TECHNICAL NOTE

ON THE DYNAMIC SNAP-THROUGH BUCKLING OF FUNCTIONALLY GRADED SPHERICAL CAPS

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Abstract

Here, the dynamic snap-through buckling characteristics of clamped functionally graded spherical caps suddenly exposed to a thermal field are studied using finite element procedure. The material properties are graded in the thickness direction. The temperature load corresponding to a sudden jump in the maximum average displacement in the time history of the shell structure is taken as the dynamic buckling temperature. Numerical study is carried out to highlight the influences of shell geometries and material gradient index on the critical buckling temperature.

Keywords *: Functionally graded, Dynamic buckling, Spherical shell, Thermal load, Power law index, Nonlinear*

Introduction

Functionally graded materials [1] are high performance heat resistant materials able to withstand ultra high temperatures and extremely large thermal gradients used in fusion reactors and aerospace industries. These materials are microscopically inhomogeneous, in which the material properties vary smoothly and continuously from one surface of the material to the other surface. Typically, these materials are made from a mixture of ceramic and metal, or a combination of different materials. Although these materials are initially designed as thermal barrier materials, they are now employed for general use as structural elements for different applications.

Among various structural elements, shell elements, in particular, spherical shells form an important class of structural components, with many significant engineering applications. These shells when subjected to dynamic load could encounter deflections of the order of the shell thickness. The dynamic response of such shells may lead to the phenomenon of dynamic snapping or dynamic buckling. The investigation of such phenomenon considering externally applied pressure load has received considerable attention in the literature [2-4]. Budiansky and Roth [2] have employed the Galerkin method whereas Simitses [3] adopted Ritz-Galerkin procedure. Haung [4], has solved using finite difference scheme. The limited studies are also available on dynamic buckling of orthotropic shallow spherical shells [5-6]. However, studies pertaining to FGM shell structures are mainly limited to thermal stress, deformation, and static analysis in the literature [7-9]. Work on vibration/dynamic stability of FGM shell structures are also reported in the work of Ng et al. [10]. To the author's knowledge, work on the dynamic buckling behavior of isotropic/orthotropic/functionally graded material spherical shells suddenly exposed to thermal environment is rather meager in the literature and such studies are important to the structural designers.

In the present work, the nonlinear dynamic thermal buckling of functionally graded spherical caps suddenly exposed to a heat flux that results in a uniform temperature change over the surface is investigated using a three-noded shear flexible axisymmetric curved shell element based on field-consistency principle [6]. Geometric nonlinearity is assumed using von Karman's strain-displacement relations. The material properties are graded in the thickness direction according to the power-law distribution in terms of volume fractions of the constituents of the material. The nonlinear governing equations derived are solved employing Newmark's numerical integration method in conjunction with the modified Newton-Raphson iteration scheme. The critical dynamic buckling temperature difference is taken as the temperature difference between the shell surfaces corresponding to a sudden jump in the maximum average displacement in the time history.

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Numerical results are presented considering different values for geometrical parameter and material power law index on the dynamic snap through thermal bucking behavior of functionally graded spherical caps.

Formulation

An axisymmetric functionally graded shell of revolution (radius a, thickness h) made of a mixture of ceramics and metals is considered with the coordinates s, θ and z along the meridional, circumferential and radial/thickness directions, respectively. The materials in outer ($z = h/2$) and inner ($z = -h/2$) surfaces of the spherical shell are ceramic and metal, respectively. The locally effective material properties are evaluated using homogenization method that is based on the Mori-Tanaka scheme [11]. The effective bulk modulus K, shear modulus G, coefficients of thermal conductivity κ and thermal expansion α of the functionally gradient material evaluated using the Mori-Tanaka estimates [11-12] are as

$$\frac{K-K_m}{K_c-K_m}=V_c\Big/\left[1+(1-V_c)\frac{3\,(K_c-K_m)}{3\,K_m+4\,G_m}\right] \tag{1}$$

$$\frac{G-G_m}{G_c-G_m}=V_c\Big/\left[1+(1-V_c)\frac{(G_c-G_m)}{G_m+f_1}\right] \tag{2}$$

where,

$$f_1=\frac{G_m\,(9\,K_m+8\,G_m)}{6\,(K_m+2\,G_m)}$$

Here, V is volume fraction of phase material. The subscripts m and c refer the ceramic and metal phases, respectively. The volume-fractions of ceramic and metal phases are related by $V_c + V_m = 1$, and V_c is expressed as

$$V_c(z)=\left(\frac{2\,z+h}{2h}\right)^k \tag{3}$$

where k is the volume fraction exponent ($k \geq 0$).

The effective values of Young's modulus E and Poisson's ratio υ can be found as from

$$E(z)=\frac{9KG}{3\,K+G}\ \text{and}\ \upsilon(z)=\frac{3\,K-2\,K}{2\,(3\,K+G)} \tag{4}$$

The locally effective heat conductivity coefficient κ is given as

$$\frac{K-K_m}{K_c-K_m}=V_c\Big/\left[1+(1-V_c)\frac{(K_c-K_m)}{3\,K_m}\right] \tag{5}$$

The coefficient of thermal expansion α is determined in terms of the correspondence as

$$\frac{\alpha-\alpha_m}{\alpha_c-\alpha_m}-\left(\frac{1}{K}-\frac{1}{K_m}\right)\Big/\left(\frac{1}{K_c}-\frac{1}{K_m}\right) \tag{6}$$

The effective mass density ρ can be given by rule of mixture as

$$\rho(z)=\rho_c V_c+\rho_m V_m \tag{7}$$

The temperature variation is assumed to occur in the thickness direction only and the temperature field is considered constant in the xy plane. In such a case, the temperature distribution along the thickness can be obtained by solving a steady-state heat transfer equation

$$-\frac{d}{dz}\left[\kappa(z)\frac{dT}{dz}\right]=0\ ,\quad T=T_c\ at\ z=h/2\ ;$$

$$T=T_m\ at\ z=-h/2 \tag{8}$$

By using the Mindlin formulation, the displacements at a point (s, θ, z) are expressed as functions of the mid-plane displacements u_o, v_o and w, and independent rotations β_s, and β_θ of the radial and hoop sections, respectively, as

$$u(s,\theta,z,t)=u_o(s,\theta,t)+z\,\beta_s(s,\theta,t)$$

$$v(s,\theta,z,t)=v_o(s,\theta,t)+z\,\beta_\theta(s,\theta,t)$$

$$w(s,\theta,z,t)=w(s,\theta,t) \tag{9}$$

where t is the time.

Using von Karman's assumption for moderately large deformation, Green's strains can be written in terms of middle-surface deformations as,

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_p^L \\ 0 \end{Bmatrix} + \begin{Bmatrix} z\varepsilon_b \\ \varepsilon_s \end{Bmatrix} + \begin{Bmatrix} \varepsilon_p^{NL} \\ 0 \end{Bmatrix} \tag{10}$$

where, the membrane strains $\{\varepsilon_p^L\}$, bending strains $\{\varepsilon_b\}$, shear strains $\{\varepsilon_s\}$ and the nonlinear in-plane strains $\{\varepsilon_p^{NL}\}$ in Eqn. (10) are written as [13]

$$\{\varepsilon_p^L\} = \begin{Bmatrix} \dfrac{\partial u_o}{\partial s} + \dfrac{w_o}{R} \\ \dfrac{u_o \sin\phi}{r} + \dfrac{w\cos\phi}{r} \\ -\dfrac{v_o \sin\phi}{r} + \dfrac{\partial v_o}{\partial s} \end{Bmatrix};$$

$$\{\varepsilon_b\} = \begin{Bmatrix} \dfrac{\partial \beta_s}{\partial s} + \dfrac{\partial u_o}{R\,\partial s} \\ \dfrac{\beta_s \sin\phi}{r} + \dfrac{u_o \sin\phi}{R\,r} \\ \dfrac{\partial v_o}{\partial s}\dfrac{\cos\phi}{r} + \dfrac{\partial \beta_\theta}{\partial s} - \dfrac{\beta_\theta \sin\phi}{r} \end{Bmatrix};$$

$$\{\varepsilon_s\} = \begin{Bmatrix} \beta_s + \dfrac{\partial w_o}{\partial s} \\ \beta_\theta \pm \dfrac{v_o \cos\phi}{r} \end{Bmatrix};$$

$$\{\varepsilon_p^{NL}\} = \begin{Bmatrix} \dfrac{1}{2}\left(\dfrac{\partial w}{\partial s}\right)^2 \\ 0 \\ 0 \end{Bmatrix} \tag{11}$$

where r, R and ϕ are the radius of the parallel circle, radius of the meridional circle and angle made by the tangent at any point in the middle-surface of the shell with the axis of revolution.

The potential energy functional U can be written in terms of the field variables u_o, v_o, w_o, β_s, β_θ and their derivatives. The kinetic energy includes the effects of in-plane and rotary inertia terms. The governing equations obtained using Lagrange's equation of motion are solved based on finite element procedure. Using Eqs. (4-11) and following the procedure outlined in the work of Rajasekaran and Murray [14], the finite element equations thus derived are

$$[M]\{\ddot{\delta}\} + \left[[K] + \frac{1}{2}[N_1(\delta)] + \frac{1}{3}[N_2(\delta)]\right]\{\delta\} = \{F\} \tag{12}$$

where $[K]$ and $[M]$ are the linear stiffness and mass matrices, respectively. $[N_1]$ and $[N_2]$ are non-linear stiffness matrices linearly and quadratically dependent on the field variables, respectively and $\{F\}$ is the load vector consisting of mechanical load $\{F_M\}$ and thermal load $\{F_T\}$. $\{\ddot{\delta}\}$, $\{\delta\}$ are the acceleration and displacement vectors, respectively. The three-noded finite element employed here has five degrees of freedom (u_o, v_o, w_o, β_s, β_θ) per node. The resulting nonlinear Eq. (12) is solved for dynamic response histories employing Newmark's numerical integration method coupled with Newton-Raphson iteration. The dynamic buckling loads are evaluated based on the displacement response histories.

The criterion suggested by Budiansky and Roth [2] is employed here as it is widely accepted. This criterion is based on the plots of the peak non-dimensional average displacement in the time history of the structure with respect to the amplitude of the thermal load (e. g. inserted figure in Fig. 2). There is a temperature difference range where a sharp jump in peak average displacement occurs for a small change in load magnitude. The inflection point of the load-deflection curve is considered as the dynamic buckling temperature difference.

Results and Discussion

Based on progressive mesh refinement, 15-element idealization is found to be adequate in modeling the spherical caps. The static thermal buckling load obtained from the present formulation for isotropic spherical shells have been compared with the analytical solutions [15] and excellent agreement was seen. For the sake of brevity, such comparisons are not presented here. Next, the nonlinear formulation developed herein is also examined for clamped isotropic spherical caps subjected to uniform external pressure of infinite duration and the results are shown in Fig. 1 along with those of results from Haung [4] for different values of the geometrical parameter $\lambda = 2\left[\ 3\ (1 - \nu^2)\right]^{1/4}$; H, a are the central shell rise and base radius, respectively. H, a are the central shell rise and base radius, respectively). It is inferred that the present results are in very good agreement with the analytical solutions. The FGM spherical shell considered here consists of aluminum and alumina. The material properties for alumina and aluminum are taken from Lanhe [16] :

For alumina : $E_c = 380\ GPa$, $\kappa_c = 10.4\ W/mK$,
$\alpha_c = 7.4 \times 10^{-6}\ (1/°C)$

For aluminum : $E_m = 70\ GPa$, $\kappa_m = 204\ W/mK$,
$\alpha_m = 23 \times 10^{-6}\ (1/°C)$

Next the results for nonlinear dynamic thermal buckling analysis of functionally graded spherical caps is presented in terms of critical bucking temperature difference ΔT_{cr} ($=T_c - T_m$). For the chosen shell parameter and power law index of FGM, the thermal buckling study is conducted for suddenly applied temperature load. The length of response calculation time

$$\tau\left(= \sqrt{\frac{E_{ef} h^2}{12\ (1 - \nu^2)\ \rho_{ef} a^4}}\ t \right)$$

in the present study is varied between 1 and 2 with the criterion that in the neighborhood of the buckling, τ is large enough to allow deflection-time curves to develop fully. E_{ef} $\left(= (1/2) \int_{-h/2}^{h/2} E\,(z)\,d\,z \right)$ corresponds to effective modulus of corresponding gradient index. The detailed investigation for dynamic buckling of clamped functionally graded spherical caps is carried out for different geometrical parameters and material power law index. A typical nonlinear axisymmetric dynamic response history with time for a clamped isotropic spherical shell parameter ($\lambda = 6$, a/h = 400 and k = 0), considering temperature loads is shown in Fig 2. Further, using such plots, the variation of maximum average displacement with applied temperature obtained is also highlighted as an insert in Fig. 2 for predicting the critical temperature difference. It is seen that there is a sudden jump in the value of the average

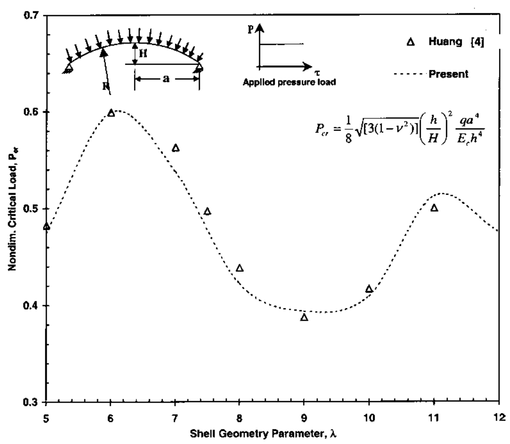

Fig.1 Comparison of axisymmetric nondimensional critical dynamic pressure for clamaped isotropic (k = 0) spherical cap under mechanical loading

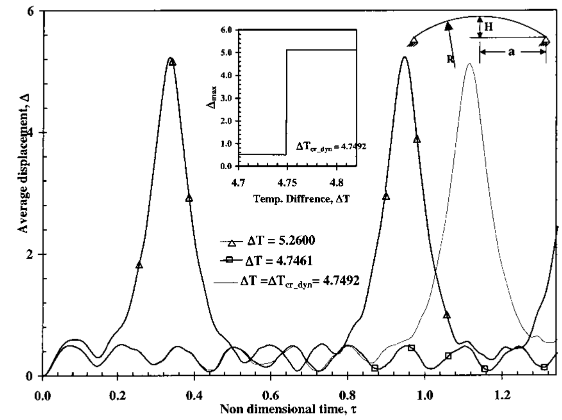

Fig.2 Average displacement versus nondimensional time for clamped isotropic spherical cap (λ = 6, k = 0) under thermal loading

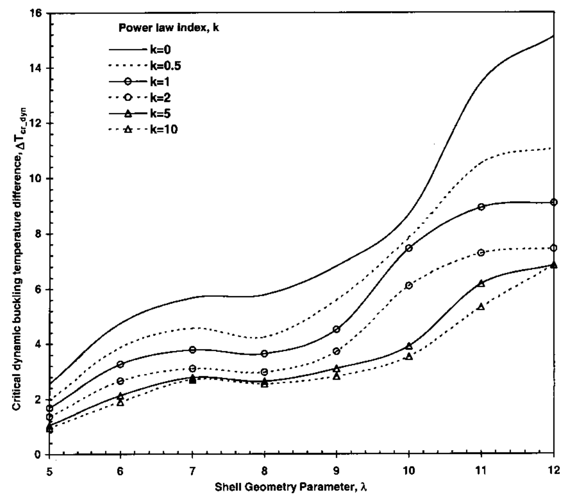

Fig.3 Variation of critical dynamic buckling temperature against shell geometry parameter (λ), of a clamped FGM spherical cap

displacement when the temperature difference reaches the value $\Delta T_{cr} = 4.7492$ for the shell considered here. Fig. 3 depicts the behavior of critical dynamic temperature difference of FGM shell pertaining to clamped case. It is revealed from this Figure that, with the increase in power law index *k*, the critical dynamic buckling temperature difference ΔT decreases, irrespective of shell geometrical parameter. This is attributed due to the stiffness reduction because of the increase in the metallic volumetric fraction and the introduction of different stiffness couplings due to elastic properties variation through the thickness of FGM shell. It can be also noted that the rate of increase in the critical dynamic buckling temperature difference highly depends on the values of shell geometric parameter and material gradient index. Furthermore, It is observed while varying the shell parameter value that the average displacement increases gradually with increase in temperature load for low values of geometrical parameter ($\lambda < 5$) indicating the absence of sudden jump in amplitude with temperature. For the sake of brevity, these studies are not reported here. Such shell may fail due to material failure. It is hoped that the present study is useful for the researchers dealing FGM shell structures.

References

1. Koizumi, M., "FGM Activities in Japan", Composites Part B: Engineering, 28, 1997, pp. 1-4.

2. Budiansky, B. and Roth R. S., "Axisymmetric Dynamic Buckling of Clamped Shallow Spherical Shells", NASA TND-510, 1962, pp. 597-609.

3. Simitses, G. J., "Axisymmetric Dynamic Snap-Through Buckling of Shallow Spherical Caps", AIAA Journal, 5, 1967, pp.1019-1021.

4. Haung, N. C., "Axisymmetric Dynamic Snap-Through of Elastic Clamped Shallow Shell", AIAA Journal, 7, 1969, pp. 215-220.

5. Chao, C. C. and Lin, I. S., "Static and Dynamic Snap-Through of OrthotropicSpherical Caps", Composite Structures, 14, 1990, pp. 281-301.

6. Ganapathi, M., Gupta, S. S. and Patel, B. P., "Nonlinear Asymmetric Dynamic Buckling of Isotropic/Laminated Orthotropic Spherical Caps." AIAA Journal, 41, 2003, pp. 1363-1369.

7. Makino, A., Araki, N., Kitajima, H. and Ohashi, K.,"Transient Temperature Response of Functionally Gradient Material Subjected to Partial, Stepwise Heating", Transactions JSME, Part B, 60, 1994, pp. 4200-4206.

8. Obata, Y. and Noda, N.,"Steady Thermal Stresses in a Hollow Circular Cylinder and Hollow Sphere of a Functionally Gradient Material", Journal of Thermal Stresses, 17, 1994, pp. 471-487.

9. Takezono, S., Tao, K., Inamura, E. and Inoue, M., "Thermal Stress and Deformation in Functionally Graded Material Shells of Revolution Under Thermal Loading Due to Fluid", JSME International Series A: Mechanics and Material Engineering, 39, 1994, pp. 573-581.

10. Ng, T. Y., Lam, K. Y., Liew, K. M. and Reddy, J.N.,"Dynamic Stability Analysis of functionally Graded Cylindrical Shells Under Periodic Axial Loading", Int. J. of Solids and Structures, 38, 2001, pp. 1295-1309.

11. Mori. T. and Tanaka. K., "Average Stress in Matrix and Average Elasticenergy of Materials with Misfitting Inclusions", Acta Metall., 21, 1973, pp. 571-574.

12. Benveniste, Y., "A New Approach to the Application of Mori-Tanaka's Theory in Composite Materials", Mechanics of Materials, 6, 1987, pp. 147-157.

13. Kraus H., "Thin Elastic Shells", John Wiley, New York, 1967.

14. Rajasekaran, S. and Murray, D. W., "Incremental Finite Element Matrices", ASCE Journal of Structures Division, 99, 197, pp. 2423-2438.

15. Ganesan N. and Ravikiran Kadoli, "A Theoretical Analysis of Linear Thermoelastic Buckling of Composite Hemispherical Shells with a Cut-out at the Apex", Composite Structures, 68, 2005, pp. 87-101.

16. Lanhe, Wu., "Thermal Buckling of a Simply Supported Moderately Thick Rectangular FGM Plate", Composite Structures, 64, 2004, pp. 211-218.