.964 | 0.850 |
| 6 | 0.511 | 1.244 | 1.087 | 0.906 |
| 7 | 1.625 | 1.737 | 1.047 | 1.358 |
| 8 | 1.096 | 1.373 | 1.403 | 1.517 |

**Table 3: Weighted FOMs for different patch distribution aspect ratios & for two values of plate aspect ratios of clamped-simply supported (longer edges) plate**

| No | Assumed weights for modes | Patch distribution | | | |
|---|---|---|---|---|---|
| | | Plate aspect ratio 1.50 | | Plate aspect ratio 2.00 | |
| | | Initial Patch Configuration | Stretched Patch Configuration | Initial Patch Configuration | Stretched Patch Configuration |
| 1. | All equal | 0.0542 | 0.0520 | 0.0536 | 0.0525 |
| 2. | Equal for only odd | 0.0790 | 0.0677 | 0.0871 | 0.0816 |
| 3. | Equal for only even | 0.0742 | 0.0791 | 0.0625 | 0.0660 |
| 4. | Equal for only first four | 0.0739 | 0.0652 | 0.0777 | 0.0671 |
| 5. | Equal for only last four | 0.0793 | 0.0811 | 0.0739 | 0.0807 |
| 6. | First four lower equal & next four higher equal | 0.0651 | 0.0662 | 0.0611 | 0.0659 |
| 7. | First four higher equal & next four lower equal | 0.0612 | 0.0546 | 0.0639 | 0.0561 |

**Table 4: Weighted FOMs for different patch distribution aspect ratios & for two values of plate aspect ratios of clamped (longer edges)-simply supported plate**

| No | Assumed weights for modes | Patch distribution | | | |
|---|---|---|---|---|---|
| | | Plate aspect ratio 1.50 | | Plate aspect ratio 2.00 | |
| | | Initial Patch Configuration | Stretched Patch Configuration | Initial Patch Configuration | Stretched Patch Configuration |
| 1. | All equal | 0.0511 | 0.0544 | 0.0470 | 0.0508 |
| 2. | Equal for only odd | 0.0818 | 0.0822 | 0.0670 | 0.0716 |
| 3. | Equal for only even | 0.0614 | 0.0713 | 0.0658 | 0.0722 |
| 4. | Equal for only first four | 0.0714 | 0.0679 | 0.0614 | 0.0692 |
| 5. | Equal for only last four | 0.0732 | 0.0850 | 0.0711 | 0.0745 |
| 6. | First four lower equal & next four higher equal | 0.0603 | 0.0694 | 0.0582 | 0.0612 |
| 7. | First four higher equal & next four lower equal | 0.0590 | 0.0569 | 0.0511 | 0.0573 |

and known equal voltage (25 volt) is supplied to all the patches in phase with the maximum deformation of each mode at a given patch location. The vibration amplitude in each mode is obtained for both activated actuator patches of initial patch configuration and stretched patch configuration of same plate aspect ratio. The excitation used for this purpose is the harmonic force of 0.1 N at appropriate locations. Assuming structural damping to be constant at 0.01%, the dynamic responses along z-direction are plotted for each mode with active actuator patches. These response diagrams are obtained by performing harmonic analysis in ANSYS©.

The data generated from the harmonic analysis is used to compare the control effectiveness of initial and stretched patch configuration in each mode of 9-patch configurations integrated with simply supported (longer edges) and clamped plates. For this purpose the measure

**Table 5: Control effectiveness comparison of initial and stretched patch configuration of simply supported (longer edges) and clamped plates of various aspect ratio**

| Modes | Plate aspect ratio 1.5 | | Plate aspect ratio 2.0 | |
|---|---|---|---|---|
| | Amplitude ratio | FOM ratio | Amplitude ratio | FOM ratio |
| 1 | 0.896 | 0.862 | 0.806 | 0.861 |
| 2 | 0.787 | 0.682 | 0.778 | 0.664 |
| 3 | 0.849 | 0.871 | 0.965 | 0.980 |
| 4 | 1.059 | 1.022 | 0.856 | 0.874 |
| 5 | 0.876 | 0.814 | 0.880 | 0.789 |
| 6 | 1.035 | 1.068 | 2.160 | 2.082 |
| 7 | 0.865 | 0.881 | 1.066 | 1.028 |
| 8 | 2.516 | 2.637 | 1.378 | 1.368 |

of control effectiveness is defined as the ratio of maximum vibration amplitude generated by active piezo patches of stretched and initial patch configuration of the same plate in each mode of vibration. The estimated values of control effectiveness as well as ratio of Figure of Merit of stretched and initial patch configuration are given in table 5. It should be noted that both initial and stretched patch configurations are equally effective if the resulting control effectiveness ratio is unity (i.e. the vibration generated with initial and stretched patch configurations are same). The stretched patch configuration is more effective, if the resulting ratio is higher than unity and it is seen from table 5 that control effectiveness in eighth mode of stretched patch configuration increases 2.5 times as compared to initial patch configuration of plate of aspect ratio 1.5. It is also seen from table 5 that control effectiveness in sixth mode of stretched patch configuration increases 2.16 times as compared to initial patch configuration of plate of aspect ratio 2.0. The frequency response plots of these cases are given in Figs. 10 and 11 respectively. The control effectiveness (ratio of maximum vibration amplitude) and ratio of figure of merit of stretched and initial patch configuration of same plate in each mode are in close agreement (table 5). This validates the process of quantification of control effectiveness in the form of figure of merit.

## Conclusions

The problem of determining more effective patch locations for higher mode vibration control of clamped-simply supported plates is investigated through a modal analysis approach. Finite element analysis using ANSYS© is carried out for both host plate as well as plate integrated with nine piezoelectric patches in many different configurations, but having similar structure. The regions of high modal curvature are identified for first eight vibration modes for both square and rectangular plates. The concept of 'stretching' the patch distribution is used to improve the effectiveness of the patch configuration for vibration control. A modal strain based figure of merit is obtained for various patch distributions, for quantifying its overall vibration control effectiveness, which is also found to depend on the plate aspect ratio. The study provides results that have the potential to help in the choice of a more effective patch configuration for a specific group of modes, using a formal optimization procedure. The control effectiveness comparison study of initial and stretched patch configuration also supports the process of quantification of control effectiveness in the form of figure of merit. The present study also provides a simple yet effective mechanism of placing patches, based on the judicious mix of modal analysis and geometry of plate type structures. This is also proposed as a method that would provide a placement of patches that is fairly close to the actual optimum, but needs much less mathematical effort. It is believed that methodology purposed in this paper is capable of providing results fairly closer to the results obtained from a formal optimization procedure.

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**Appendix**: **Material Properties**

*Aluminum material data,*

| | |
|---|---|
| Density of material | = 2800 Kg/m$^3$ |
| Poisson's ratio | = 0.32 |
| Young's modulus | = 6.8 x 10$^{10}$ N/m$^2$ |
| PZT 5H Density | = 7500 Kg m$^{-3}$ |

Piezoelectric strain matrix,

$$h^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 17 \\ 0 & 0 & 0 & 0 & 17 & 0 \\ -6.5 & -6.5 & 23.3 & 0 & 0 & 0 \end{bmatrix} Cm^{-2}$$

Elastic stiffnes matrix

$$C^E = \begin{bmatrix} 12.6 & 7.95 & 8.41 & 0 & 0 & 0 \\ & 12.6 & 8.41 & 0 & 0 & 0 \\ & & 11.7 & 0 & 0 & 0 \\ & & & 2.33 & 0 & 0 \\ & & & & 2.3 & 0 \\ & & & & & 2.3 \end{bmatrix} \times 10^{10}\ Nm^{-2}$$

Dielectric matrix at constant strain,

$$b = \begin{bmatrix} 1.503 & 0 & 0 \\ 0 & 1.503 & 0 \\ 0 & 0 & 1.3 \end{bmatrix} \times 10^{-8}\ Fm^{-1}$$