QUALITY ASSISTED THREE FRINGE PHOTOELASTICITY FOR AUTONOMOUS EVALUATION OF TOTAL FRINGE ORDER

K. Ashokan* and K. Ramesh**

Abstract

Three fringe photoelasticity (TFP) is a technique for determining the total fringe order from a single colour isochromatic fringe field. In TFP, the total fringe order is obtained by comparing the colour of the unknown photoelastic fringe pattern with a calibration specimen. Comparison is conventionally done by minimising the colour difference error. This leads to incorrect evaluation of fringe order in some regions showing itself as noise when these values are plotted as an image. In this paper, a new autonomous quality based approach is developed to identify and remove this noise. The method is explained by evaluating the total fringe order for the problem of a disc under diametral compression. The application of this methodology is demonstrated by solving two other example problems.

***Keywords**: isochromatics, three fringe photoelasticity, total fringe order, quality map*

Introduction

Digital Photoelasticity is a whole field technique, which provides the information of the magnitude of principal stress difference (isochromatics) and the principal stress direction (isoclinics) at every pixel in the image domain based on intensity processing [1]. Among the various experimental techniques, Photoelasticity has the unique advantage of providing stress information in colour. This has been exploited by the development of techniques such as Three Fringe Photoelasticity (TFP) [2] / RGB Photoelasticity (RGBP) [3]. TFP is one of the techniques in digital photoelasticity which uses a single image acquired in white light to get the total fringe order. In TFP, the colour information is used to assign approximately the total fringe order. The colours tend to merge beyond fringe order three and hence the technique is termed as three fringe photoelasticity. Since R, G and B values of a colour image are used, it is also termed as RGB photoelasticity. Since TFP gives the total fringe order from a single colour isochromatic fringe field, it is very much helpful in situations where one attempts to analyse a time varying phenomena. TFP also comes in handy for situations where the number of fringes available for data interpretation is only a few such as in stress frozen slices.

One of the difficulties faced in three dimensional photoelasticity is the fabrication of 3D models. With the developments in Rapid Prototyping (RP) particularly the method of Stereolithography is attractive to photoelasticians as the model generated is directly useful for conducting a detailed three dimensional photoelastic studies. The Computer Aided Design (CAD) model used for model generation in RP can also be used for numerical modeling of the problem by Finite Element (FE) analysis. The recent developments on post processing of FE results [4, 5] for photoelastic fringe plotting has provided a convenient method for comparing the results with the experimental technique of photoelasticity. Although numerical analysis is suitable for a parametric study, in complex problems the numerical modeling has to be verified with experiments at least for one set of parameters. Thus, fringe order evaluation by TFP assumes significance to effect this comparison easily. Another variation of photoelasticity to study prototypes made of different materials is the use of bi-refringent coatings. Here again, the photoelastic fringes are not many and use of TFP to analyse such problems is gaining prominence. The focus of this paper is to present a reliable method to evaluate total fringe order by TFP effectively.

Ramesh et al. in 1996 [2] attempted to attenuate the noise present in the total fringe order obtained by TFP. They observed that the scanning direction plays an important role in noise removal. Govindarajan [6] in 1997 used

* Research Scholar ** Professor and Head
Department of Applied Mechanics, Indian Institute of Technology Madras, Chennai-600 036, India; Email : kramesh@iitm.ac.in
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B-spline curves for data smoothing. Though it improved the results to some extent, it required more than one image for data interpretation. Ajovalasit et al. [3] proposed the use of normalised RGB values in TFP for smoothing the intensity data. Quiroga et al. [7, 8] have extended this approach and introduced a regularisation term to minimise noise in the determination of fringe order.

Madhu et al. [9] proposed a new noise immune colour difference formula for total fringe order evaluation using TFP. But the method requires the user to select a seed point in a region free of noise and a factor '*K*' which is problem dependent. The entire procedure is christened as refined TFP (RTFP). A greater simplification to extend the use of TFP to industrial problems is brought about by Madhu et al. recently [10], in which they have proposed a colour adaptation technique, which can in practice use just one colour calibration table for the use of different application specimen materials. The procedure results in some noise which is removed by employing RTFP.

In this paper, a new approach is proposed to identify and remove the noise autonomously based on a quality measure. The models used for this study have been taken from various batches of materials over a period of time. Thus there will be some tint variation which will require the use of colour adaptation approach recently developed, and the resulting noise is removed by the new approach. For completeness, the methodologies of TFP, the colour adaptation approach and quality map are briefly summarised.

Methodology of Conventional TFP

In TFP/RGBP the fringe order at a given data point is determined by comparing the RGB values of that point with the calibrated RGB values assigned with known fringe orders. For any data point, an error term '*e*' is defined as [1]

$$e = \sqrt{(R_e - R_c)^2 + (G_e - G_c)^2 + (B_e - B_c)^2} \tag{1}$$

where, subscript '*e*' refers to the experimentally measured RGB values for the data point and '*c*' denotes the RGB values in the calibration table. The calibration table is to be searched until the error e is a minimum. For the R_c, G_c and B_c values thus determined, the calibration table provides the total fringe order. Ideally, RGB values have to be unique for any fringe order. However, in view of experimental difficulties, the RGB values corresponding to a data point may not exactly coincide with the RGB values in the calibration table. The calibration table is usually prepared from a specimen with known fringe order variation such as a beam under four point bending. In this, the fringe order is zero at the centre of the specimen and increases linearly to a maximum at the edges. The beam is loaded such that the edge is at a fringe order of 3 as beyond fringe order three, the colours tend to merge. From the image recorded, a calibration table is prepared containing the R, G and B values of each pixel along with its fringe order.

Colour Adaptation Approach

Since TFP use color information, any tint variation in the specimen due to aging or thermal cycling such as annealing or stress freezing can affect the performance unless a fresh calibration table is made for that tint. Madhu et al. [10] proposed the concept of an equivalent calibration table. In this, the tint of the application specimen is recorded in unloaded condition and let their average RGB values be R_{ze}, G_{ze} and B_{ze}. The tint of the specimen from which calibration table is made is also recorded in unloaded condition and let their RGB values be R_{zc}, G_{zc} and B_{zc}. From these measurements, the RGB intensities of the equivalent calibration table is obtained as [1]

$$R_{mi} = \frac{R_{ze}}{R_{zc}} R_{ci} \; ; \; G_{mi} = \frac{G_{ze}}{G_{zc}} G_{ci} \; ; \; B_{mi} = \frac{B_{ze}}{B_{zc}} B_{ci} \tag{2}$$

where R_{mi}, G_{mi} and B_{mi} are the RGB values in the modified calibration table at i^{th} row, R_{ci}, G_{ci} and B_{ci} are the RGB values at i^{th} row of the original calibration table. The fringe order *N* for each row is not modified in the equivalent table. The modified table thus obtained serves as the calibration table for evaluating Eq. (1) for every point in the model.

Noise in Conventional TFP

Any noise present in the calculated value of the total fringe order can be visually appreciated if the fringe order over the domain is represented as an image. The fringe order data is represented as grey level image by using the following equation

$$g(x, y) = \text{INT}\left[\frac{255}{B} \times f(x, y)\right] = \text{INT}[R] \tag{3}$$

where $f(x, y)$ is the fringe order at point (x, y), *B* is the maximum fringe order of the calibration table (three in most cases), $g(x, y)$ is the grey level value at the point (x, y) and INT [*R*] is the nearest integer of *R*.

Quality Map

There have been several statistical measures proposed that decides on the reliability of the phase value of a pixel. The application of this to the whole domain results in a map called quality map. These are arrays of values that indicate the quality of the phase data. The pixels in the phase map are scaled to yield quality values that range from 0 to 1 where ‘0’ refers to the lowest quality and ‘1’ denotes the highest quality. Among the available quality maps, use of phase derivative variance to obtain quality map has been used in this work as it is found to be satisfactory for processing isochromatic data in phase shifting techniques [11].

In phase derivative variance the quality is obtained as

$$\frac{\sqrt{\Sigma\,(\Delta^{x}\,i,j-\overline{\Delta}^{x}\,m,n)^2}+\sqrt{\Sigma\,(\Delta^{y}\,i,j-\overline{\Delta}^{y}\,m,n)^2}}{k^2} \qquad (4)$$

where for each sum, the indices (i,j) range over the $k \times k$ neighborhood of each center pixel (m,n) and $\Delta^x\ i,j$ and $\Delta^y\ _{i,j}$ are the partial derivatives of the total fringe order. The terms $\overline{\Delta}^x\ _{m,n}$ and $\overline{\Delta}^y\ _{m,n}$ are the averages of these partial derivatives in the $k \times k$ windows. The phase derivative variance map indicates the "badness" rather than the goodness of the fringe order data. The quality value is negated consequently to represent goodness.

The quality map is represented as a grey level image by using the following equation

$$g\,(x,y) \;=\; \mathrm{INT}\left[\frac{255}{B-A}\times(q\,(x,y)-A)\right] \;=\; \mathrm{INT}\,[R] \qquad (5)$$

where $q\,(x, y)$ is the quality value, B is the maximum quality value (one in most cases), A is the minimum quality value, $g\,(x, y)$ is the grey level value at the point (x, y) and INT $[R]$ is the nearest integer of R.

Quality Assisted TFP

An epoxy circular disc under diametral compression (diameter = 60 mm, thickness = 6 mm, load = 492 N, F_σ = 11.54 N/mm/fringe) is chosen to explain the methodology of quality assisted TFP. Dark-field colour isochromatics of the circular disc under diametral compression is recorded using a 3CCD colour camera (Sony XC-003) which is shown in Fig. 1a. The fringe orders have been evaluated in the whole model using Eq. (1). Instead of preparing a calibration table afresh for this model material, the calibration table already available for epoxy material in the laboratory is used for obtaining an equivalent calibration table. An approach like this helps in simplifying the experimental efforts and also accounts for tint variation due to aging of epoxy sheet over a period of time. Boundary data extracted from a circle drawn around the boundary is used to restrict the processing of data within the model domain. The fringe orders obtained are represented as a grey level image in Fig. 1b. Ideally one should get a smooth variation of grey levels. However, this is not the case in Fig. 1b. The distinct patches are due to incorrect evaluation of fringe orders in these zones [9]. From the phase values obtained, quality values of the pixels in the domain are calculated and the quality map is plotted as a grey level image.

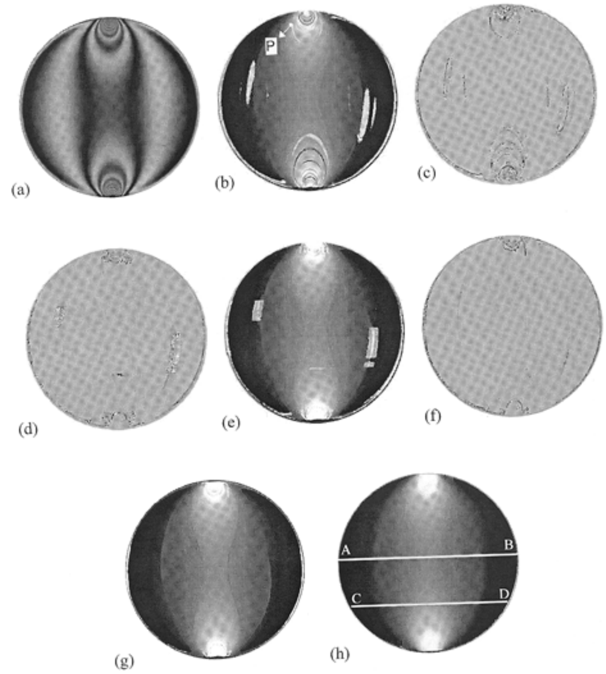

Fig.1 Disc under diametral compression
(a) Dark field colour isochromatics. Image representation of
(b) Total fringe order variation obtained by conventional TFP
(c) Initial quality map (d) Quality map after 5 iterations
(e) Fringe order variation after 5 iterations
(f) Quality map after 20 iterations
(g) Fringe order variation after 20 iterations
(h) Fringe order variation obtained after smoothing

Fig. 1c shows the quality map obtained from the fringe order data. It is observed from Fig. 1c that the quality map identifies the boundary of the noisy regions as low quality regions. However, the pixels inside the erroneous regions have been identified as high quality regions. This is because of uniform variation in the fringe order data of the pixels inside the erroneous regions. On scrutiny it is found that the quality values obtained in high quality regions are close to 1 (greater than 0.9999) whereas the values of low quality pixels are less than 0.99. The value of low quality pixels varies and the variation depends on the number of bad quality neighborhood pixels in the 3 x 3 window considered. In order to remove the noise, in this paper a method is proposed to improve the values of low quality pixels in the quality map by selecting the suitable fringe order from the calibration table with the use of quality values of the neighborhood pixels which is discussed next.

The fringe order data is scanned pixelwise starting from the origin (0, 0) along the horizontal direction leaving the boundary pixels. The pixels inside the model domain having low quality values are identified. For example, in Fig. 1b the pixel at coordinate P (95, 329) having a quality value of 0.9602 is considered (Fig. 2a). The fringe order for this pixel is 2.75 (Fig. 2b). Instead of taking the fringe order corresponding to the least value of 'e' here (Eq. (1)), the error values obtained at point P are arranged as an array of ascending order and from these the first 20 values are taken up for further study which are stored in a separate table called Error Distribution Table (Table-1). The correct fringe order is then selected from this table based on the principle of stress continuity which is adjudged by the new quality value as explained below.

An immediate neighborhood pixel is identified by using the following two conditions (i) quality value is greater than 0.99 (ii) the difference of fringe order data between the neighborhood pixel and the scanned pixel is greater than 0.3. This ensures that a pixel having correct fringe order is selected. In fact the conditions imply that a pixel just outside the incorrect region is selected by this procedure. In this case, the pixel at coordinate Q (95, 327) is found to be having high quality value (0.9999) and the corresponding fringe order is 1.78 (Figs. 2c and 2d).

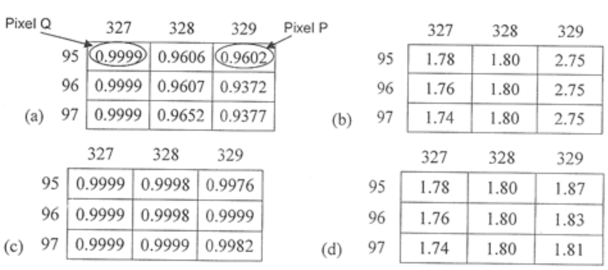

Fig.2 (a) Initial qualaity values for 3 x 3 window
(b) Fringe order corresponding to pixel positions shown in 'a'
(c) Quality values for 3 x 3 window after correction

Following the principle of stress continuity, the current pixel cannot have a fringe order of 2.75 as the difference is quite high. The Error Distribution Table is checked for a fringe order which is closest to this value. For the present case, the fringe order of 1.87 in the Error Distribution Table (Table-1) is found to be the closest and hence it is selected. The quality of the present pixel is recalculated and it is found to be greater than 0.99. The same procedure is applied to all the pixels of the domain having low quality values to complete one iteration. This process is repeated a number of times so that the low quality boundary region gets reduced and removed completely in subsequent steps. In most cases studied, 20 iterations are found to be sufficient. The quality map after 5 iterations and 20 iterations are shown in Figs. 1d and 1f and the corresponding total fringe order variations are shown in Figs. 1e and 1g. One can notice from Fig. 1g that after 20 iterations the erroneous regions have been removed completely except the

Table-1 : Least error values and the corresponding fringe order for the pixel coordinate P (95, 329) shown in Fig.2a

Sl. No.	Least Error Values	Corresponding Fringe Order
1	6.63	2.75
2	6.71	2.76
3	9.27	2.74
4	14.18	2.78
5	16.25	2.79
6	16.31	2.71
7	20.98	2.81
8	23.75	2.82
9	26.34	2.83
10	**26.42**	**1.87**
11	29.14	2.85
12	34.03	2.86
13	37.46	2.88
14	43.46	2.89
15	49.77	2.90
16	51.64	2.92
17	57.18	1.91
18	64.31	2.93
19	66.29	1.93
20	67.08	2.60

regions close to the loading point. The noise removal scheme will not work there because the fringe order is greater than 3 close to the loading point. The fringe orders obtained (Fig. 1g) are then smoothened [12] to get continuous smooth data which is shown in Fig. 1h. Figs. 3a and 3b show the comparison of the total fringe order variation obtained by quality assisted TFP after smoothing with theory [1] and conventional TFP along the lines AB (y/R = 0) and CD (y/R = -0.55) in Fig. 1h respectively. The graph shows that the results obtained by this approach compare well with theory. The average error in fringe order along the lines AB and CD are 0.02 and 0.03 respectively. The maximum error in fringe order along the lines AB and CD are 0.12 and 0.10 respectively. The software for the above said methodology is developed using VC++. The methodology has also been developed along vertical scanning direction so that the combination of horizontal and vertical scanning directions will be useful for effective handling of noise in different classes of problems.

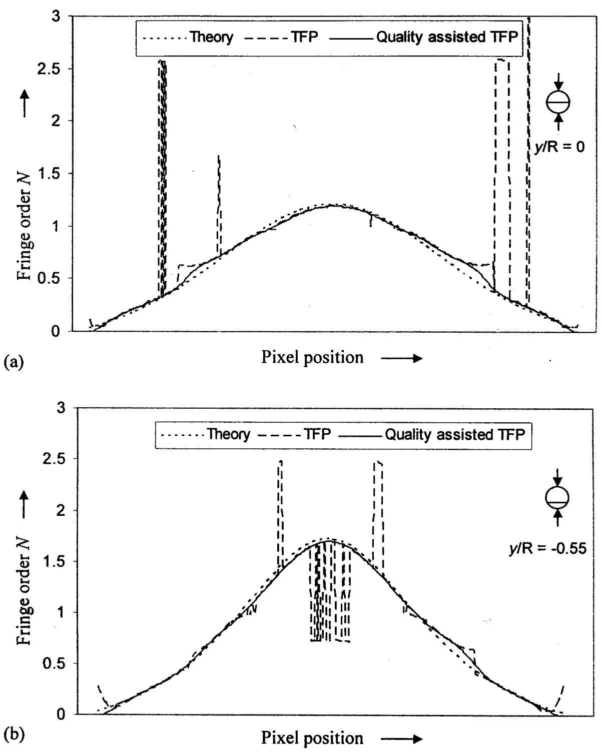

Fig.3 Total fringe order plot for disc under diametral compression by theory, conventional TFP and quality assisted TFP after smoothing along (a) the line AB (y/R = 0) shown in Fig.1h (b) the line CD (y/R = -0.55) shown in Fig.1h

Application of the Methodology to Practical Problems

The technique is extended for evaluating the fringe order in multiply connected bodies such as a ring under diametral compression, plate with a circular hole etc. The multiply connected region is divided into an assembly of simply connected regions by inserting straight line boundaries along horizontal and vertical diameters. The methodology is then applied to each of the simply connected regions.

Ring Under Diametral Compression

Dark field colour isochromatics of an epoxy ring under diametral compression (inner diameter = 40 mm, outer diameter = 80 mm, load = 328 N and F_σ = 11.54 N/mm/fringe) is recorded (Fig. 4a). The total fringe order variation is obtained by conventional TFP using the equivalent calibration table approach as shown in Fig. 4b and from the fringe order calculated, the quality map is evaluated (Fig. 4c). Fig.4c clearly shows that the boundary of the presence of erroneous regions have been identified by quality map. The noisy region is progressively shrunk by the newly developed quality assisted TFP. It is found that the noisy regions are removed after 20 iterations in each of the four simply connected regions considered. The quality map and the corresponding fringe order variation after 20 iterations are shown in Figs. 4d and 4e. Fig.4f shows the fringe order variation obtained after smoothing. The fringe order variation obtained by quality assisted

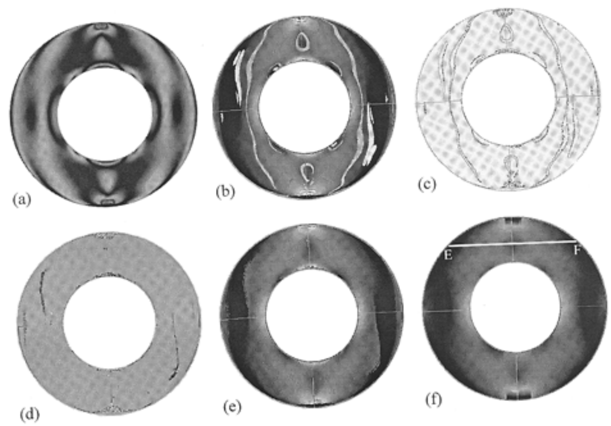

Fig.4 Ring under diametral compression (a) Dark field colour isochromatics. Image representation of (b) Total fringe order variation obtained by conventional TFP (c) Initial quality map (d) Quality map after 20 iterations (e) Fringe order variation after 20 iterations (f) Fringe order variation after smoothing

TFP is compared with theory and conventional TFP along the line EF in Fig.4f (y/R = 0.68) which is shown in Fig.5. The average error and maximum error in fringe order along the line EF are 0.03 and 0.10 respectively.

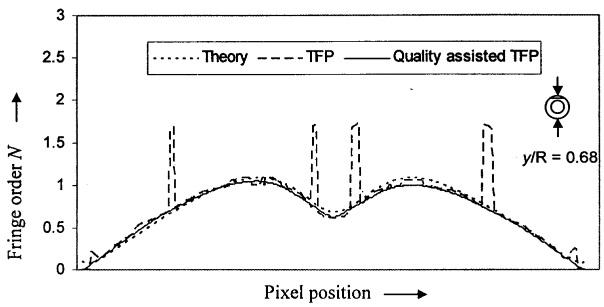

Fig.5 Total fringe order plot for ring under diametral compression by theory, conventional TFP and quality assisted TFP after smoothing along the line EF (y/R = 0.68) showin in Fig.4f

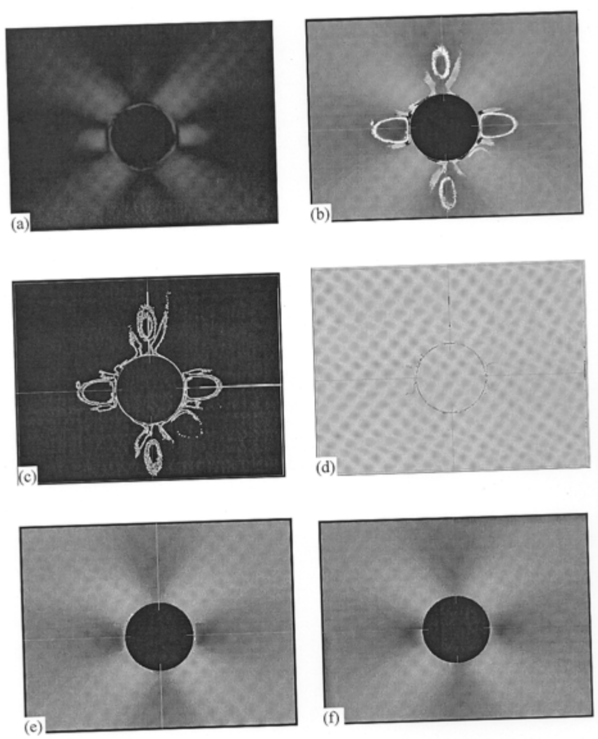

Fig.6 Plate with a circular hole (a) Dark field colour isochromatics. Image representation of (b) Total fringe order variation obtained by conventional TFP (c) Initial quality map (d) Quality map after 20 iterations (e) Fringe order variation after 20 iterations (f) Fringe order variation after smoothing

Plate with a Circular Hole

Figure 6a shows the experimentally grabbed dark field colour isochromatics of an epoxy plate with a circular hole under tensile load (width = 36 mm, height = 230 mm, thickness = 6 mm, load = 492 N). Total fringe order variation obtained by conventional TFP using equivalent calibration table is shown in Fig. 6b. The methodology is applied to the total fringe order data obtained. Initial quality map is shown in Fig. 6c. The results obtained after convergence (20 iterations) are shown in Figs. 6d and 6e. The fringe order variation obtained after smoothing is shown in Fig. 6f.

Conclusion

Autonomous quality assisted approach is proposed for the identification and correction of erroneous region in the total fringe order evaluated using conventional TFP. The methodology is explained by using the benchmark problem of circular disc under diametral compression and the results are comparing well with the theory. Also, the applications of this methodology to the problems of ring under diametral compression and plate with a circular hole are demonstrated.

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