# ESTIMATION OF PARAMETERS WITH PARTITIONED DATA FROM LARGE AMPLITUDE MANEUVERS OF AN AIRCRAFT

V. Parameswaran*, Girija, G* and J.R. Raol*

**Abstract**

*In this paper data partitioning method is applied to concatenated large amplitude manoeuvres (LAM) of a fighter aircraft to estimate its stability and control derivatives. Stepwise multiple linear regression (SMLR) is utilized to determine the model structure within each partitioned data set. It is shown that the procedure with data generated from a research simulator, yields a useful method for estimating the pitch up tendency exhibited by the aircraft during certain angle of attack (AOA) range. The SMLR is also applied to estimate non-linear aerodynamic derivatives of the aircraft from LAM data. The method is applied to real LAM data of the aircraft and gives reasonably consistent estimates of the aerodynamic derivatives. Results using model error method for the longitudinal derivatives from LAMs are also presented.*

## Introduction

The estimation of aerodynamic stability and control derivatives from flight data of an aircraft forms a very important part of flight data analysis. In addition to providing validation of the theoretical and wind tunnel predictions of aerodynamic derivatives, the estimated derivatives from flight data are required for improving the stability augmentation and flight control system design and for development of high fidelity simulators. Parameter estimation techniques have evolved to a stage where, by conducting planned flight tests and using the measured input and output responses, it is possible to estimate the aerodynamic derivatives from a few dynamic manoeuvres on an aircraft.

In general, parameter estimation methods are applied to small manoeuvres about trim flight conditions. Linear aerodynamic models are assumed for analysis of these small perturbation manoeuvres. However, when the trim condition is in a region of rapidly changing aerodynamic characteristics, non-linear aerodynamic models may be required for the analysis of the small perturbation manoeuvres. When analysing LAMs that involve large variations in angle of attack, angle of sideslip or control positions, bilinear/non-linear aerodynamic models would be mostly required [1]. Large manoeuvres could occur during certain regimes perhaps due to loss of stability, damping or control effectiveness. In order to estimate the aerodynamic derivatives under such conditions, it would be necessary to use an appropriate methodology for the analysis of LAMs.

Output error / maximum likelihood method is the most popular method used for estimating stability and control derivatives from flight data [2,3]. The method requires accurate apriori knowledge of the model structure. In cases where the model structure is not known, which might often be the case, it would be required to try several alternative models. A two step method known as Estimation Before Modelling [4] has been quite extensively used in flight data analysis. Here, the state estimation is done using the EKF and model structure is determined using a Stepwise Multiple Linear Regression (SMLR) method [5]. Since the time does not appear explicitly, the measured data points can be arranged in arbitrary order leading to a procedure known as data partitioning [1].

Scientists, Flight Mechanics and Control Division (FMCD), National Aerospace Laboratories, Post Box No.1779 Kodihalli, Bangalore-560 017, India; e-mail : jrraol@yahoo.com
Manuscript received on 29 Apr 2004; Paper reviewed, revised and accepted on Aug 2004

**Table-1: The Estimation Procedures for LAM Analysis**

| | LAM Analysis ( Data Partitioning ) | LAM Manoeuvre ( Without Partitioning ) | LAM Manoeuvre ( Without Partitioning ) |
|---|---|---|---|
| Data handling | * Concatenated data from several LAMs<br>* Divided into subspaces | * Single LAM data<br>* Entire data handled | * Single LAM data<br>* Entire data handled |
| Analysis method | SMLR within each subspace | SMLR for the entire data | Model Error Method for the entire data |
| Application | * Simulated data of longitudinal LAM (Fig. 2-5,7,8)<br>* lateral – Directional LAM (Fig. 6,9,10,11 ) | * Simulated longitudinal data (Table-2, Row 1&2 )<br>* Real data (Fig. 12,13; Table-3, Row 3) | * Simulated longitudinal data (Table-3)<br>* Real data (Table-3 and 4) |
| Benefits | * If trim at certain AOA not possible then very useful<br>* Model Selection possible<br>* Pitch up tendency can be captured | * If trim at certain AOA not possible then very useful<br>* Model Selection possible<br>* Saves Flight Time | * Estimation of deficiency in the postulated model with single data set<br>* Model selection possible<br>* Saves Flight Time |

The conventional method of parameter identification from flight test consists of first trimming the test airplane to some given equilibrium condition and perturbing the aircraft slightly from trim position by giving a control input to one or more control surfaces. It may not be possible to trim a given airplane at certain AOAs. For such situation, using LAMs and data partitioning, it is possible to generate aerodynamic derivatives over the AOA range covered by the LAM. The method for analysing these manoeuvres consists of partitioning i.e. dividing the LAM that covers a large AOA range to several bins or subsets, each of which spans a smaller range of the AOA and SMLR to determine the structure of the aerodynamic model within each bin. Usual practice is to include linear aerodynamic terms of the Taylor's series expansion of the aerodynamic forces and moments first into the model. Non-linear aerodynamic models are represented by extending the linear terms to include higher-order terms. The LAM data are first corrected for scale factor and bias errors using kinematic consistency check on the data [2] before using the data for computing the aerodynamic forces and moments.

This paper presents results of analysis of the LAM data of an inherently unstable/augmented aircraft using data partitioning and SMLR to estimate aerodynamic derivatives. The LAM data is generated using a research simulator of the fighter aircraft. Since, the fighter aircraft is relatively highly unstable, a feedback controller is used to stabilize the aircraft. It is mandatory to have sufficient number of data points in each partitioned data set for estimation of the aerodynamic derivatives using SMLR. Hence several LAMs with varying input amplitudes are concatenated before partitioning w.r.t. AOA for analysis. In addition, results of application of SMLR method to estimate the non-linear aerodynamic derivatives of the aircraft directly (without partitioning) from analysis of single LAM manoeuvre data are also presented. The method is applied to real LAM data of the fighter aircraft. Since the real flight data is generated with the feedback controller, the AOA excursion in the LAM is limited. Results are presented in terms of estimated derivatives w.r.t. angle of attack and compared with the reference values. The models are checked by cross-validation wherein the estimated aerodynamic model is used to predict the responses for a flight data set that is not used in the identification analysis, see Fig.1. Table-1 brings out the features of the estimation procedure followed in this paper for LAM analysis.

## LAM Data Generation and Data Partitioning

### Longitudinal LAM Data

An aircraft is trimmed at a chosen flight condition. LAM data in the longitudinal axis are generated by giving doublets and multi-step 3211 inputs with

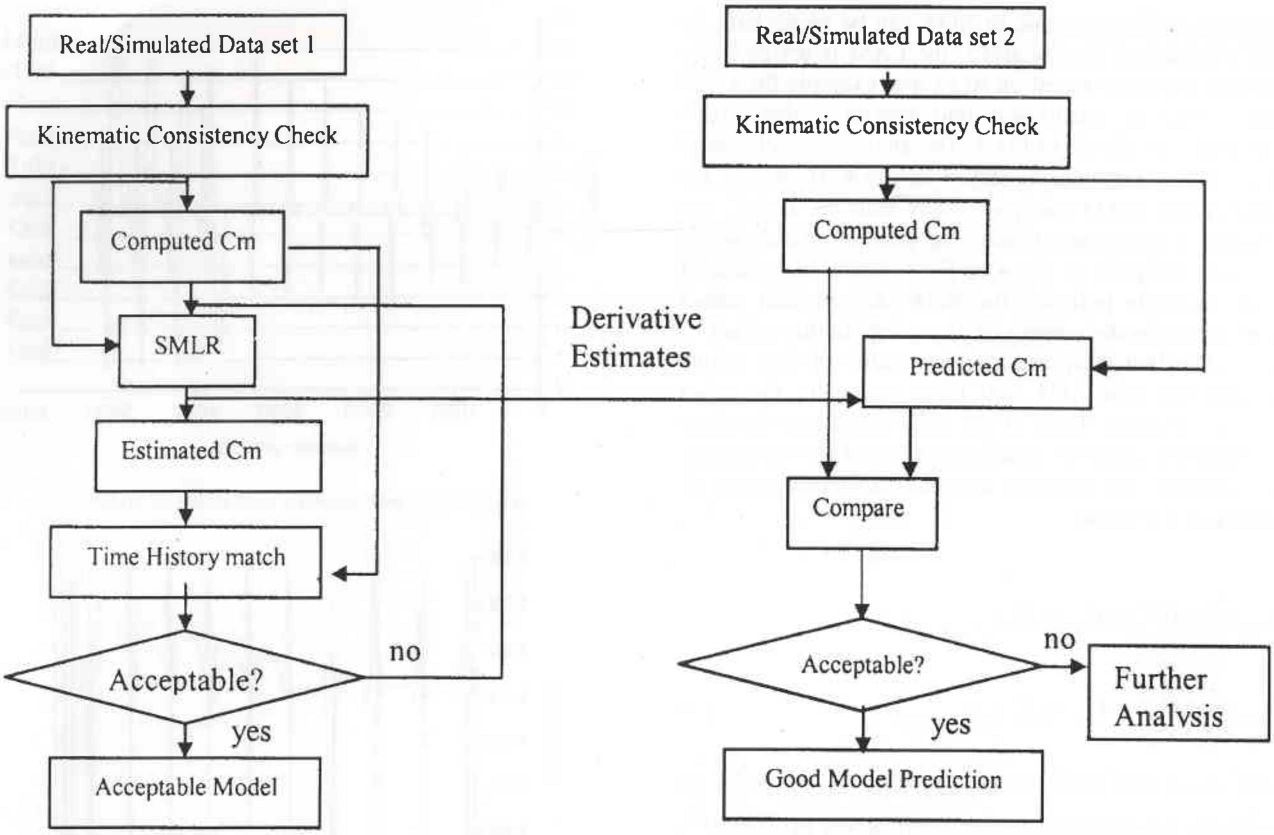


*Fig.1 Model estimation and validation for LAM analysis- A schematic*

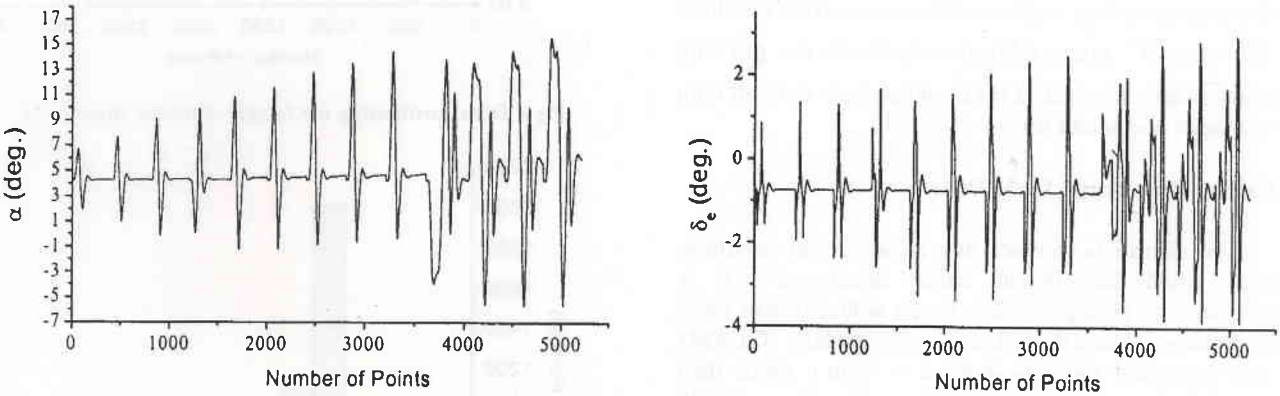


*Fig.2 Large amplitude maneuver - Longitudinal axis (simulated data)*

different large amplitudes to the pitch stick. 13 LAMs were generated and concatenated to form a single data set consisting of 5213 points. Sampling interval was 25 msec. Fig.2 shows the time history of the elevator deflection and angle of attack. The LAM shows an angle of attack variation from –6 to 16 deg.

Data partitioning procedure involves dividing a manoeuvre that covers a large range of some variable into several portions, each of which spans a smaller range of that variable [1]. The principle behind partitioning is that in the range of AOA defined by each subspace, the variation in the aerodynamic force and

moment coefficients due to AOA can be neglected. In the longitudinal data analysis, the LAM is divided into several partitions based on AOA. For example the LAM data could be partitioned into several 2 deg. AOA subspaces as shown in Fig.3. The angle of attack signal for $3<\alpha<5$ subspace is shown in Fig.4. However, for longitudinal LAM analysis, AOA bins of 1 deg. are chosen. A histogram showing the number of data points in each subspace is given in Fig.5 where the width of each rectangle indicates the AOA range in that subset and the height the number of data points in that subset.
It is clear that there are a large number of data points around the trim AOA and fewer points in the other regions of AOA. Each of the longitudinal aerodynamic coefficients could be analysed in each of the subspaces. For instance, the pitching moment coefficient would be analysed as follows:

$$C_m(\overline{\alpha}=10^0)=C_m(q,\delta_e)_{9^0<\alpha<11^0}$$

$$C_m(\overline{\alpha}=12^0)=C_m(q,\delta_e)_{11^0<\alpha<13^0} \qquad (1)$$

and so on for other values of AOA i.e. in order to estimate the pitching moment derivatives at $\overline{\alpha}=10^0$, all data in the subspace corresponding to $9^0<\alpha<11^0$ are combined into one group for analysis. Similarly, data corresponding to parts of the manoeuvres in which $11^0<\alpha<13^0$ are combined to estimate the pitching moment derivatives at $\overline{\alpha}=12^0$ and so forth until all data have been accounted for.

**Lateral Directional LAM Data**

The aircraft is trimmed at a chosen flight condition and LAM data in the lateral directional axis is generated by giving doublets inputs with different large amplitudes to the roll stick and rudder pedals. 10 LAMs were generated and concatenated to form a single data set consisting of 8010 data points. Sampling was 25 msecs. In this case, it was found that despite large excursions of the roll stick and rudder pedals, the feedback controller limits the AOA excursions. Hence, in order to generate the data for LAM analysis covering large excursions in angle of attack and angle of sideslip, the simulator is flown without the feedback controller in the loop. This resulted in an AOA variation from –3 to 20 deg at this flight condition.

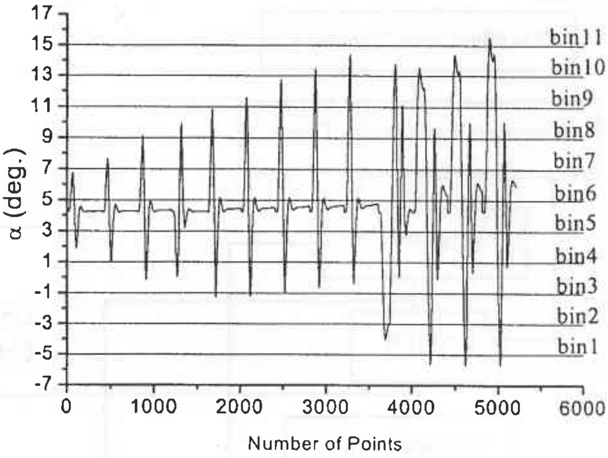


*Fig.3 Data partitioning w.r.t angle of attack*

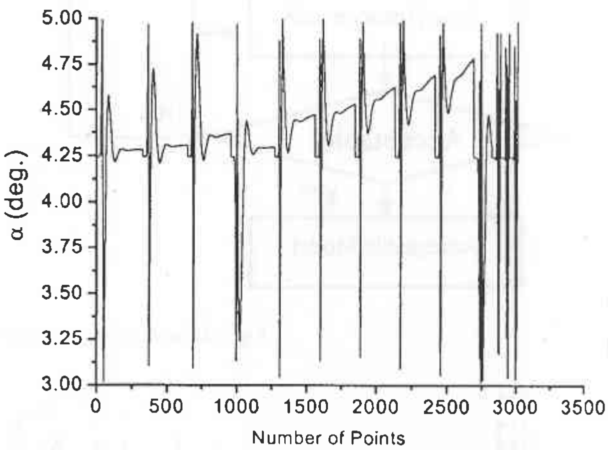


*Fig.4 Data partitioning w.r.t angle of attack (bin no. 5)*

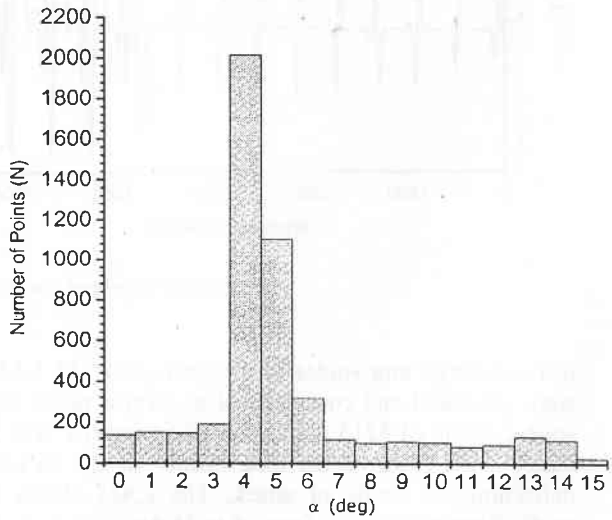


*Fig.5 Number of data points in subsets using partitioning of data from longitudinal maneuvers*

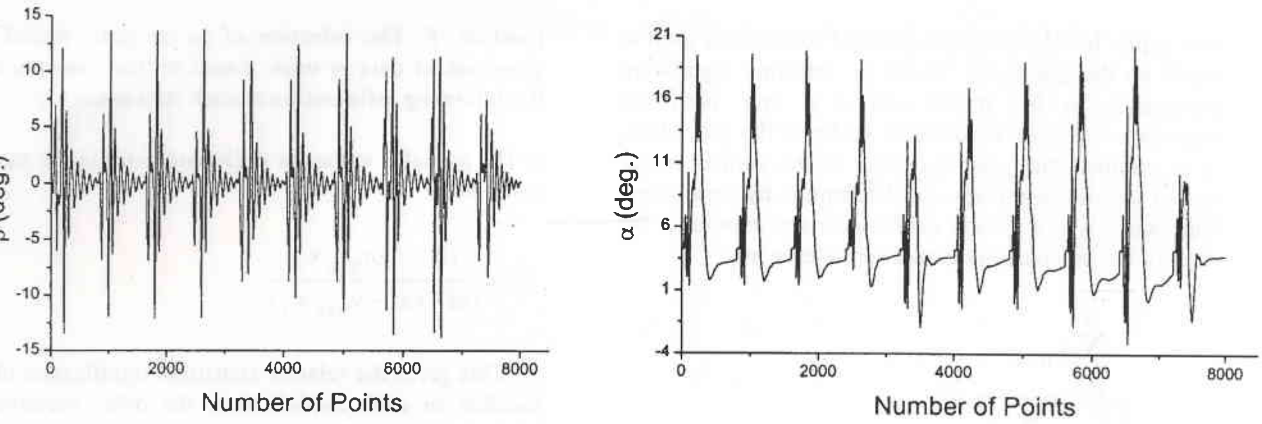


*Fig.6 Large amplitude maneuver - lateral axis (simulated data)*

Figure 6 shows the time history of angle of sideslip and angle of attack. The resulting ensemble of 8010 data points was then partitioned into subsets according to the values of AOA. The modelling of lateral parameters is carried out at 2 deg. AOA sub spacing as shown below:

$$C_l(\overline{\alpha} = 10\,deg.) = C_l(\beta, p, r, \delta_{control})_{9\,deg} < \alpha <_{11\,deg}$$

$$C_l(\overline{\alpha} = 12\,deg.) = C_l(\beta, p, r, \delta_{control})_{11\,deg} < \alpha <_{13\,deg}$$

$$C_n(\overline{\alpha} = 10\,deg.) = C_n(\beta, p, r, \delta_{control})_{9\,deg} < \alpha <_{11\,deg}$$

$$C_n(\overline{\alpha} = 12\,deg.) = C_n(\beta, p, r, \delta_{control})_{11\,deg} < \alpha <_{13\,deg} \quad (2)$$

**Analysis Procedure**

The methodology adopted for analysing the LAMs to estimate the aerodynamic parameters consists of the following three steps:

(i) The data are corrected for scale factor and bias errors using kinematic consistency check on the data [2]

(ii) The aerodynamic forces and moments are computed using the corrected data

(iii) The aerodynamic derivatives are estimated using SMLR. Details of steps (i) and (ii) can be found in Ref. 2. The step (iii) is described next. For all the data analysis results presented in this paper, the three steps are used.

## SMLR Method

The form of the aerodynamic model is given by

$$y(t) = \theta_0 + \theta_1 x_1(t) + \ldots + \theta_{n-1} x_{n-1}(t) + \varepsilon(t)\,,$$
$$t = 1, 2, \ldots, N \quad (3)$$

Here y is the dependent variable which could be one of the aerodynamic force or moment coefficients, $x_1, x_2, \ldots, x_{n-1}$ are the regressors formed by the aircraft input and output response variables or their combinations, $\varepsilon(t)$ is the equation error with zero mean and variance $\sigma^2$. $\theta_0, \theta_1, \ldots, \theta_{n-1}$, the stability and control derivatives, $\theta_0$ is the value of any particular coefficient corresponding to the initial steady-flight conditions. The equation error is considered to be in the equation between y(t) and the right hand side terms collectively. So it is gross error and not the error in particular one parameter. If $N$ observations for $y(t)$ and x(t) are available, then the least square estimates of θ is given by:

$$\hat{\theta} = (x^T x)^{-1} x^T y \quad (4)$$

Here $\hat{\theta}$ is the estimate of $\theta$, y is the (Nx1) vector of measured values of $y(t)$, and x is the (N x n) matrix of measured independent variables.

For selecting an appropriate model structure for a given data set, stepwise regression has been adopted in this paper. In this technique, partial F statistics is used to build up the parameter vector by selecting significant parameters in the model one at a time until the regression equation is satisfied. To begin the procedure, it is assumed that only the mean of the data is in the model and the estimates are determined by regression. Subsequently correlation coefficients are computed for each of the independent model variables using:

$$r_{x_j y} = \frac{\sum_{i=1}^{N} x_{ij} y_i}{\sqrt{\sum_{i=1}^{N} x_{ij}^2 \sum_{i=1}^{N} y_i^2}} \quad (5)$$

The $x_j$ with the largest $r_{x_j y}$ is chosen to enter the regression first. The fitted model is then given by:

$$\hat{y} = \hat{\theta}_1 + \hat{\theta}_j x_j + \hat{\varepsilon} \quad (6)$$

In the next step, the correlation coefficient for each remaining $x_i (i = 2,3,..., j-1, j+1,..., nl)$ is computed on $x_j$ and $\hat{y}$. The correlation coefficient is given by:

$$r_{yx_k x_j} = \frac{\sum_{i=1}^{N} (x_{ik} - x_{ij} \hat{\theta}_j - \hat{\theta}_1)(y_i - \hat{y}_i)}{\sqrt{\sum_{i=1}^{N} (x_{ik} - x_{ij} \hat{\theta}_j - \hat{\theta}_1)^2 \sum_{i=1}^{N} (y_i - \hat{y}_i)^2}} \quad (7)$$

Here $r_{yx_k x_j}$ is the partial correlation of y on $x_k$ given that $x_j$ is in the regression. The $x_k$ yielding the largest value of $r_{yx_k x_j}$ is selected to enter the model so that the regression model becomes

$$y = \hat{\theta}_1 + \hat{\theta}_j x_j + \hat{\theta}_k x_k \quad (8)$$

Again partial correlation coefficients $r_{yx_l x_j x_k}$ is computed and the $x_l$ giving the largest value enters the model. This process of computation is continued until the remaining variables entering the regression do not offer any statistical improvement in the model at significance level of F statistics selected apriori. These F values depend upon the number of data points, the number of parameters in the model and the selected risk level of F. The selection of an adequate model for a given set of data is made based on the examination of the following information at each step in the regression:

i) The partial F value for each parameter in the model is given by:

$$F_p = \frac{(N - nl) r_{yx_k x_j}}{(nl - 1)(1 - y_{yx_k x_j})} \quad (9)$$

This gives the relative statistical significance of each variable in each model when the other variables are present. Since $F_p$ is the inverse of the relative parameter variance, for an adequate model, it should have maximum value.

ii) The total F value $F_{total}$ = (Regression mean square)/(Residual mean square). The model with the maximum F value is the best one for a given set of data.

iii) The value of the squared multiple correlation coefficient given by:

$$R^2 = \frac{\sum_{i=1}^{N} [\hat{y}_i - \bar{y}]^2}{\sum_{i=1}^{N} [y(i) - \bar{y}]^2} \quad (10)$$

measures the proportion of variation explained by the terms other than $\theta_0$ in the model. The value of $R^2$ varies from 0 to 1 for a perfect set. It is generally expressed as a percentage and the improvement in $R^2$ due to the addition of a new parameter should be significant and should not reflect only the effect of increased number of parameters. $\bar{y}$ is mean of y signal.

iv) The residual sum of the squares (RSS) for the *l*th model is given by:

$$RSS_l = \sum [y(i) - \hat{y}(i)_l]^2 \quad (11)$$

This value is the measure of goodness of fit and any new parameter entering the model should contribute significantly to reduce its value.

v) The value of the residual variance estimated from:

$$s^2(\varepsilon) = (\text{residual sum of squares})/(N - n1) \qquad (12)$$

This value should be small. Examining the accuracy of the estimated parameters is also considered for verification of the model.

Since the subsets are selected based on the value of AOA, the data in a given subset may not be contiguous in time. The subsets may include several sections of the manoeuvre as well as data from several large manoeuvres. The model structure determination and parameters estimation are carried out by applying modified stepwise regression to each bin or subset of partitioned data. The determination of an adequate model for the aerodynamic coefficients includes three steps: the postulation of terms that might enter the model, the selection of an adequate model, and the verification of the model selected.

Several criteria have been used to judge the goodness of the estimated aerodynamic model using SMLR:

i) relatively low standard deviations of estimates
ii) Good time history match between computed and estimated aerodynamic coefficients
iii) plausibility of the estimates from physical interpretation of the estimates and comparison with reference values where possible
iv) model predictive capability.

**Results and Discussions**

### Longitudinal Data Analysis using Data Partitioning and SMLR

Figure 1 shows the schematic of the model estimation and validation for LAM analysis. Reference values are liberalised values based on wind tunnel predictions. The method of data partitioning and SMLR for analysis of longitudinal LAM is illustrated for estimation of pitching moment derivatives from the data generated from the simulator and partitioned into subspaces as shown in Fig. 3.

In each of the subsets, a linear model is postulated

$$C_{ml} = C_{m0} + C_{m\alpha}\alpha + C_{mq}\frac{q\bar{c}}{2V}q + C_{m\delta_e}\delta_e \qquad (13)$$

Here pitching moment derivatives $C_{m0}, C_{m\alpha}, C_{mq}$ and $C_{m\delta_e}$ are to be estimated.

It was observed that there were considerable variations in some of the lateral response variables during longitudinal LAMs. These cross coupling effects could arise due the kinematics or due to aerodynamics. In order to arrive at the appropriate model structure, the following model was adopted for accounting for the cross-coupling terms (subscript augmented by k) [5]

$$C_{mk} = C_{m\beta}\beta^2 + C_{m\beta\alpha}\beta^2\alpha + C_{m\delta a}|\delta_a| + C_{m\delta r}\delta r^2 + C_{m_{p^2}}p^2\frac{b}{2V} + C_{mp}\left|p\frac{b}{2V}\right| + C_{mr}\left|r\frac{b}{2V}\right| \qquad (14)$$

Finally the model $C_m = C_{ml} + C_{mk}$ that includes both linear as well as bi-linear/coupling aerodynamic effects with a total of 10 regressors is used for estimation. The plot of $\%R^2$ and F statistics for the coefficient $C_m$ at $\alpha = 12\,\text{deg}$. is shown in Fig. 7 as each of the 10 regressors enter the model for Cm. From Fig. 7, it is clear that the F statistics reaches the maximum value at step number 5 and the $\%R^2$ remains more or less constant after step number 5. Hence variables $\alpha, q, \delta_e, \delta_r^2$ *and* $p$ entered the model resulting in the estimation of $C_{m\alpha}, C_{mq}, C_{m\delta e}, C_{m\delta r}, C_{m_{p^2}}$ derivatives. It is emphasized here that linear terms due to $\alpha, q, \delta_e$ got automatically entered. The estimate of the derivatives $C_{m\alpha}, C_{mq}, C_{m\delta e}$ are plotted in Fig. 8a, 8b and 8c.

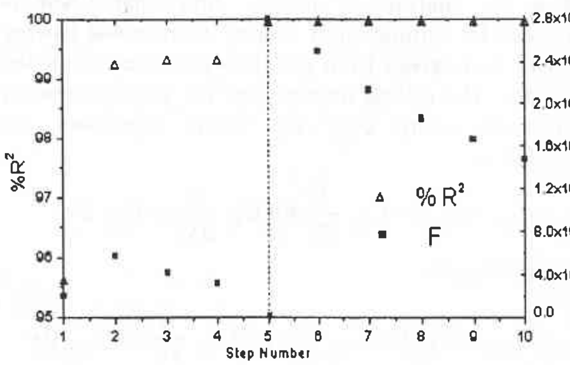


*Fig. 7 F statistics and %R² for $C_m$*

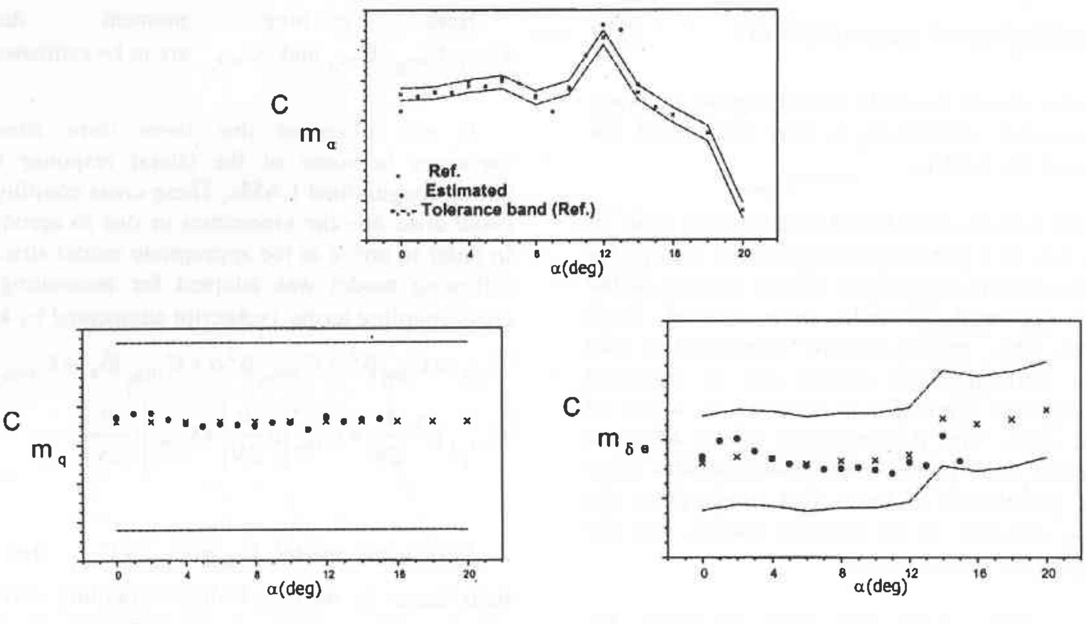


*Fig.8 The longitudinal derivatives*

The estimated derivatives fall within the tolerance bound and are reasonably well determined. The estimates are found to have low standard deviations. The bands plotted are the wind tunnel tolerances on the derivatives. Since the concatenated data of LAMs covered the alpha range of –6 to 16 deg, the estimated derivatives are available up to 15 deg. AOA. The pitch up tendency exhibited by the unstable aircraft seems estimated quite accurately using this procedure.

## Lateral-Directional LAM Analysis with Data Partitioning and SMLR

In this section, the method of data partitioning and SMLR for analysis of lateral- directional LAM is illustrated for estimation of rolling moment and yawing moment derivatives from the data generated from the simulator. The rolling moment and the yawing moment coefficients using only the linear regressors are modelled as:

$$C_{ll} = C_{l_0} + C_{l_\beta}\beta + C_{l_p}\frac{b}{2V}p + C_{l_r}\frac{b}{2V}r + C_{l_{\dot\beta}}\dot\beta + C_{l_{\delta a}}\delta a + C_{l_{\delta r}}\delta r$$

$$C_{nl} = C_{n_0} + C_{n_\beta}\beta + C_{n_p}\frac{b}{2V}p + C_{n_r}\frac{b}{2V}r + C_{n_{\dot\beta}}\dot\beta + C_{n_{\delta a}}\delta a + C_{n_{\delta r}}\delta r \quad (15)$$

The linear model for $C_{ll}$ & $C_{nl}$ are augmented by aerodynamic coupling terms given by:

$$C_{nk} = C_{n_q}\frac{\bar{c}}{2V}q + C_{n_\alpha}\alpha \quad (16)$$

Additional candidate combinations involving

$$\beta^2, \beta^3, |\beta|\beta, |\beta|p, |\beta|r, \beta^2 r, \delta_a^2, \delta_a^3, \delta_r^2, \delta_r^3 \text{ and } |\beta|\delta_r \quad (17)$$

are considered for accounting for any possible bi-linear/coupling terms in the aerodynamic coefficients within each subset [2]. Initially this model with 19 regressors (Eqs. 15,16 and 17) was used to estimate the aerodynamic derivatives for $C_l$ and $C_n$ .

The plot of $\%R^2$ and F statistics for the coefficient $C_l$ at $\bar{\alpha} = 12$ deg. is shown in Fig. 9. It is clear that the F statistics for $C_l$ reaches the maximum value at step number 7 and $\%R^2$ remains more or less constant after step numbers 7. At that stage, 7 variables $\beta, p, r, \delta_a, \delta_r, \delta_a^3, |\beta|\delta_r$ enter the model for $C_l$

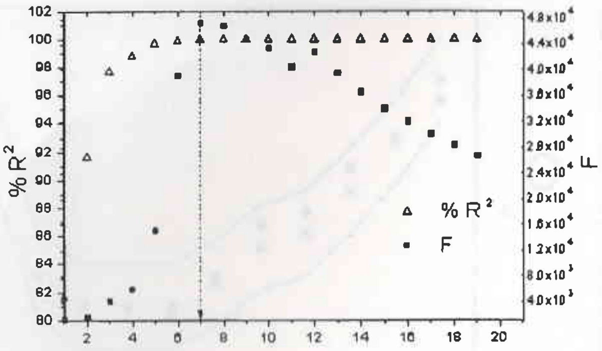


*Fig. 9 F Statistics and $\%R^2$ for $C_l$*

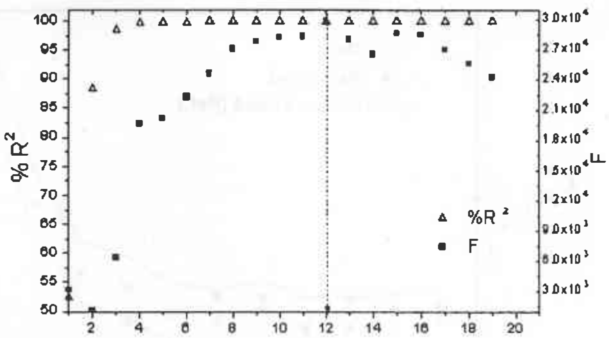


*Fig. 10 F Statistics and $\%R^2$ for $C_n$*

**Table 2: The estimates from LAM/SMLR – Longitudinal data w/o partitioning**

| Trim angle of attack (deg.) | $C_{m_\alpha}$ | | $C_{mq}$ | | $C_{m_{\delta e}}$ | | $\%R^2$ (only linear terms) | $\%R^2$ (with additional terms) |
|---|---|---|---|---|---|---|---|---|
| | Ref. | Estm. | Ref. | Estm. | Ref. | Estm. | | |
| 11.90 | 0.0771 (0.018) | 0.0714 (0.004) | -1.1733 (0.422) | -1.2945 (0.018) | -0.3905 (0.040) | -0.3752 (0.002) | 98.81 | 99.88 |
| 4.30 | -0.0144 (0.016) | -0.0102 (0.002) | -1.3195 (0.612) | -1.1884 (0.058) | -0.4151 (0.040) | -0.3975 (0.006) | 98.68 | 99.67 |
| 11.55* | 0.0771 (0.018) | 0.0580 (0.007) | -1.1733 (0.422) | -0.9916 (0.074) | -0.3905 (0.040) | -0.3935 (0.006) | 97.84 | 98.40 |

* Large amplitude manoeuvre results from real flight data// (.) Standard deviation absolute

resulting in the estimation of the derivatives $C_{l_\beta}, C_{l_p}, C_{l_r}, C_{l_{\delta a}}, C_{l_{\delta r}}, C_{l_r}, C_{l_{\delta a^3}} C_{l|\beta|\delta r}$. Fig. 10 shows the plot of F and $\%R^2$ for $C_n$. F statistics reaches the maximum value at step number 12 and these variables enter the model resulting in derivative estimates for

$$C_{n_\beta}, C_{n_p}, C_{n_r}, C_{n_{\delta a}}, C_{n_{\delta r}}, C_{n_{\beta^3}}, C_{n|\beta|\beta}, C_{n|\beta|r}$$

$$C_{n_{\delta r^3}}, C_{n_{\beta^2 r}}, C_{n_{\delta a^3}} \text{ and } C_{n|\beta|\delta r}.$$

Some of the estimated lateral-directional derivatives from large amplitude manoeuvres using partitioning method are compared with the reference wind tunnel values in Fig. 11. The estimates seem very reasonable and are well within the aerodynamic tolerance bounds. From the results of LAM analysis using data partitioning and SMLR, it has been possible to estimate all the longitudinal and lateral directional derivatives covering a large AOA variation about the trim value at each flight condition. The method is particularly advantageous to estimate the aerodynamic derivatives at those flight regimes where it may not be possible to trim the aircraft and perform the conventional small perturbation manoeuvres.

## Estimation of Longitudinal Parameters from LAMs using SMLR

Linear model of the aircraft is sufficient enough for small perturbations from trim conditions at low angles of attack. As mentioned in the introduction, the

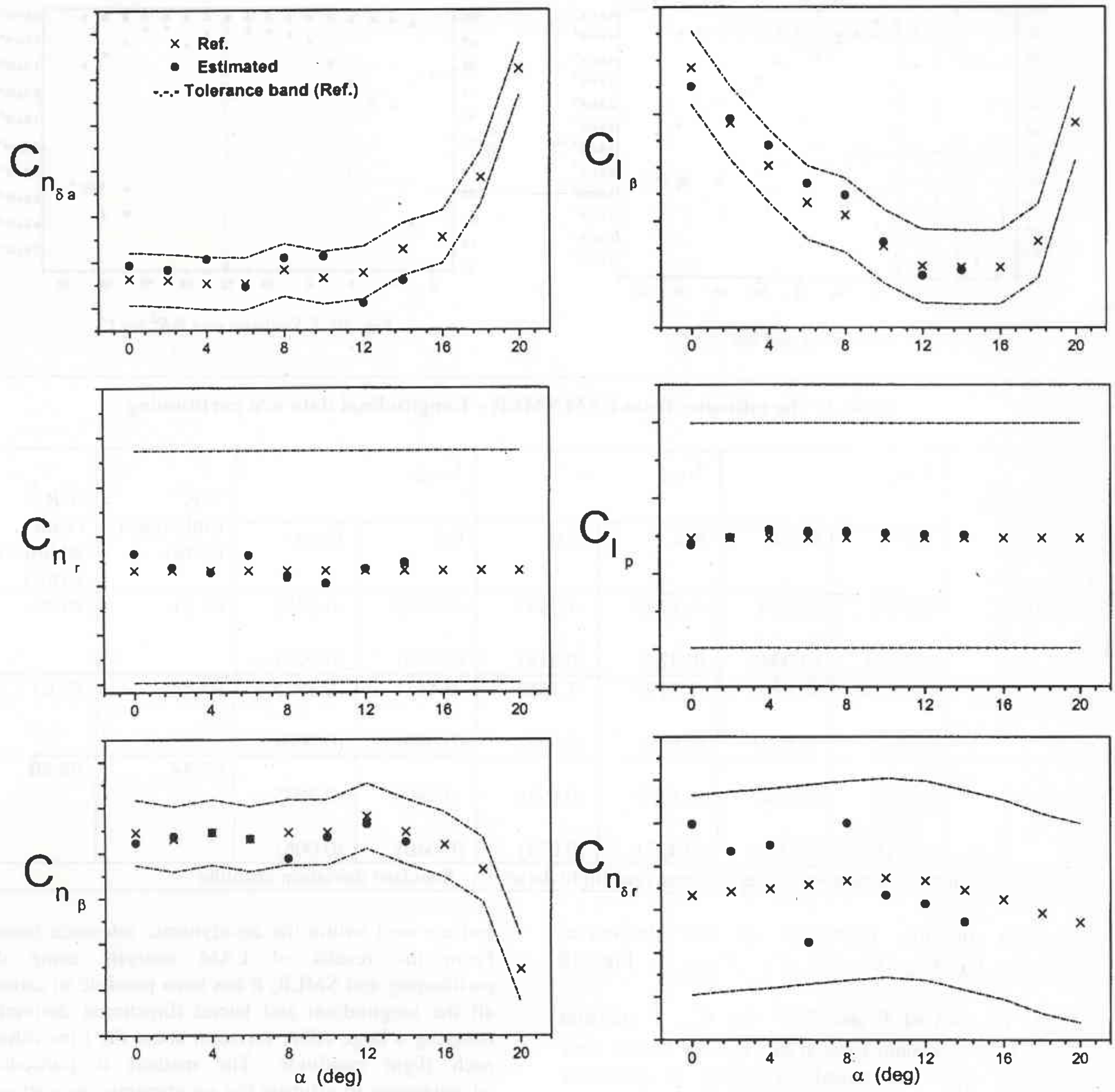


*Fig.11 : Lateral Directional parameter estimates*

development of estimation methods for non-linear aerodynamic models would be essential to analyse unanticipated LAMs in flight. Hence, some studies have been carried out in this paper to estimate the parameters in the longitudinal mode from large amplitude manoeuvres without partitioning at two typical flight conditions. The flight conditions are such that (1) AOA=4.3 deg., U/C up and (2) AOA=11.9 deg., U/C down. Pitch stick input of large amplitude is given to elevator to generate the data.

As in the partitioning method, the pitching moment related parameters are estimated using stepwise regression method. Apart from linear parameters, candidate combinations $\alpha^2$, $\alpha q$, $\alpha\delta_e$ signifying the slope of linear parameters w.r.t. AOA and

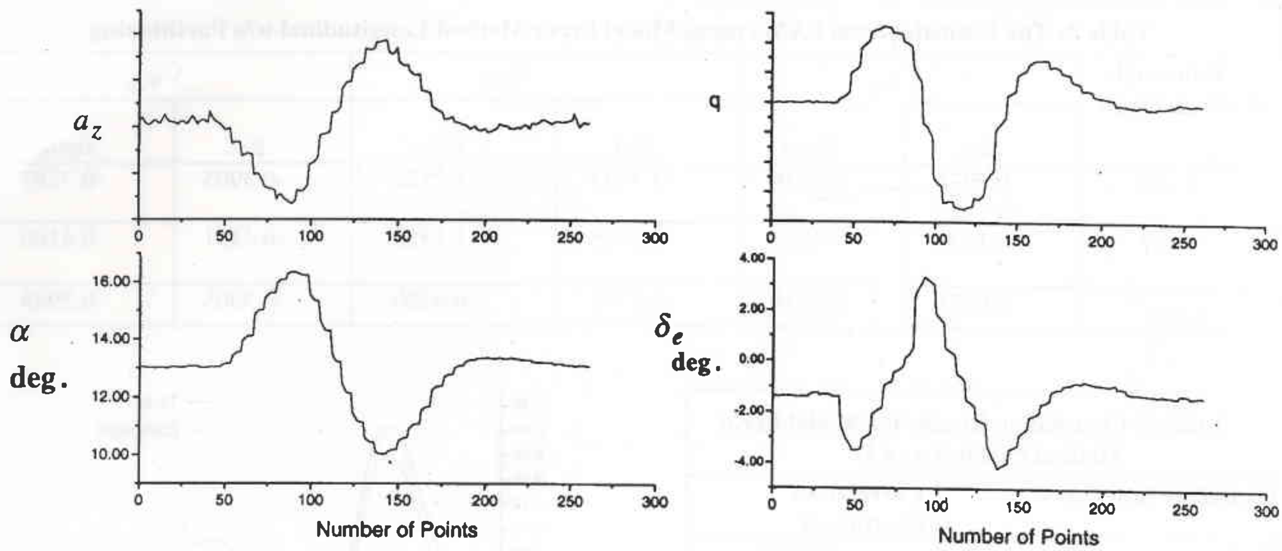


*Fig.12 : Real flight trajectories of Large amplitude (Longitudinal Axis / w/o Partitioning)*

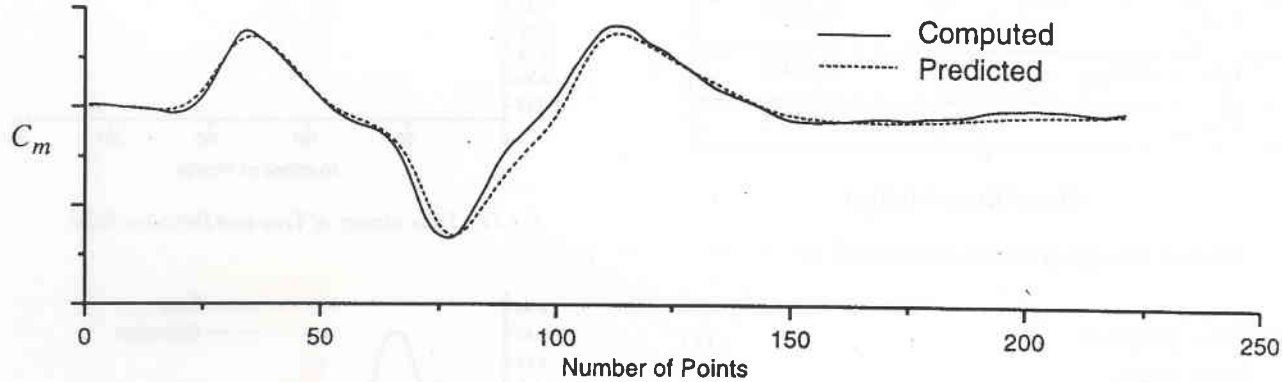


*Fig.13 : Model Validation (Real Flight Data / w/o Partitioning)*

$\alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6, \alpha^7$ signifying higher order terms (since longitudinal motion is generally dependent on α) were considered as candidate aerodynamic derivatives for estimation of parameters. The results of parameter estimation of linear aerodynamic derivatives using stepwise regression method for the two flight conditions are given in Table 2. The candidate variables that entered the model are: $C_{m_{\delta e}}, C_{m_q}, C_{m_\alpha}, C_{m_{\alpha^3}}$ and $C_{m_{\alpha\delta_e}}$. The $\%R^2$ values are also given in Table 2. It is clear from the Table that the estimated derivatives compare well with the reference values.

The SMLR method is applied to the analysis of real LAM data of the unstable/augmented aircraft. Fig. 12 shows the measured flight trajectories of the longitudinal axis variables. The kinematically consistent flight data is used for estimation of pitching moment aerodynamic derivatives which are given in row 3 of Table 2. The control effectiveness derivative $C_{m_{\delta e}}$ compares well with the reference values whereas the estimates of $C_{m_\alpha}$ and $C_{m_q}$ from flight are lower than the reference values. In Fig. 13, the estimated model is used to predict the $C_m$ for another LAM from flight indicative of the adequacy of the estimated aerodynamic model .

**Table 3: The Estimates from LAMs using Model Error Method Longitudinal w/o Partitioning**

| Trim angle of attack (deg.) | $C_{m_\alpha}$ | | $C_{mq}$ | | $C_{m_{\delta e}}$ | |
|---|---|---|---|---|---|---|
| | Ref. | Estm. | Ref. | Estm. | Ref. | Estm. |
| 11.90 | 0.0771 | 0.0810 | -1.1733 | -1.2522 | -0.3905 | -0.3787 |
| 4.30 | -0.0144 | -0.0201 | -1.3195 | -1.1558 | -0.4151 | -0.4106 |
| 11.55* | 0.0771 | 0.0610 | -1.1733 | -0.9220 | -0.3905 | -0.3988 |

**Table 4: Correlation Results for Model Error Method (Table 3, row3)**

| Candidate function | Correlation Coefficient |
|---|---|
| $Cm_{\alpha^3}$ | 0.7010 |
| $Cm_{\alpha^2} + Cm_{\alpha^3}$ | 0.6584 |
| $Cm_{\alpha^2} + Cm_{\alpha^3} + Cm_{\alpha q}$ | 0.6577 |
| $Cm_{\alpha^2}$ | 0.6555 |
| $Cm_{\alpha^2} + Cm_{\alpha q}$ | 0.6505 |
| $Cm_{\alpha q}$ | 0.0579 |

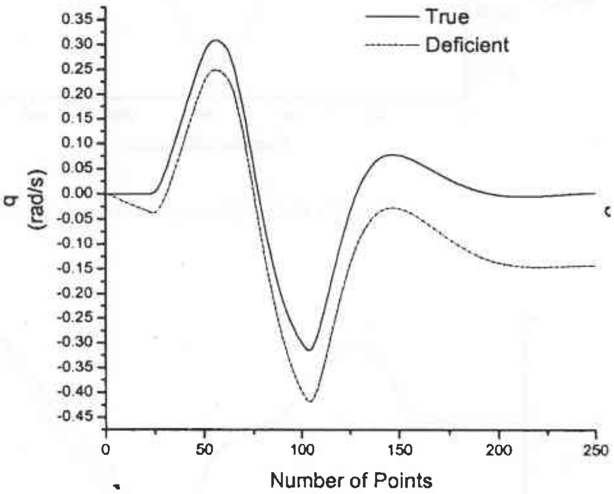

*Fig.14 : Time history of True and Deficient State*

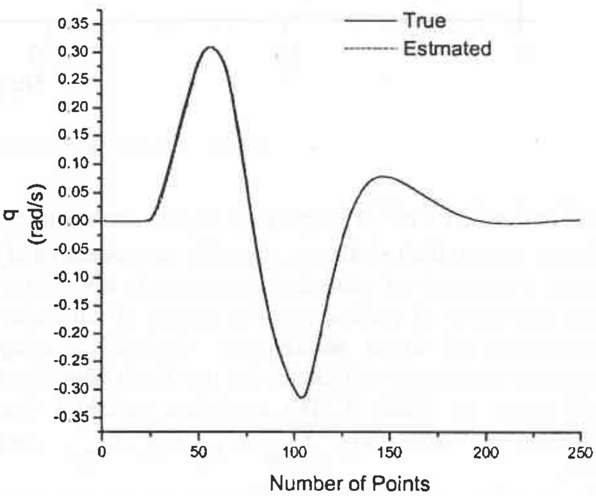

*Fig.15 : Time history of True and Estimated State*

**Model Error Method**

The true non-linear system is described as:

$$\dot{X}(t) = g(X(t), t) \quad (18)$$
$$Y(t) = HX(t)$$

g is true representation of the dynamical system. The observables Y are obtained for interval $t_0 < t < \tau$. Eqn. (A1) is recast to explicitly express the deterministic model error as follows:

$$\dot{x}(t) = f(x(t), t) + d(t) \quad (19)$$
$$y(t) = H(t)x(t) + v(t)$$

Here f denotes the nominal model, v is additive measurement noise and the vector 'd' is the model discrepancy. The vector 'd' is to be estimated in the sense of minimum model error criterion:

$$J = \int_0^{\tau} [(y(t) - H(t)x(t)^T Q_l(t)(y(t) - H(t)x(t)) + d^T(t)Q_2 d(t)]dt \tag{20}$$

Tuning of the algorithm is done by selecting appropriate values of matrices $Q_l$ and $Q_2$. The model discrepancy is provided in the form of time histories. These time histories, when parameterised in the least squares sense, yield the coefficients of the models by which the true models were deficient.

Recursive algorithm based on the technique of invariant embedding is used for estimation of deterministic model errors in non-linear systems [6]. These algorithms have features similar to the extended Kalman filter.

The estimates using model error method are given in Table 3. Table 4 gives the candidate models. Fig. 14 shows the cross plot of pitch rate when the model is deficient without bilinear/non-linear terms. Fig. 15 shows the cross plot of pitch rate when the deficiency in the model is estimated (row 3 of Table 3). The matching is good [7].

## Conclusions

In this paper, the procedure to capture the pitch up tendency occurring in inherently unstable/augmented aircraft has been described. The pitching moment related parameters are estimated by concatenating large amplitude manoeuvres using stepwise regression technique and compared with predicted values. The estimated values match reasonably well with the predicted values for most of the angle of attack region. Also the estimates of lateral-directional derivatives from large amplitude manoeuvres using stepwise regression technique match reasonably with predicted values. The parameter estimates related to pitching moment coefficient from large perturbation manoeuvres without partitioning and using only SMLR and the model error method are satisfactory for simulated / real flight data. However, the physical interpretation of other derivatives estimated using the methods has not been attempted.

## Acknowledgements

The authors are grateful to Aeronautical Development Agency for technical & financial support. Also they are grateful to Flight Simulation team of FMCD, NAL.

## References

1. James G. Batterson and V. Klein. Partitioning of flight Data for Aerodynamic Modelling of Aircraft at High Angles of Attack, Journal of Aircraft, Vol. 26, No.2, 1989.

2. Girija G, Parameswaran V, Raol J.R. and Balakrishna S. Estimation of Aircraft Performance and Stability Characteristics by Dynamic Manoeuvres using Maximum Likelihood estimation Techniques, in preprints of 9th IFAC Symposium on Identification and System Parameter Estimation, Budapest, Hungary, July 1991.

3. R.V. Jategaonkar. Identification of DLR research aircraft ATTAS from flight test data. DLR-FB 90-40, July 1990.

4. Girija G. and Raol J.R. Estimation of Aerodynamic Parameters from Dynamic Manoeuvres using Estimation Before Modelling Procedure. Journal of Aero. Soc. of India, Vol. 48, No. 2, May, pp. 110-127, 1996.

5. V. Klein. Estimation of Aircraft Aerodynamic Parameters from Flight Data, Progress in Aerospace Science, Vol. 26, pp. 1-77, 1989.

6. V. Parameswaran and J.R. Raol. Estimation of model error for non-linear system identification, IEE Proc.-Cont. Theory Appl., Vol. 141, No. 6, Nov. 1994.

7. V. Parameswaran, Girija G and J.R.Raol. Estimation of parameters from large amplitude manoeuvres with partitioned data for an aircraft, AIAA Conference and exhibit on Atmospheric Flight Mechanics, Austin, Texas, U.S., Aug.11-17, 2003.