FREE VIBRATION OF FUNCTIONALLY GRADED SKEW PLATES SUBJECTED TO THERMAL ENVIRONMENT

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Abstract

Here, the free vibration behavior of functionally graded skew plates is investigated using finite element procedure. Temperature field is assumed to be a uniform distribution over the plate surface and varied in thickness direction only. Material properties are assumed to be temperature dependent and graded in the thickness direction according to simple power law distribution. For the numerical illustrations, silicon nitride/stainless steel is considered as functionally graded material. The variation in frequencies is highlighted considering gradient index, temperature, thickness and aspect ratios, and skew angle of the plate.

Keywords: *Functionally graded plate, frequency, aspect ratio, temperature, gradient index.*

Introduction

Functionally graded materials (FGMs) are the new generation of composite materials in which the micro-structural details are spatially varied through non-uniform distribution of the reinforcement phase. This can be achieved by using reinforcements with different properties, sizes and shapes, as well as by interchanging the role of reinforcement and matrix phases in a continuous manner. The result is a microstructure that produces continuously varying thermal and mechanical properties at the macroscopic or continuum level. Due to recent advances in material processing capabilities, that aid in manufacturing wide variety of functionally graded materials, their use in application involving severe thermal environments is gaining acceptance in composite community, the aerospace and aircraft industry [1-4]. For instance, in a thermal protection system, FGMs take advantage of heat and corrosion resistance typical of ceramics, and mechanical strength and toughness typical of metals. In view of these, there is an increased interest among researchers to study the dynamic and stability behaviors of the structural components made of these materials.

It is seen from the literature that the amount of work carried out on the vibration characteristics of isotropic plates and composite laminates are exhaustive. However, the investigations of linear free behaviors of FGM plates under thermo-mechanical environment are limited in number and are discussed briefly here. Tanigawa et al. [5] have examined transient thermal stress distribution of FGM plates induced by unsteady heat conduction with temperature dependent material properties. Reddy and Chin [6] have dealt with many problems, including transient response of plate due to heat flux. In Ref. [7], three-dimensional analysis of transient thermal stress in functionally graded plates has been carried out adopting Laplace transformation technique and power series method. He et al. [8] presented finite element formulation based on thin plate theory for the shape and vibration control of FGM plate with integrated piezoelectric sensors and actuators under mechanical load whereas Liew et al. [9] have analyzed the active vibration control of plate subjected to a thermal gradient using shear deformation theory. Yang and Shen [10] have analyzed dynamic response of thin FGM plates subjected to impulsive loads using Galerkin procedure coupled with modal superposition method whereas, by neglecting the heat conduction effect, such plates and panels in thermal environments have been

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Manuscript received on 24 Sep 2004; Paper reviewed and accepted on 07 Nov 2004

examined based on shear deformation with temperature dependent material properties in Ref. [11]. Cheng and Batra [12] studied the steady state vibration of a simply supported functionally graded polygonal plate with temperature independent material properties. Sills et al. [13] have presented different modeling aspects and also simulated the dynamic response under a step load. It may be concluded from the existing literature that all the studies have been mainly dealt with rectangular plates, and the knowledge pertaining to such investigations of FGM skew plate structure is meager. Due to the increasing utilization of skewed-type of FGM structural components in the design of flight vehicle structures, understanding their vibration characteristics are important for the structural designers.

Here, an eight-noded shear flexible quadrilateral plate element developed based on consistency approach [14] is used to analyze the free vibration of FGM plate. The temperature field is assumed to be constant in the plane and varied only in the thickness direction of the plate. The material is assumed to be temperature dependent and graded in the thickness direction according to the power-law distribution in terms of volume fractions of the constituents. The influences of thickness and aspect ratios, thermal load and skew-angle of the plate on the vibration of functionally graded skew plates are brought out.

Theoretical development and formulation

A functionally graded rectangular plate (length *a*, width *b,* and thickness *h*) made of a mixture of ceramics and metals is considered with the coordinates x, y along the in-pane directions and z along the thickness direction. The material in top surface $(z = h/2)$ of the plate and in bottom surface $(z = -h/2)$ of the plate is ceramic and metal, respectively. The effective material properties P, such as Young's modulus E, and thermal expansion coefficient α, can be written as [15]

$$P = P_c V_c + P_m V_m \tag{1}$$

where P_c and P_m are the material properties of the ceramic rich top surface and metal rich bottom surface, respectively. V_c and V_m are volume-fractions of ceramic and metal respectively and are related by

$$V_c + V_m = 1 \tag{2}$$

The properties of the plate are assumed to vary through the thickness. The property variation is assumed to be in terms of a simple power law. The volume fraction V_c is expressed as

$$V_c(z) = \left(\frac{2z+h}{2h}\right)^k \tag{3}$$

where k is the volume fraction exponent $(k \geq 0)$. The material properties P that are temperature dependent can be written as

$$P = P_0(P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3) \tag{4}$$

where P_0, P_{-1}, P_1, P_2 and P_3 are the coefficients of temperature $T(K)$ and are unique to each constituent.

From Eqs. (1) - (4), the modulus of elasticity E, the coefficient of thermal expansion α, the density ρ and the thermal conductivity K are written as

$$E(z,T) = (E_c(T) - E_m(T))\left(\frac{2z+h}{2h}\right)^k + E_m(T)$$

$$\alpha(z,T) = (\alpha_c(T) - \alpha_m(T))\left(\frac{2z+h}{2h}\right)^k + \alpha_m(T)$$

$$\rho(z) = (\rho_c - \rho_m)\left(\frac{2z+h}{2h}\right)^k + \rho_m$$

$$K(z) = (K_c - K_m)\left(\frac{2z+h}{2h}\right)^k + K_m$$

(5)

Here the mass density ρ and thermal conductivity K are assumed to be independent of temperature. The Poisson's ratio ν is assumed to be a constant $\nu(z) = \nu_0$.

The temperature variation is assumed to occur in the thickness direction only and the temperature field is considered constant in the *xy* plane. In this case, the temperature through thickness is governed by the one-dimensional Fourier equation of heat conduction:

$$\frac{d}{dz}\left[K(z)\frac{dT}{dz}\right]=0, \qquad T=T_c \ at \ z=h/2 \qquad (6)$$
$$T=T_m \ at \ z=-h/2$$

The solution of Eq. (6) is obtained by means of polynomial series [16] and given by

$$T(z)=T_m+(T_c-T_m)\eta(z) \qquad (7)$$

where

$$\eta(z)=\frac{1}{C}\left[\left(\frac{2z+h}{2h}\right)-\frac{K_{cm}}{(k+1)K_m}\left(\frac{2z+h}{2h}\right)^{k+1}+\right.$$
$$\frac{K_{cm}^2}{(2k+1)K_m^2}\left(\frac{2z+h}{2h}\right)^{2k+1}-\frac{K_{cm}^3}{(3k+1)K_m^3}\left(\frac{2z+h}{2h}\right)^{3k+1}+$$
$$\left.\frac{K_{cm}^4}{(4k+1)K_m^4}\left(\frac{2z+h}{2h}\right)^{4k+1}-\frac{K_{cm}^5}{(5k+1)K_m^5}\left(\frac{2z+h}{2h}\right)^{5k+1}\right];$$

$$C=1-\frac{K_{cm}}{(k+1)K_m}+\frac{K_{cm}^2}{(2k+1)K_m^2}-\frac{K_{cm}^3}{(3k+1)K_m^3}+$$
$$\frac{K_{cm}^4}{(4k+1)K_m^4}-\frac{K_{cm}^5}{(5k+1)K_m^5};$$

and $K_{cm}=K_c-K_m$

Using Mindlin formulation, the displacements u,v,w at a point (x,y,z) in the plate (Fig. 1a) from the medium surface are expressed as functions of mid-plane displacements u_0,v_0 and w, and independent rotations θ_x and θ_y of the normal in xz and yz planes, respectively, as

$$\begin{aligned} u(x,y,t)&=u_0(x,y,t)+z\theta_x(x,y,t)\\ v(x,y,t)&=v_0(x,y,t)+z\theta_y(x,y,t)\\ w(x,y,t)&=w(x,y,t)\end{aligned} \qquad (8)$$

where t is the time. The strains in terms of mid-plane deformation can be written as

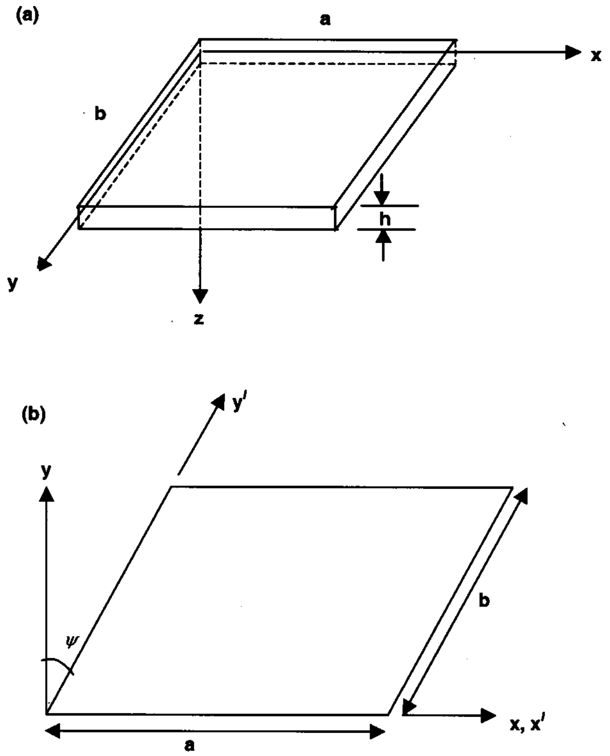

Fig. 1 (a) Configuration and coordinate system of a rectangular FGM plate
(b) Coordinate system of a skew plate

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_p\\0\end{Bmatrix}+\begin{Bmatrix}z\varepsilon_b\\\varepsilon_s\end{Bmatrix} \qquad (9)$$

The mid-plane strain $\{\varepsilon_p\}$, bending strains $\{\varepsilon_b\}$ and shear strains $\{\varepsilon_s\}$ in Eq. (9) are written as

$$\{\varepsilon_p^L\}=\begin{Bmatrix}u_{0,x}\\v_{0,y}\\u_{0,y}+v_{0,x}\end{Bmatrix} \qquad (10)$$

$$\{\varepsilon_b\}=\begin{Bmatrix}\theta_{x,x}\\\theta_{y,y}\\\theta_{x,y}+\theta_{y,x}\end{Bmatrix} \qquad (11)$$

$$\{\varepsilon_s\}=\begin{Bmatrix}\theta_x+w_{,x}\\\theta_y+w_{,y}\end{Bmatrix} \qquad (12)$$

where the subscript comma denotes the partial derivative with respect to the spatial coordinate succeeding it.

The membrane stress resultants $\{N\}$ and the bending stress resultants $\{M\}$ can be related to the membrane strains $\{\varepsilon_p\}$ and bending strains $\{\varepsilon_b\}$ through the constitutive relations by

$$\{N\} = \begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = [A_{ij}]\{\varepsilon_p\} + [B_{ij}]\{\varepsilon_b\} - \{N^T\} \quad (13)$$

$$\{M\} = \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = [B_{ij}]\{\varepsilon_p\} + [D_{ij}]\{\varepsilon_b\} - \{M^T\} \quad (14)$$

where the matrices $[A_{ij}],[B_{ij}]$ and $[D_{ij}]$ $(i,j=1,2,6)$ are the extensional, bending-extensional coupling and bending stiffness coefficients and are defined as $[A_{ij}, B_{ij}, D_{ij}] = \int_{-h/2}^{h/2} [\bar{Q}_{ij}](1,z,z^2)dz$.

The thermal stress resultant $\{N^T\}$ and moment resultant $\{M^T\}$ are

$$\{N^T\} = \begin{Bmatrix} N_x^T \\ N_y^T \\ N_{zy}^T \end{Bmatrix} = \int_{-h/2}^{h/2} [\bar{Q}_{ij}] \begin{Bmatrix} \alpha(z,T) \\ \alpha(z,T) \\ 0 \end{Bmatrix} \Delta T(z)\, dz \quad (15)$$

$$\{M^T\} = \begin{Bmatrix} M_x^T \\ M_y^T \\ M_{zy}^T \end{Bmatrix} = \int_{-h/2}^{h/2} [\bar{Q}_{ij}] \begin{Bmatrix} \alpha(z,T) \\ \alpha(z,T) \\ 0 \end{Bmatrix} \Delta T(z)\, zdz \quad (16)$$

where the thermal coefficient of expansion $\alpha(z,T)$ is given by Eq. (5), and $\Delta T(z) = T(z) - T_0$ is temperature rise from the reference temperature T_0 at which there are no thermal strains.

Similarly the transverse shear force $\{Q\}$ representing the quantities $\{Q_{xz}, Q_{yz}\}$ is related to the transverse shear strains $\{\varepsilon_s\}$ through the constitutive relations as

$$\{Q\} = [E_{ij}]\{\varepsilon_s\} \quad (17)$$

where $E_{ij} = \int_{-h/2}^{h/2} [\bar{Q}_{ij}]\kappa_i \kappa_j dz$

Here $[E_{ij}]$ $(i,j=4,5)$ are the transverse shear stiffness coefficients, κ_i is the transverse shear coefficient for non-uniform shear strain distribution through the plate thickness. $\bar{Q}_{ij}$ are the stiffness coefficients and are defined as

$$\bar{Q}_{11} = \bar{Q}_{22} = \frac{E(z,T)}{1-\nu^2};\ \bar{Q}_{12} = \frac{\nu\ E(z,T)}{1-\nu^2};$$

$$\bar{Q}_{16} = \bar{Q}_{26} = 0;\ \bar{Q}_{44} = \bar{Q}_{55} = \bar{Q}_{66} = \frac{E(z,T)}{2(1+\nu)} \quad (18)$$

where the modulus of elasticity $E(z,T)$ is given by Eq. (5).

The strain energy functional U is given as

$$U(\delta) = (1/2)\int_A \Big[\{\varepsilon_p\}^T [A_{ij}]\{\varepsilon_p\} + \{\varepsilon_p\}^T [B_{ij}]\{\varepsilon_b\} + \{\varepsilon_b\}^T [B_{ij}]\{\varepsilon_p\} + \{\varepsilon_b\}^T [D_{ij}]\{\varepsilon_b\} + \{\varepsilon_s\}^T [E_{ij}]\{\varepsilon_s\} - \{\varepsilon_p^0\}^T \{N^T\} - \{\varepsilon_b\}^T \{M^T\} \Big]\, dA \quad (19)$$

where δ is the vector of the degree of freedom associated to the displacement filed in a finite element discretisation.

The kinetic energy of the plate is given by

$$T(\delta) = (1/2)\int_A \left[p\left(\dot{u}_0^2 + \dot{v}_0^2 + \dot{w}_0^2\right) + I\left(\dot{\theta}_x^2 + \dot{\theta}_y^2\right) \right] dA \quad (20)$$

where $p = \int_{-h/2}^{h/2} \rho(z)dz,\quad I = \int_{-h/2}^{h/2} z^2 \rho(z)dz$ and $\rho(z)$ is mass density which varies through the thickness of the plate and is given by Eq. (5). A dot over the variable denotes derivative with respect to time.

The plate is subjected to temperature filed and this, in turn, results in-plane stress resultants $\left(N_{xx}^{th}, N_{yy}^{th}, N_{xy}^{th}\right)$. Thus, the potential energy due to pre-buckling stresses $\left(N_{xx}^{th}, N_{yy}^{th}, N_{xy}^{th}\right)$ developed under thermal load can be written as

$$V(\delta) = \int_A \left\{ \frac{1}{2}\left[N_{xx}^{th}\left(\frac{\partial w}{\partial x}\right)^2 + N_{yy}^{th}\left(\frac{\partial w}{\partial y}\right)^2 + 2N_{xy}^{th}\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right) \right. \right.$$
$$+ \frac{h^3}{24}\left[N_{xx}^{th}\left\{\left(\frac{\partial \theta_x}{\partial x}\right)^2 + \left(\frac{\partial \theta_y}{\partial x}\right)^2\right\} + N_{yy}^{th}\left\{\left(\frac{\partial \theta_x}{\partial y}\right)^2 + \left(\frac{\partial \theta_y}{\partial y}\right)^2\right\} \right.$$
$$\left.\left.\left. + 2N_{xy}^{th}\left\{\left(\frac{\partial \theta_x}{\partial x}\right)\left(\frac{\partial \theta_x}{\partial y}\right) + \left(\frac{\partial \theta_y}{\partial x}\right)\left(\frac{\partial \theta_y}{\partial y}\right)\right\}\right]\right]\right\} dA \quad (21)$$

Substituting Eqs. (19-21) in Lagrange's equation of motion, one obtains the governing equations as

$$([M]\{\ddot{\delta}\} + ([K] + [K_G]^{th})\{\delta\} = 0 \quad (22)$$

where [M] is the consistent mass matrix and $\{\ddot{\delta}\}$ is the acceleration vector. $[K_G]^{th}$ is the geometric stiffness matrices due to thermal load. $[M]$ is the consistent mass matrix.

For the harmonic vibration $\{\ddot{\delta}\} = -\omega^2\{\delta\}$, Eq. (22) leads to

$$\left([\mathbf{K}] + [\mathbf{K_G}]^{th}\right)\{\delta\} - \omega^2[M]\{\delta\} = \{\mathbf{o}\} \quad (23)$$

where, ω is the natural frequency. The natural frequencies are extracted using standard eigenvalue algorithm.

Element Description

The plate element employed here is a C^0 continuous shear flexible element and needs five nodal degrees of freedom $u_0, v_0, w, \theta_x, \theta_y$ at eight nodes in QUAD-8 element. If the interpolation functions for QUAD-8 are used directly to interpolate the five variables u_0 to θ_y in deriving the shear strains and membrane strains, the element will lock and show oscillations in the shear and membrane stresses. Field consistency requires that the transverse shear strains and membrane strains must be interpolated in a consistent manner. Thus θ_x and θ_y terms in the expressions for $\{\varepsilon_s\}$ given by Eq. (12) have to be consistent with field functions $w_{,x}$ and $w_{,y}$. This is achieved by using field redistributed substitute shape functions to interpolate those specific terms, which must be consistent, as described in Ref. [14]. This element is free from locking syndrome and has good convergence properties. For the sake of brevity, these are not presented here, as they are available in the literature [14]. Since the element is based on field consistency approach, exact integration is applied for calculating various strain energy terms.

Skew Boundary Transformation

For skew plates supported on two adjacent edges, the edges of the boundary elements may not be parallel to the global axes (x, y, z). In such a situation, it is not possible to specify the boundary conditions in terms of the global displacements u_o, v_o, w_o, etc. In order to specify the boundary conditions at skew edges, it is necessary to use edge displacements u_o^l, v_o^l, w_o^l, etc. in local coordinates (x^l, y^l, z^l) as shown in Fig. 1(b). It is thus required to transform the element matrices corresponding to global axes to local edge axes with respect to which the boundary conditions can be conveniently specified. The relation between the global and local degrees of freedom of a node can be obtained through the simple transformation rules [17] and the same can be expressed as

$$d_i = L_g d_i^l \quad (25.a)$$

in which d_i, d_i^l are generalized displacement vectors in the global and local coordinate system, respectively of node i and they are defined as

$$d_i = [u_o \quad v_o \quad w_o \quad \theta_x \quad \theta_y]^T \tag{25.b}$$

$$d_i^l = [u_o^l \quad v_o^l \quad w_o^l \quad \theta_x^l \quad \theta_y^l]^T \tag{25.c}$$

The nodal transformation matrix for a node i, on the skew boundary is

$$L_g = \begin{bmatrix} c & s & 0 & 0 & 0 \\ -s & c & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & c & s \\ 0 & 0 & 0 & -s & c \end{bmatrix} \tag{26}$$

in which $c = \cos(\psi)$ and $s = \sin(\psi)$, where ψ is the angle of the plate. It may be noted that for the nodes, which are not lying on the skew edges, the node transformation matrix has only nonzero values for the principal diagonal elements, which are equal to 1. Thus, for the complete element, the element transformation matrix is written as

$$[T]_e = diag\left\langle L_g \quad L_g \quad L_g \quad L_g \quad L_g \quad L_g \quad L_g \quad L_g \right\rangle \tag{27}$$

For those elements whose nodes are on the skew edges, the element matrices are transformed to the local axes using the element transformation matrix T_e and then the global matrices/vectors are obtained using standard assembly procedures.

Results and Discussion

In this section, we use the above formulation to investigate the effect of parameters like gradient index, aspect and thickness ratios, skew angle, and thermal gradient on the vibration characteristics of functionally graded skew plates. Based on progressive mesh refinement, 8 x 8 mesh idealization is found to be adequate to model the full plate for the present analysis, as shown in Table-1 for isotropic skew case [18]. The non-dimensional frequency used for the validation study is given by: $\varpi = \left(\frac{\omega a^2}{\pi^2 h}\right)\sqrt{\frac{\rho}{E}}$. Fig. 2 shows the variation of the volume fractions of ceramic and metal respectively, in the thickness direction z for the FGM plate. The top surface is ceramic rich and the bottom surface is metal rich. The FGM plate considered here consists of Silicon nitride (Si_3N_4) and stainless steel (SUS304). The temperature coefficients corresponding to Si_3N_4 / SUS304 are listed in Table-2 [19]. The mass density and thermal conductivity are: $\rho_c = 2370\ kg/m^3$, $K_c = 9.19\ W/mK$ for Si_3N_4; and $\rho_m = 8166\ kg/m^3$, $K_m = 12.04\ W/mK$ for SUS304. Poisson's ratio ν is assumed to be a constant and equals to 0.28. Transverse shear coefficient is taken as 0.91. The plate is of uniform thickness and boundary conditions considered here for the simply supported case are:

$u_0^L = w_0^L = \theta_Y^L = 0$ on $x^L = 0, a$ and $v_0^L = w_0^L = \theta_X^L = 0$ on $y^L = 0, b$

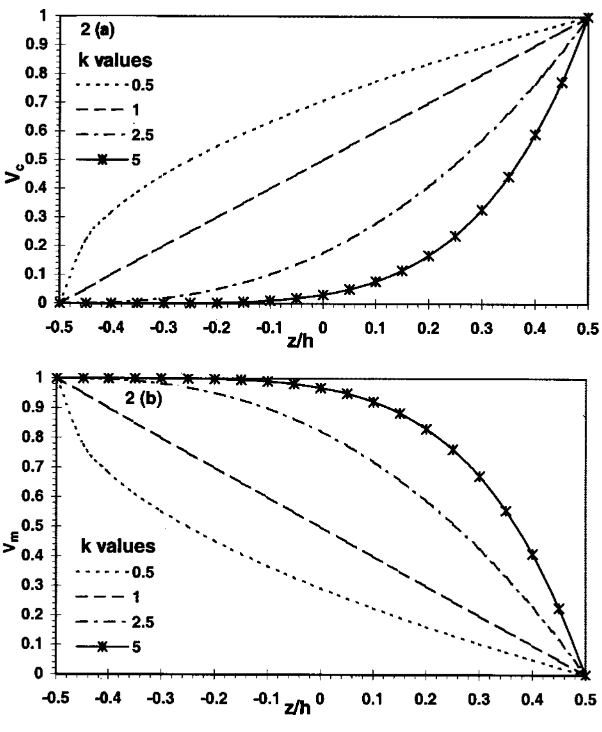

Fig. 2 Variation of volume fractions through thickness: a) Ceramic; b) Metal

Table 1. Validation of Free Vibration of Isotropic Skew Plates

Skew angle	Mesh size	Frequencies, $\overline{\omega}_i$ *					
		1	2	3	4	5	6
0°	4x4	2.0066	5.0988	5.0988	8.4122	10.7563	10.7563
	6x6	2.0013	5.0197	5.0197	8.0817	10.1504	10.1504
	8x8	1.9999	4.9996	4.9996	7.9986	9.9964	9.9964
	Wang [18]	2.0000	5.0000	5.0000	8.0000	10.0000	10.0000
30°	4x4	2.5856	5.4616	7.5844	9.1233	13.4879	13.8567
	6x6	2.5583	5.3601	7.3875	8.6366	12.6864	12.8799
	8x8	2.5310	5.3327	7.2850	8.4950	12.4360	12.4384
	Wang [18]	2.5293	5.3333	7.2815	8.4967	12.4445	12.4446
45°	4x4	3.7986	6.9615	11.3642	11.7695	17.1202	19.1882
	6x6	3.7094	6.7667	10.4442	11.3419	14.9844	17.5681
	8x8	3.5903	6.7141	10.1740	10.9937	14.2508	17.0383
	Wang [18]	3.5800	6.7154	10.1759	10.9724	14.2675	17.0530

$$*\,\overline{\omega} = \left(\frac{\omega b^2}{\pi^2 h}\right)\sqrt{\frac{\rho}{E}}$$

Table 2 Temperature dependent coefficients for material Si_3N_4/SUS304, Ref. [19]

Materials	Properties	P_0	P_{-1}	P_1	P_2	P_3	P (T=300K)
Si_3N_4	E (Pa)	348.43e+9	0.0	-3.070e-4	2.160e-7	-8.946e-11	322.2715e+9
	α (1/K)	5.8723e-6	0.0	9.095e-4	0.0	0.0	7.4746e-6
SUS304	E (Pa)	201.04e+9	0.0	3.079e-4	-6.534e-7	0.0	207.7877e+9
	α (1/K)	12.330e-6	0.0	8.086e-4	0.0	0.0	15.321e-6

Table 3 Comparison of non-dimensional frequencies of simply supported FGM plate (a/b=1, a/h=8)

Temperature	k	Frequencies					
		Ω_{11}		Ω_{12}		Ω_{22}	
		Ref [20]	Present	Ref [20]	Present	Ref [20]	Present
Tc=400 Tm=300	0.0	12.397	12.311	29.083	29.016	43.835	44.094
	0.5	8.615	8.483	20.215	19.979	30.530	30.391
	1.0	7.474	7.444	17.607	17.511	26.590	26.648
	2.0	6.693	6.679	15.762	15.706	23.786	23.894
	10.0	---	5.742	---	13.560	---	20.609
Tc=600 Tm=300	0.0	11.984	11.888	28.504	28.421	43.107	43.343
	0.5	8.269	8.150	19.784	19.534	29.998	29.836
	1.0	7.171	7.131	17.213	17.101	26.109	26.139
	2.0	6.398	6.376	15.384	15.314	23.327	23.410
	10.0	---	5.423	---	13.146	---	20.100

Table-4 : Natural Frequencies $\left(\overline{\Omega}_i\right)$ of Simply Supported Thin FGM Skew Plates under Uniform Temperature against Aspect Ratio and Gradient Index, K for a/h=100, T_{cer}=300, T_{met}=300

a/b	Skew angle	Mode No.	gradient index, *k*					
			0	0.5	1	2	5	10
1	0°	1	45.6450	31.5100	27.6770	24.8770	22.5760	21.4680
		2	114.4200	78.8110	69.1050	62.0810	56.4450	53.7540
		3	114.4200	78.8110	69.1050	62.0810	56.4450	53.7540
	15°	1	48.3280	33.3640	29.3070	26.3420	23.9050	22.7310
		2	111.8000	77.0130	67.5310	60.6690	55.1580	52.5260
		3	130.3200	89.7610	78.7060	70.7070	64.2870	61.2220
	30°	1	58.3410	40.2800	35.3840	31.8050	28.8600	27.4410
		2	122.1700	84.1670	73.8130	66.3140	60.2830	57.4010
		3	168.3200	115.9400	101.6600	91.3320	83.0370	79.0750
	45°	1	84.5690	58.3880	51.2920	46.1030	41.8350	39.7780
		2	154.2000	106.2500	93.1970	83.7330	76.1040	72.4550
		3	237.8000	163.9100	143.8000	129.2000	117.4000	111.7500
	60°	1	164.4100	113.5000	99.6990	89.6110	81.3190	77.3260
		2	246.6300	170.0200	149.1700	134.0400	121.7800	115.9100
		3	361.5700	249.3100	218.7900	196.5900	178.5700	169.9400
2	0°	1	114.0500	78.7520	69.1870	62.1900	56.4260	53.6480
		2	182.8400	126.0400	110.5800	99.3600	90.2780	85.9290
		3	299.4400	206.3300	180.9800	162.5900	147.7800	140.6900
	15°	1	121.2500	83.7290	73.5610	66.1220	59.9920	57.0380
		2	191.5500	132.0500	115.8700	104.1100	94.5890	90.0280
		3	309.2400	213.1000	186.9200	167.9300	152.6200	145.3000
	30°	1	147.7300	102.0100	89.6290	80.5670	73.0940	69.4910
		2	223.5100	154.1100	135.2400	121.5200	110.3900	105.0500
		3	345.6000	238.1800	208.9400	187.7300	170.5900	162.4000
	45°	1	215.2800	148.6700	130.6200	117.4100	106.5200	101.2700
		2	304.4300	209.9700	184.3000	165.6100	150.4000	143.1100
		3	437.9200	301.8900	264.8800	238.0000	216.2200	205.8000
	60°	1	413.4300	285.5000	250.8400	225.4700	204.5500	194.4700
		2	539.5100	372.2400	326.8200	293.7000	266.6400	253.6400
		3	706.3000	487.1300	427.5600	384.1800	348.8800	331.9700

Before proceeding for the detailed study for the free flexural vibration of functionally graded skew plates, the formulation developed herein is validated against the available results [20] pertaining to the free vibrations square FGM plates in Table-3. Here, the calculated non-dimensional linear frequency is defined as $\Omega = \omega\left(\frac{a^2}{h}\right)\left(\frac{\rho_m\left(1-\nu^2\right)}{E_m}\right)^{1/2}$, where ρ_m and E_m are the mass density and Young's modulus of metal, respectively. The results are found to be in good agreement with the existing solutions.

Table-5 : Natural Frequencies $\left(\overline{\Omega}_i\right)$ of Simply Supported Thin FGM Skew Plates under Uniform Temperature against Aspect Ratio and Gradient Index, K for a/h=20, T_{cer}=300, T_{met}=300

a/b	Skew angle	Mode No.	gradient index, *k*					
			0	0.5	1	2	5	10
		1	45.2580	31.2560	27.4440	24.6650	22.3810	21.2820
	0°	2	112.0100	77.1820	67.6510	60.7630	55.2280	52.5940
		3	112.0100	77.1820	67.6510	60.7630	55.2280	52.5940
		1	47.8940	33.0780	29.0450	26.1050	23.6860	22.5220
	15°	2	109.5000	75.4550	66.1400	59.4070	53.9940	51.4170
		3	127.2000	87.6510	76.8270	69.0030	62.7140	59.7230
		1	57.6990	39.8380	34.9960	31.4530	28.5360	27.1320
1	30°	2	119.4300	82.2760	72.1530	64.8080	58.8950	56.0800
		3	163.1200	112.3700	98.5330	88.4920	80.4140	76.5760
		1	83.1370	57.4040	50.4270	45.3180	41.1090	39.0870
	45°	2	149.8600	103.2600	90.5690	81.3470	73.9080	70.3670
		3	227.4000	156.7600	137.5300	123.5200	112.1600	106.7600
		1	158.6500	109.5500	96.2230	86.4450	78.3900	74.5390
	60°	2	235.6100	162.3900	142.4600	127.9300	116.1800	110.6000
		3	337.4500	232.7200	204.2100	183.3700	166.3900	158.3500
		1	111.6900	77.1580	67.7620	60.8980	55.2350	52.5140
	0°	2	176.8900	121.9800	106.9800	96.0970	87.2720	83.0680
		3	283.5600	195.5000	171.4300	153.9400	139.7900	133.0800
		1	118.5900	81.9280	71.9530	64.6630	58.6480	55.7580
	15°	2	185.0300	127.6100	111.9200	100.5300	91.2920	86.8910
		3	292.3800	201.5900	176.7700	158.7400	144.1400	137.2100
		1	143.7800	99.2990	87.2460	78.4040	71.1010	67.5940
2	30°	2	214.6500	148.0000	129.8800	116.6500	105.9100	100.7900
		3	324.7900	223.8900	196.4200	176.3700	160.1200	152.4100
		1	206.9200	142.9100	125.5600	112.8200	102.2900	97.2440
	45°	2	287.9700	198.6200	174.3100	156.5400	142.0600	135.1800
		3	405.1500	279.3600	245.1200	220.0700	199.7000	190.0600
		1	383.9800	265.2300	233.0300	209.3000	189.6500	180.3000
	60°	2	489.3300	337.4100	295.9400	265.4800	240.8100	229.3100
		3	537.5700	374.4800	326.7800	289.4800	258.8300	246.4000

Next, the free vibration behavior of FGM thin skew plate (a/h=100) subjected uniform temperature is investigated considering two values of aspect ratio and the results are presented in Table-4. Here, the calculated non-dimensional linear frequency is defined as $\overline{\Omega}=\omega\left(\frac{a^2}{h}\right)\left(\frac{12\rho_m\left(1-\nu^2\right)}{E_m}\right)^{1/2}$, where ρ_m and E_m are the mass density and Young's modulus of metal, respectively. It can be noticed that the frequency increases with increasing in skew angle, as expected. It is also observed that, with the increase in power law index k up to certain value, the rate of decrease in the frequency value is high, and further increase in k leads to less reduction in the frequency, i.e. monotonically

Table-6 : Natural Frequencies $\left(\overline{\Omega}_i\right)$ of Simply Supported Thin FGM Skew Plates under Uniform Temperature against Aspect Ratio and Gradient Index, K for a/h=20, T_{cer}=700, T_{met}=300

a/b	Skew angle	Mode No.	gradient index, *k*					
			0	0.5	1	2	5	10
1	0°	1	29.4420	17.8590	14.3350	11.3190	7.6342	4.3255
		2	96.5150	64.7180	55.8030	49.1390	43.2740	40.1020
		3	96.5150	64.7180	55.8030	49.1390	43.2740	40.1020
	15°	1	32.2480	19.9020	16.2070	13.1180	9.5720	6.8399
		2	94.0360	63.0020	54.2980	47.7840	42.0350	38.9180
		3	111.5100	75.0900	64.9150	57.3410	50.7450	47.2320
	30°	1	42.5450	27.2540	22.8490	19.3280	15.7240	13.4180
		2	103.8300	69.7510	60.2580	53.1500	46.9210	43.5800
		3	146.9400	99.5320	86.4080	76.6650	68.3190	63.9800
	45°	1	68.5670	45.4990	39.0470	34.0990	29.5090	26.9260
		2	133.7900	90.4450	78.4400	69.5010	61.7830	57.7400
		3	209.9800	143.0300	124.6000	110.9600	99.3780	93.4970
	60°	1	143.8100	97.6650	84.9740	75.4780	67.2230	62.9620
		2	217.9000	148.5000	129.4000	115.2600	103.2700	97.1800
		3	317.7300	217.3800	189.8600	169.4900	152.3200	143.7700
2	0°	1	96.1740	64.6740	55.9120	49.2770	43.2640	39.9890
		2	160.3500	108.8500	94.5790	84.0060	74.9310	70.2380
		3	265.0100	181.0000	157.8200	140.7600	126.4100	119.2000
	15°	1	102.9900	69.4000	60.0720	53.0240	46.6720	43.2400
		2	168.3600	114.3800	99.4430	88.3770	78.8910	74.0020
		3	273.6500	186.9700	163.0600	145.4600	130.6600	123.2400
	30°	1	127.8700	86.5880	75.2220	66.6580	59.0600	55.0410
		2	197.5000	134.4700	117.1300	104.2700	93.2940	87.6910
		3	305.4300	208.8300	182.3300	162.7500	146.3000	138.0800
	45°	1	190.0200	129.5500	112.9800	100.5900	89.8230	84.2920
		2	269.5900	184.2600	160.8600	143.5200	128.8400	121.4600
		3	384.1900	263.2000	230.0500	205.5500	185.0000	174.8300
	60°	1	363.6700	249.4600	218.3000	195.0900	175.2800	165.4200
		2	467.1200	320.5700	280.4000	250.6400	225.7500	213.5700
		3	527.4300	367.2900	320.3300	283.5000	253.1400	240.7600

decreasing trend. This is attributed due to the stiffness degradation occurs because of the increase in the metallic volumetric fraction. It is further seen that the rate of decrease in frequency value is almost same even for the higher modes. The rate of change in frequencies obtained here with respect to gradient index k is somewhat similar for all skew angles considered for the analysis. Tables-5 and 6 exhibit qualitatively similar variation in frequency for fairly thick FGM skew plates (a/h=20) for both uniform and non-uniform surface temperature cases. It can be viewed from Tables-4 and 5 that the non-dimensional frequency decreases with the increase in thickness of the FGM plate. Furthermore, it is revealed from Tables 5 and 6 that the increase in surface temperature difference results in reduction of the natural frequency values, as expected, and its effect is significant with the increase in gradient index due to change in the stiffness of FGM plate. Similar trend is highlighted while varying the aspect ratio. It is opt to mention here that the critical temperature difference □T_{cr} (= T_c-T_m) of FGM plate decreases with the increase in the value of *k*, and increases with the aspect ratio and skew angle of the plate.

Conclusion

Vibration behavior of functionally graded skew plates subjected to thermal environment is examined using eight-noded plate element based on shear flexible theory. Numerical experiment conducted here highlights that the parameters such as gradient index, aspect and thickness ratios, and skew angle can significantly influence the vibration characteristics of the FGM plate.

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