# AN IMPROVED FINITE ELEMENT MODEL TO STUDY STRESS CONCENTRATION AROUND AN ELLIPTICAL CUTOUT IN PRESSURE VESSEL:VALIDATION : PART-I

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**Abstract**

*A finite element model to the analysis of thin shell structures with cutouts/inclusions is developed based on shear deformation theory. An eight-noded isoparametric shell element is considered with six degrees of freedom at each node. A convergence study is under taken to arrive at a suitable finite element mesh. The loading conditions include uniform tension, uniform pressure. The finite element results presented in this work are shown to be in good agreement with available analytical and experimental data and the direction for future work is also given.*

***Keywords*** *: shell, cutout, inclusion, finite element, stress concentration factor*

## Nomenclature

R, t = radius and thickness of the shell
L = half the length of the shell
2a, 2b = lengths of major and minor axes, of the elliptical cutout
k = axis ratio of the ellipse, b/a
K = stress concentration factor (SCF)
$\nu$ = Poisson's ratio of the shell
$\nu_c$ = Poisson's ratio of the elastic cover
E = Young's modulus of the shell
$E_c$ = Young's modulus of the elastic cover
$\beta$ = a non dimensional curvature parameter, $\beta^2 = \frac{a^2}{8\,Rt}\left[12\,(1-\nu^2)\right]^{1/2}$
$\eta$ = elliptic coordinate with respect to axis of the shell

## Introduction

Shell structures with cutouts are problems of the utmost importance in aerospace applications, pressure vessel and piping technology and nuclear engineering. The development of efficient finite elements for modelling shell structures has been, and continues to be active research area [1, 2]. The finite element method has proved to be an extremely powerful tool for the solution of problems involving complex geometries, arbitrary loadings and rather general material properties. Considerable advances have been made in the development and application of the finite element method to shell structures. There is a substantial body of literature devoted to shell finite elements and we will review this literature in order to identify formulations suitable for reliable, accurate, and efficient analysis of cutouts in shell structures. Much of the work reviewed by Gallagher [3,4]. The main objective of finite element analysis here described is the determination of the stress distribution around the cutout, where there are steep stress gradients in the immediate vicinity of the hole boundary. A very fine mesh of lower order elements near the hole will be required to accurately recover the steep stress gradients. Alternatively, the same accuracy can be achieved using coarse mesh of high precision elements at a reduced computational cost. For the realistic modelling of arbitrary shaped cutouts in general shells, triangular and quadrilateral elements with curved edges are preferable.

Several analytical, numerical and experimental studies have been conducted during the past sixty years to determine stress distributions in cylindrical shells with a cutout and subjected to various types of loadings; such as, axial tension and compression, torsion, and internal and external pressure. Pioneering analytical work was conducted

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by Lurie [5, 6], to investigate the effect of axial tension and internal pressure, and shell curvature, on the stress concentrations around a circular cutout in the 1940s. Many years later, analytical studies were presented by Lekkerkerker [7], Van Dyke [8], Ashmarin [9], Murthy et. al [10], Guz et. al [12] and Van Tooren et. al [13], that further investigated the effects of various factors on the stress concentrations around a cutout in a cylindrical shell. Similarly, experimental investigations have been conducted by Pierce and Chou [14], Bull [15], and Zirka and Chernopiskii [16] and numerical studies have been conducted by Key [17], Lindberg and Olson [18] and Liang et.al [20]. In 1964 and 1972, respectively, Hicks [21] and Ebner and Jung [22] summarized the resuls obtained from several of these previous studies and provided extensive list of references related to this problem.

Key [17] applied a quadrilateral shell element with four nodes and 36 degrees of freedom (dof) to the problem of a circular cutout in a homogeneous isotropic cylindrical shell subjected to axial tension. The element formulation is based on Discrete Kirchhoff Theory and hence transverse shear deformation effects are not considered. Lindberg and Olson [18] applied a high precision triangular shell element with three nodes and 36 dof to the analysis of circular hole in a cylindrical shell under axial tension and later to the problem of rectangular cutout [19].

Most of these previous studies are for isotropic cylindrical shells with a circular cutout. Only a few of these studies, such as those presented by Murthy et. al [10], and by Pierce and Chou [14] addressed the effects of cutout shape (elliptical cutouts) on the stress concentrations. The selected numerical results for circular cylindrical shells with either circular or elliptical cutouts/inclusions and subjected to either tension, or pressure loads are presented. Moreover, validated finite element model that enable rapid parametric studies would be very valuable to structural designers and for the development of new design criteria and design concepts.

According to these reviews, the analyses of thin shells with cutouts/inclusions are of the more difficult problems that have been attempted with the finite element method. The requirements upon valid minimum potential energy solutions in thin shell finite element analysis are extremely difficult to satisfy, and the satisfactory formulations are therefore relatively complicated. Further, for tackling problems involving description of arbitrary boundaries quadrilateral elements are obviously best suited. Mitigation of high stress concentrations by tailoring shell-wall thickness, material orthotropy and anisotropy, and cutout reinforcement are also important considerations in the design of aerospace structures made of lighweight composite materials.

It is more appropriate to use commercial FEM systems. They are widely distributed, well documented and user friendly. However there is a need to verify the accuracy of material models, finite elements and analysis procedures in such systems before using them for intended application. The predictability issue itself demands a convergence study. This in fact, is the aim and scope of the present study. The proposed finite element model is very attractive to practitioners since a general purpose ANSYS in which the isoparametric family of elements suitable for the analysis of shell-type structures taking into account membrane-bending coupling and transverse shear deformation effects is available can be used without major modifications. The proposed model can handle arbitrary material models. However, a rigorous interpretation of the finite element solutions necessitates an analytical study.

This paper will restrict its review to such works where the finite element method (FEM) is applied to the determination of the stress concentration around cutouts. While finite element modelling of shell structures of any type with cutouts of any shape does not pose any special problems, pressure loading cases require special attention. This aspect is discussed in the next section with a specific example of a pressurized cylindrical shell with an elliptical cutout. The presentation concludes by identifying directions for further work.

## Pressure Loading of Shells with Holes

Consider the specific problem of a pressurized cylindrical shell with an elliptical cutout. During the analysis the hole must be considered to have a suitable closure to transmit the pressure load over the area of the cutout, to the edge of the opening in the shell. Such a closure could be simply a cover bent to the same curvature as the uncut shell. In the majority of the earlier analyses, the elastic behavior of the cover has been ignored and for the hole boundary conditions all the forces and moments along the hole boundary were assumed to be zero, with the exception of transverse shear force. For this shear force, various assumptions have been made regarding its distribution around the hole. The most widely used assumption has been that the shear force is uniform. While this assumption is in order for a circular cutout in a spherical shell, it cannot be justified for any other case. In view of this, the elliptical hole problem in a cylindrical pressure vessel was analysed by Murthy [11] using various shear force distributions suggested in the literature. It was found that these assumptions resulted in stress distributions very different from each other. Here the stress distribution is highly sensitive to the force transmission from the cover to the shell and in order to get realistic prediction of stresses, the elastic behavior of the cover must also be taken into account. The computed results presented in this work are shown to be in good agreement with the available analytical results and experimental data. Reported work on elliptical inclusions is an effort in this direction.

## Governing Equations

### Curved Shell Element

The constitutive equation of three-dimensional elasticity written with respect to the lamina coordinate system in shell element.

$$\sigma^l = D^l \varepsilon^l \tag{1}$$

the superscript 'l' is used to emphasize that the components are in the lamina dependent. The ordering of the components is given as follows :

$$\sigma^l = \left\{\sigma^l_l\right\} = \begin{Bmatrix} \sigma^l_{11} \\ \sigma^l_{22} \\ \sigma^l_{12} \\ \sigma^l_{23} \\ \sigma^l_{31} \\ \sigma^l_{33} \end{Bmatrix} \tag{1.1}$$

$$\varepsilon^l = \left\{\varepsilon^l_l\right\} = \begin{Bmatrix} \dfrac{\partial u^l_1}{\partial x^l_1} \\ \dfrac{\partial u^l_2}{\partial x^l_2} \\ \dfrac{\partial u^l_1}{\partial x^l_2} + \dfrac{\partial u^l_2}{\partial x^l_1} \\ \dfrac{\partial u^l_2}{\partial x^l_3} + \dfrac{\partial u^l_3}{\partial x^l_2} \\ \dfrac{\partial u^l_3}{\partial x^l_1} + \dfrac{\partial u^l_1}{\partial x^l_3} \\ \dfrac{\partial u^l_3}{\partial x^l_3} \end{Bmatrix} \tag{1.2}$$

The constitutive Eqn.(1) needs to be modified in order to enforce the zero normal stress condition in three directions of the lamina system. The result is

$$\tilde{\sigma}^l = \tilde{D}^l \tilde{\varepsilon}^l \tag{2}$$

Equation (2) is known as Reduced Constitutive Equation. where,

$$\tilde{\sigma}^l = \begin{Bmatrix} \sigma^l_1 \\ \sigma^l_2 \\ \sigma^l_3 \\ \sigma^l_4 \\ \sigma^l_5 \end{Bmatrix} \tag{2.1}$$

$$\tilde{\varepsilon}^l = \begin{Bmatrix} \varepsilon^l_1 \\ \varepsilon^l_2 \\ \varepsilon^l_3 \\ \varepsilon^l_4 \\ \varepsilon^l_5 \end{Bmatrix} \tag{2.2}$$

$$\tilde{D}^l = \left[\tilde{D}^l_{IJ}\right], \ 1 \le I, J \le 5$$

$$\tilde{D}^{l}_{IJ} = \tilde{D}^{l}_{IJ} - \frac{D^{l}_{I1} D^{l}_{6J}}{D^{l}_{66}} \tag{2.3}$$

The reason for placing 33-components in the last entries of σ and ε should now be apparent. For Homogeneous Linear Element Model :

$$\tilde{D}^{l} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{2.4}$$

where $E$ is the Young's modulus and ν is Poisson's ratio.

To attain results consisted with classical bending theory, shear correction factors need to be introduced. The formulation uses a value of 5/6 for the shear correction factors as deafult.

$$\tilde{D}^{l} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & \chi\frac{1-\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & \chi\frac{1-\nu}{2} \end{bmatrix} \tag{2.5}$$

where $\chi$ ( = 5/6 or $\pi^2/12$) is the shear correction factor.

## Elements

Finite element modelling of circular cylindrical shell employs two types of elements namely the regular shell element and rigid beam element. The shell elements may be quadrilateral or triangular shape. Rigid beam elements are used around the circumference of the cylindrical shell as a constraint to transmit an equivalent pressure load as a force.

Curved Isoparametric Shell Element of Quadrilateral Shape (QUAD 8) is shown in Fig. 1.

- The top, middle and bottom surfaces of the element are curved are illustrated in Fig.1(a), where sections across the thickness are generated by straight lines.
- The geometric description requires specification of two vectors at each of the eight mid surface nodes. One is the position vector $R_I$ of the node I, with three global Cartesian components $X_i$, $Y_i$, $Z_i$, where the subscript 'i' identifies the node number i, the other is the unit normal vector $V_3^i$ along with the shell wall thickness $t_i$ of the same nodes.
- The element carries six enginering degrees of freedom (three translations and three rotations) at each of the mid side surface nodes; the nodal degrees of freedom are illustrated in Fig.1(b).
- The shape functions, takaing value of unity at node 'i' and zero at all other nodes, are derived as interpolation functions of a parent element shown in Fig.1(c).
- The vector $V_3^i$ is computed using coordinates of the eight mid-surface nodes.

Curved Isoparametric Shell Element of Triangular Shape (TRIA 6) is shown in Fig. 2.

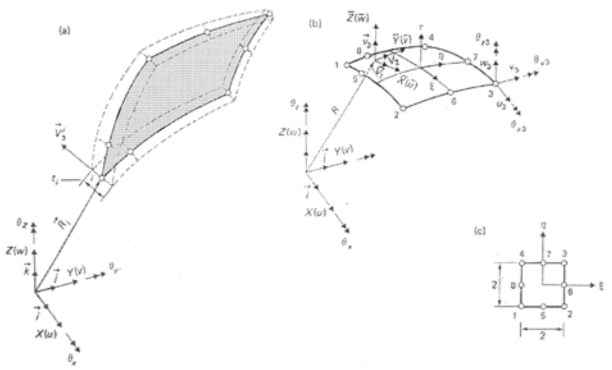

*Fig.1 Curved isoparametric shell element of quadrilateral shape*

- The element shown in Fig.2 has six nodes and six engineering degres of freedom at each node.

The matrices and vectors for this element are computed as follows.

- The edge 1-4-8 of the QUAD 8 element (Fig.1) is collapsed and nodes 4 and 8 are co-located with node 1.
- Nodes 1, 4 and 8 are tied together to have the same DOF using multipoint constraint (MPC) elements.
- The vector $V_3^i$ is computed in the solver using coordinates of the six mid-surface nodes.

Commercial FEA programs offer capabilities in their post-processor to display contours of the stress (with respect to an element local Cartesian coordinate system) at Top/Middle/Bottom surfaces of the shell. This is an important pre-requisite to capture stress levels across the thickness of each layer in composite laminated shell structures.

## Typical Meshes

The typical Finite Element Model (with computational domain) of the analysis is shown in Fig.3 and Fig.4. Total number of Elements (Shell) =1200; Total number of Elements (Inclusion) = 768. Total number of Nodes (Shell) = 3805; Total number of Nodes (Inclusion) = 2369. Total number of Rigid Elements = 100.

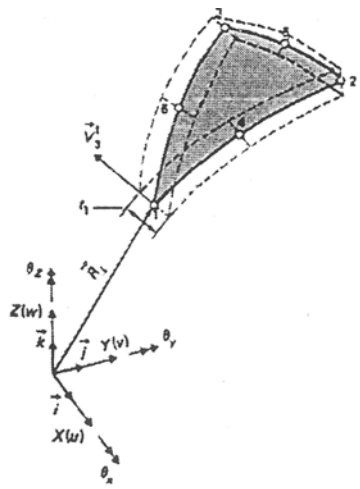


*Fig.2 Curved isoparametric shell element of triangular shape*

*Fig.3 Discretised cylindrical shell with tranversely elliptical cutout*

*Fig.4 Distretised cylindrical shell with elliptical inclusion*

## Isotropic Cylindrical Shell with Cutout Under Axial Tension

An isotropic cylindrical shell of radius R and thickness ‘t’ with transverse elliptical cutout subjected to uniform axial tension loading as shown in Fig. 5. As shown in Fig.5, the cylindrical shell contains a cutout. The shape of the cutout is defined such that if the shell is cut along a generator and flattened into a plane, the cutout becomes an ellipse with major and minor axes denoted by a and b, respectively. For simplicity and convenience, the cutout is refered to herein as an "elliptical" cutout. The major axis of the ellipse is taken to be perpendcular to the longitudinal axis of the shell.

The shell is considered to be isotropic, thin and of constant thickness. Analysis is carried out for a shell that is closed round the circumference and extends to infinity along its length on either side of the hole.

The physical parameters defining the problem are shown in Table-1.

### Target Solution

The target solution for an elliptic hole gives a plot of the stress concentration factor based on the maximum total stress against β for various values of k at intervals of 0.1 is shown in Fig. 6.

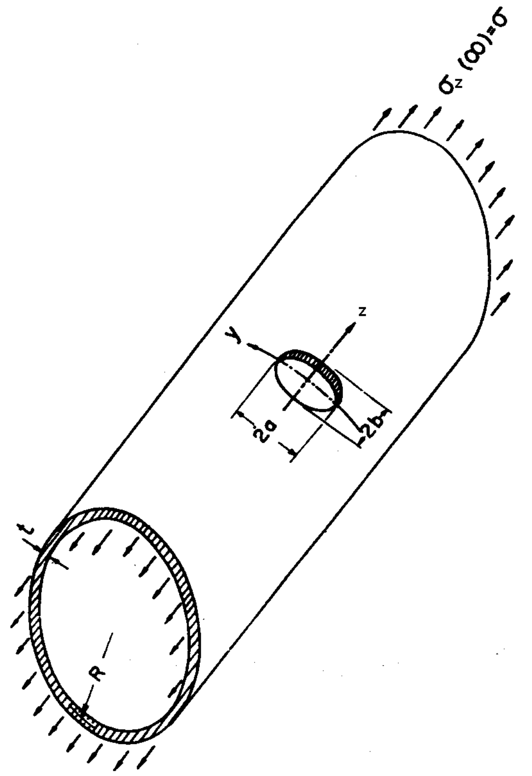


*Fig.5 Cylindrical shell under axial tension*

| Table-1 | | |
|---|---|---|
| Material properties | Geometric parameters | Loading |
| E = 68950 MPa<br>ν = 0.3 | R = 100 mm<br>t = 1 mm<br>L = 100 mm | σ = 1 MPa |

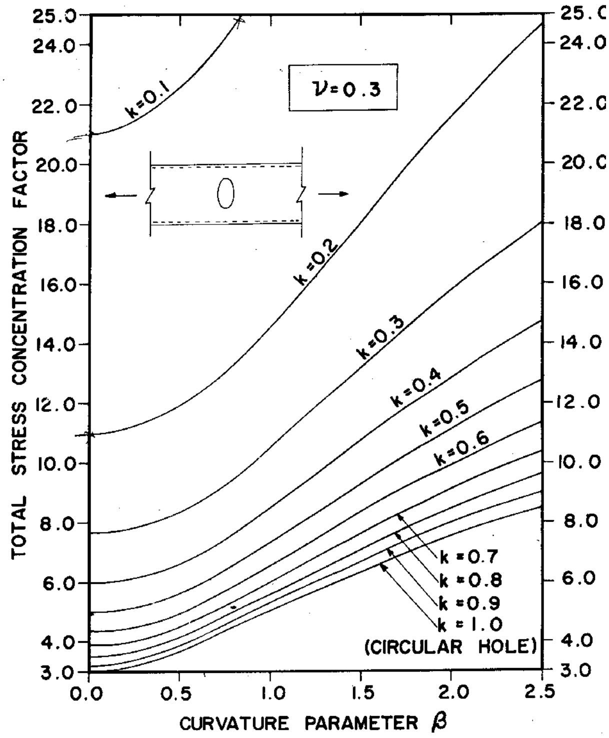


*Fig.6 Target solution [10]*

The first successful solution for the problem of an elliptical hole in a cylindrical shell without any restriction on the axis ratio k was given by Murthy et. al [10]. The loading considered was uniform axial tension and the minor axis of the hole was taken to be parallel to shell longitudinal axis of the shell. The analysis was later extended to the other symmetric orientation of the hole with respect to the shell axis.

The target solution [10] was obtained from the governing differential equation for the shell in the form of an infinite series in terms of Mathieu and Modified Mathieu Functions by the method of separation of variables, and is expressed in the form of a Fourier series.

### Discussion of Results

Results are obtained, for Poisson's ratio of 0.3 in terms of the two dimensionless parameters β and k. The range of parameter β considered : $0 < \beta \leq 2.5$; at the interval of 0.5. An inspection of the Fig.7 shows that the target solution can be generally considered to be valid for β up to 1.5 with an error of 1 percent could be admitted; the upper limit of β can be raised to 2.5. Overall, the comparisons indicate very good agreement (less than 10% differ-

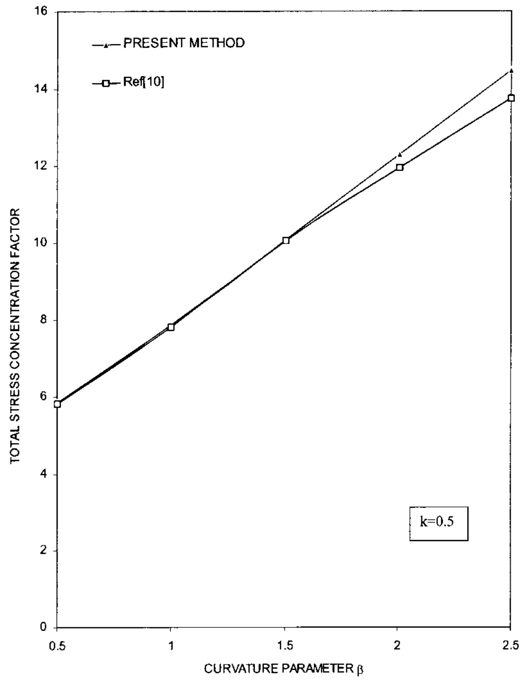


*Fig.7 Range of validity*

ence) between the corresponding results produced by the two analysis methods. Fig. 8 shows the observed behavior the shell for the case of $\beta = 2.0$. For shells with large $\beta$, differences of approximately 10% were obtained and found to be the result of insufficient mesh refinement in the finite element models.

### Isotropic Cylindrical Shell with Cutout Under Internal Pressure

Figure 9 shows the geometry of a thin-walled, homogeneous isotropic circular cylindrical shell of radius R and thickness 't' with transverse elliptical cutout located at the shell mid-length for $\beta = 2.0$. The origin of the global Cartesian coordinate system (X,Y, Z) is located at the end point of the longitudinal axis of the shell. As shown in Fig.9, the Z-axis coincides with the longitudinal axis of the shell. The X and Y coordinates span the cross-sectional plane. A curvilinear coordinate system is attached to the mid-surface of the cylindrical shell. The coordinates of points in the normal-to-the surface (transverse), circumferential (tangential) and longitudinal directions of the shell are denoted by (x, y, z). As shown in Fig.9 the cylindrical shell contains transverse elliptical cutout with major and minor axes denoted by a and b, respectively. In the analysis domain, the major and minor axes of the ellipse are aligned with a curvilinear coordinate system,

*Fig.8 Observed behaviour for the case of $\beta = 2.0$*
*a) Stress contours b) Displacement contours*

(y, z), whose origin is located at the center of the cutout. The elliptical coordinate, $\eta$ representing hyperbola, is utilized in order to obtain the stress resultant distribution in the direction tangent to the cutout boundary. The coordinate, $\eta$, varying from 0 to $2\pi$, is known as the ecentric angle and is related to the (y, z) coordinate system by $y = a \cos \eta$ and $z = b \sin \eta$. The shell is subjected to uniform internal pressure loading with an assumption of uniform shear force along the hole boundary.

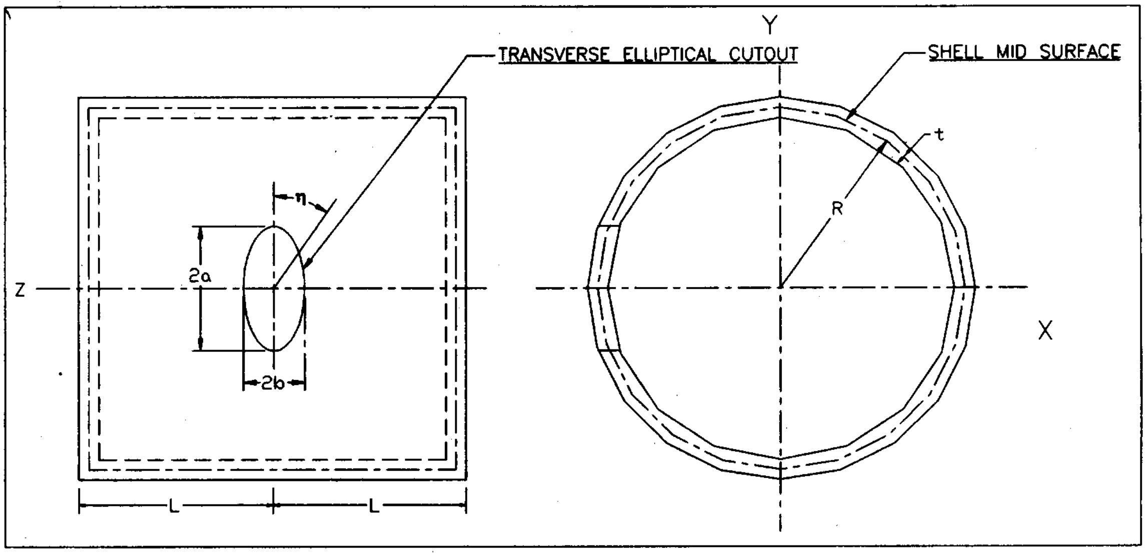


*Fig.9 Geometry, coordinate system for thin-walled cylindrical shell with an elliptical cutout*

### Results Comparison

Some typical results obtained based on the assumption of uniform shear force along the hole boundary and are shown in Fig.11. It is seen from Fig.10 and Fig.11, the results for assumption regarding shear force do not differ much. Our analysis results are in good agreement with those of Murthy [11]. Our result yields slight variation in stress concentration factors (5-8%), which suggest that we used a finer mesh density to more accurately capture the stress concentration factors. It was observed that the degree of disagreement among these results is along the hole boundary between $\eta = 0°$ to $20°$ on inner surface.

The elliptical hole problem in a pressurized cylindrical pressure vessel was analyzed by Murthy [11] using various shear force distributions suggested in the literature. It was found that these assumptions resulted in stress distributions very different from each other. Here, the stress distribution is highly sensitive to the force transmission from the cover to the shell and in order to get a realistic prediction of stresses, the elastic behavior of the cover must also be taken into account. The most widely used assumption has been that the shear force is uniform. Typical stress and displacement results are shown in Fig.12.

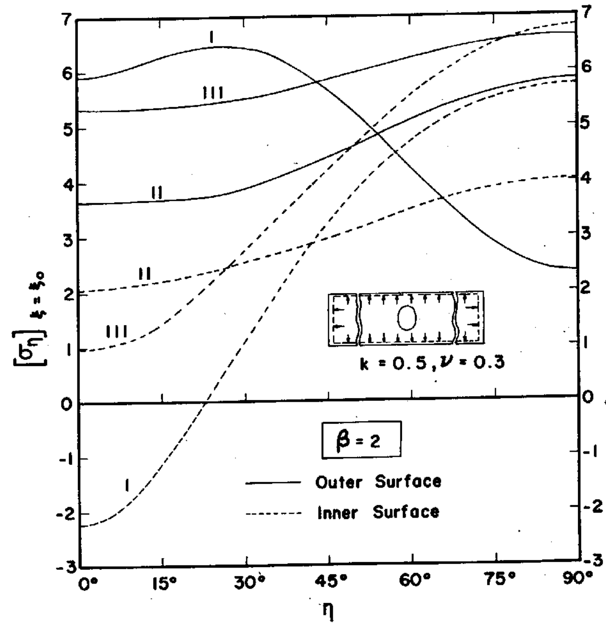


*Fig.10 Target solution [11]*

*Influence of various assumptions regarding Kirchhoff shear distribution along the hole boundary for a pressurized cylinder with an elliptical cutout. (I) Uniform Kirchhoff shear; (II) Mansfeld's idealization of transverse shear; (III) Kirchhoff shear distribution as in a clamped flat elliptic plate under uniform normal distributed load*

### Discussion of Results

Figure 11 shows a comparison on tangential stress along hole boundary for a pressurized cylinder with an elliptical cutout. The stress problem of a transverse elliptic hole in a pressurized cylindrical shell was solved by the present method. The maximum stress of 632MPa occurs approximately at $\eta = 25°$ on outer surface which is valid as shown in Fig.11. When this stress is compared with that of plain cylindrical shell of the same R/t ratio, it yields a

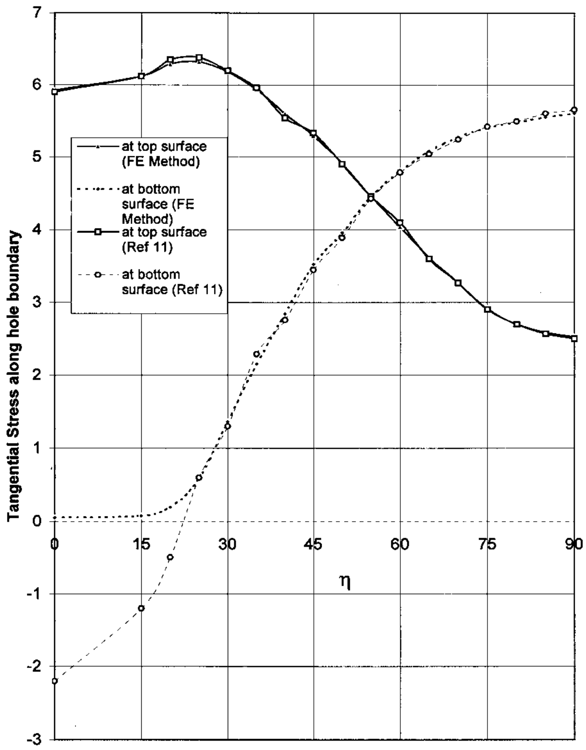


*Fig.11 Range of validity for β = 2.0*

stress concentration factor, K equal to 6.32. The smaller peak on outer surface, which occurs at $\eta$ = 90° is also positive and a value of 272 MPa. Similarly the total stress on inner surface reaches its extreme value at $\eta$ = 90° and has the magnitude of 560 MPa which is less in magnitude of stress on outer surface whereas the smaller peak which occurs at $\eta$ = 0° which is negligibly small. The stress and displacement contours along the hole boundary are shown in Fig.12. Wall bending takes place in such a way that, in highly stressed regions on the hole boundary.

## Pressurized Cylindrical Shell with Edge Bonded Elastic Cover

In this section, we consider the analytically difficult problem of a pressurized cylindrical shell with an elliptical opening, which is closed by perfectly edge bonded elastic inclusion as shown in Fig.13. This could be considered to represent, typically, a reasonably idealization of the conditions at transparent windows in pressurized or vacuum cabins introduced either for visibility as in aircraft fuselages or for radiation of heat as in environmental chambers.

Numerical solutions are obtained for the extreme cases of a rigid inclusion and an inclusion with its elastic modulus negligible in comparison with that of the shell. Results are obtained for a 2:1 basic stress field occurring in pressurized cylindrical shells with both ends closed. The major axis of the ellipse is taken to be perpendicular to the shell axis as this should generally be the preferred orientation where the hoop stress is the major principal stress.

In both the cases, free hole in a isotropic circular cylindrical shell is covered by edge bonded inclusion. The inclusion has the same curvature as the uncut shell. Along the hole boundary the inclusion is perfectly edge bonded. The middle surfaces of the shell and the inclusion are assumed to be parts of the same cylindrical surface. In other words, the inclusion is bonded symmetrically with reference to the shell middle surface. Here the analysis is

***a)*** ***b)***

*Fig.12 Observed behavior for β = 2.0*
*a) Tangential stress distribution along hole boundary b) Deformed cylindrical shell*

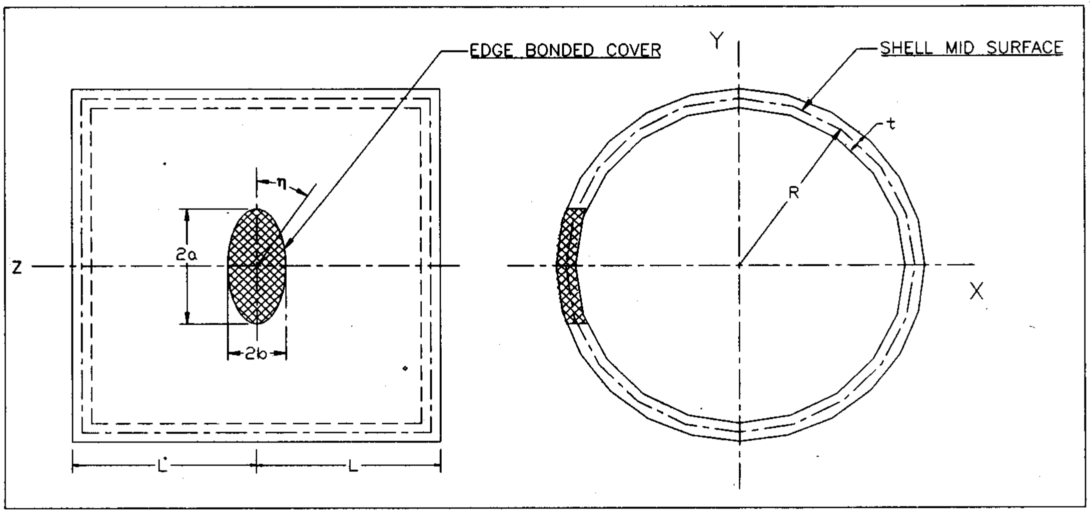


*Fig.13 Circular cylindrical shell with transverse elliptic inclusion*

carried out for the isotropic shell with low modulus inclusion and high modulus inclusion separately.

Typical geometric parameters and material properties are: 1) Low modulus elliptic inclusion (Table-2) and 2) High modulus elliptic inclusion (Table-3).

Numerical results are presented here to examine the accuracy and usefulness of the proposed finite element model when applied to solve inclusion problems in isotropic cylindrical shells. The configuration and loading condition is considered in the analysis is shown in Fig. 13. For a homogeneous isotropic cylindrical shell, the results

**Table-2 : Input data**

| Material properties | Geometric parameters | Loading |
|---|---|---|
| E = 68950 MPa<br>$E_c$ = 28670 MPa<br>ν = 0.3<br>$\nu_c$ = 0.36 | R = 100 mm<br>L = 100 mm<br>a = 31.15 mm | p = 1 MPa |

**Table-3 : Input data**

| Material properties | Geometric parameters | Loading |
|---|---|---|
| E = 68950 MPa<br>$E_c$ = 208670 MPa<br>ν = 0.3<br>$\nu_c$ = 0.3 | R = 100 mm<br>L = 100 mm<br>a = 31.15 mm | p = 1 MPa |

presented for two different inclusions. Typical results are presented to assess the effect of changes in inclusion material parameters on stress concentration factor for a pressurized cylindrical shell.

### Discussion of Results

From the finite element analysis results obtained, the influence of elastic inclusion on the stress field created by it in the shell is studied. All the computations are carried out for the axis ratio k = 0.5 and for the given hole size. Parametric studies are carried out with varying thickness of the shell for different thickness ratio (R/t). Poisson's ratio of the shell is taken as 0.3 throughout. Fig. 14 and Fig.15 shows the plot of stress concentration with the high modulus inclusion on the top surface and bottom surface

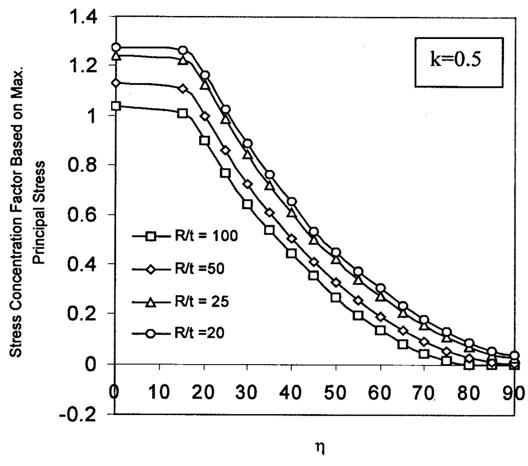


*Fig.14 SCF along hole boundary in a shell with high modulus elliptic inclusion (Top surface)*

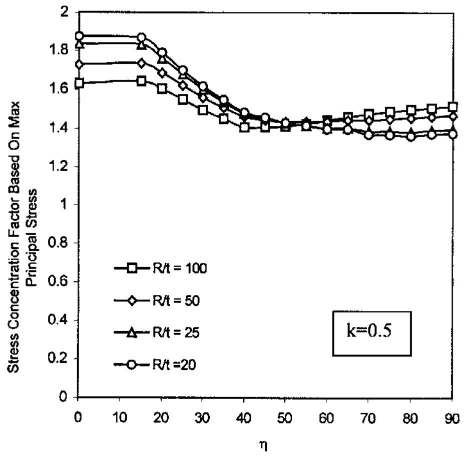


*Fig.15 SCF along hole boundary in a shell with high modulus elliptic inclusion (bottom surface)*

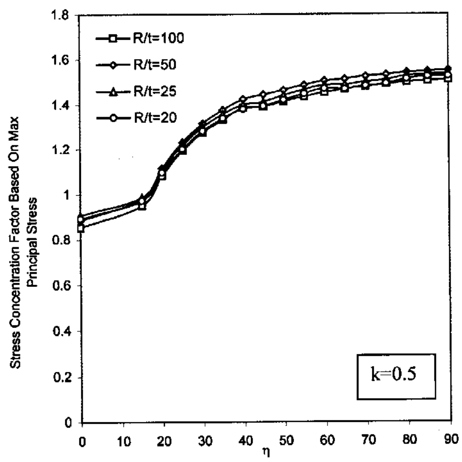


*Fig.16 SCF along hole boundary in a shell with low modulus elliptic inclusion (top surface)*

of the shell at the junction between shell and inclusion in terms of thickness ratio (R/t) and the axis ratio (k). It was observed that the stress concentration is more on the inner surface of the shell and at the ends of major axis. Similarly Fig. 16 and Fig.17 shows the plot of stress concentration with the low modulus inclusion on the top surface and bottom surface of the shell at the junction between shell and inclusion in terms of curvature parameter and the axis ratio. The stress concentration factor is based on maximum principal stress in the shell. It was observed that the maximum stress is always found to occur on the inner surface of the shell and at the ends of the minor axis. An interesting feature of the results shown in Fig. 15 and Fig.17 is that as the thickness of the shell increases the stress concentration factor on the inner surface of the shell

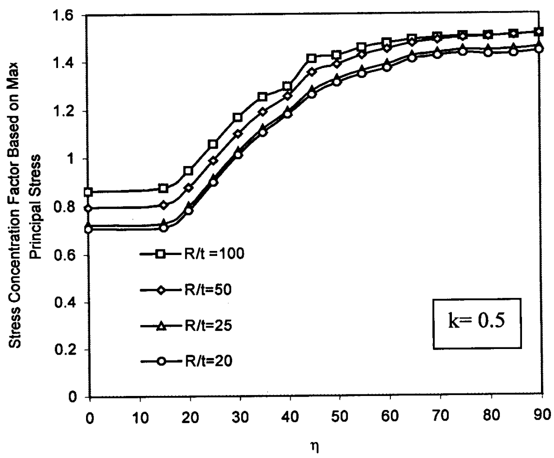


*Fig.17 SCF along hole boundary in a shell with low modulus elliptic inclusion (bottom surface)*

decreases slightly up to certain limit but later increases rapidly as the thickness is further increased.

## Conclusions

A limited series of validation studies were conducted in the present study to determine the accuracy of results obtained by using analysis method presented herein. The FE method using general purpose FE code has been presented that can be used to investigate the behavior of thin, circular cylindrical shells made of laminated-composite materials and with a cutout/inclusion, efficiently and parametrically. In particular, the effects of radius of curvature; non-circular cutout, inclusions and wall-thickness variations; and loading conditions on the stress-resultant concentration near the cutout have been presented for a isotropic shell subjected to uniform tension, and uniform pressure. In addition, studies that were conducted to validate the analysis method have been described. Overall, the results demonstrate that the analysis approach is a powerful means for developing design criteria for laminated-composite shells.

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