A CP-FDTD PREPROCESSOR IN CARTESIAN AND RECTANGULAR COORDINATES

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Abstract

In this paper we present a versatile mesh generator in Cartesian and Rectangular co-ordinates for the Contour path Finite Difference Time Domain method wherein the grid is adjusted locally to follow the contour of the geometric boundaries. The information about geometry, curved interfaces (dielectric/dielectric, PEC/air etc) material parameters, angles are obtained from the geometry of the intersecting interfaces at the mesh stage itself and does not need any particular area calculation for implementation at the FDTD solver stage. When run on a stair stepped or a contour path FDTD solver, the conformal approximation permits modeling of sloped and curved boundaries, while the stepped approximation permits modeling of linear material media. Additionally to model thin layers smaller than the spatial step size a local sub girding approach that allows fine geometries to be modeled is used at the mesh stage itself. This means structures that are less than the grid discretization such as thin plates and small bumps are incorporated into the mesh at negligible costs.

Nomenclature

B	= magnetic induction vector
CPFDTD	= contour path finite difference time domain method
D	= electric displacement vector
E	= electric field intensity vector with components E_x, E_y, E_z
EMI/ EMC	= electromagnetic interference/electromagnetic compatibility
ε_0	= free space permittivity
μ_0	= free space permeability
Δ	= space step
Δt	= time step
FDTD	= finite difference time domain
H	= magnetic field intensity vector with components H_x, H_y, H_z
i,j,k	= the x, y and z indices of the spatial co-ordinates
PEC	= perfect electric conductor
RCS	= radar cross section
$v_{x,y,x}$	= x, y and z co-ordinates of the vertex in the grid
$t_{x,y,z}$	= x, y and z co-ordinates of the testing point in the grid
ds	= distance between the testing point and the grid point

Introduction

The FDTD scheme, since its inceptive advent in 1966 [1] has established formidable modeling goals and has radically advanced simulation standards for a wide range of electromagnetic scattering, guidance and propagation problems. The FDTD algorithm is a numerical solution of Maxwell's equations where the equations for the algorithm are derived from the differential form of Faraday's and Ampere's laws in conjunction with the constitutive relations $\mathbf{B} = \mu\mathbf{H}$ and $\mathbf{D} = \varepsilon\mathbf{H}$. These are implemented on an array of electrically small, spatially orthogonal contours and the contours *mesh* (intersect) in the manner of links in a chain providing a geometrical interpretation of the coupling of the two laws.

Notwithstanding the algorithmic elegance and its simplicity, the geometric modeling and meshing of the FDTD method however can be time consuming and error prone for manually manipulating complex geometries like an aircraft. Moreover the classical FDTD method presents a principal hindrance in its ability to successfully cope with abrupt curvatures and complicated geometries in open-region problems. It renders the well-known "stair-casing" procedure [2] inadequate to provide consistent numerical simulations and generates severe discretization and lattice reflection errors.

Considerable effort has been expended since the early 1980s to make the FDTD method more versatile and as a

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Manuscript received on 19 Nov 2003, Paper reviewed, revised and accepted on Aug 2004

result two principal versions are now available. The first of these is the *global non-orthogonal FDTD algorithm* [3]. The second is a *Contour Path FDTD* approach [4], which uses a nearest neighbour approximation to truncate the contour locally or the use of local extended contours, which overlap on a predominantly rectangular grid at the curved boundaries or angles. The global curvilinear FDTD method has the advantage of having uniform formulas and therefore the core code is universal. For different practical problems, only the particular metric tensor needs to be specified. Nonetheless, the global curvilinear grid needs many times more memory than the rectangular FDTD to store the metric matrix tensor and it also consumes much more computational time, since many transformations have to performed between convariant and contra-variant components. The CPFDTD technique on the other hand requires information only local to the intersection points between the surface of the target being modeled and the Cartesian/Rectangular grid while retaining the original uniform format for the grid, both in the interior and the exterior of the object. This offers considerable run time savings for large electrical length problems or intricate geometries for a given mesh compared with stepped edge modeling.

In this paper we present a versatile *mesh generator* for the CPFDTD method in *Cartesian and Rectangular Coordinates* with a facility to model either with a body conforming FDTD approximation or a stepped (uniform) approximation. In the CPFDTD approach the geometric boundaries described with curves and angles are not distorted to follow the geometric boundaries, rather the grid is adjusted to follow the contour of the geometric boundaries in two and three dimensions. The approach is similar to the BoR algorithm described by Jurgens and Harfoush [5] for calculating the wake fields in particle accelerators.

The advantages of the mesh generator developed are that information about geometry, curved interfaces (dielectric/dielectric, PEC/air etc) material parameters, angles are obtained from the geometry of the intersecting interfaces at the mesh stage itself and does not need any particular area calculation for implementation at the solver stage. When run on a stair stepped or a contour path FDTD solver, the conformal approximation permits modeling of sloped and curved boundaries, while the stepped approximation permits modeling of linear material media. Additionally to model thin layers smaller than the spatial step size a local sub girding approach that allows fine geometries to be modeled is used. This means structures that are less than the grid discretization such as thin plates and small bumps are incorporated into the mesh at negligible costs.

Theory

In the CPFDTD algorithm the equations for the FDTD updates are derived from Ampere and Faraday's integral laws. The discretization for the electric and magnetic field components in both space and time is the same as for the uniform FDTD and in regions where the curved media interfaces exists, the CPFDTD update algorithms equations are distorted from the uniform mesh. These are done in separate ways for a PEC/free space and for dielectric interfaces as follows:

PEC Curved Surfaces: Figure 1 shows the geometry of the intersection between a curved PEC object and the FDTD mesh. In the CPFDTD technique, the electric field update equations (*Ampere grid*) remain unchanged from that in the uniform scheme, but the magnetic field (*Faraday Grid*) is updated differently. For instance, for the H_z component for the contour C_1 we use:

$$H_z^{n+1/2}(i+1/2, j+1/2) = H_z^{n-1/2}(i+1/2, j+1/2) + \frac{\Delta t}{A\mu_z} x\,(l_2 E_y^n(i+1, j+1/2) - l_1 E_y^n(i, j+1/2) - \Delta E_x^n(i+1/2, j+1)$$

where A is the area within contour C_1. The contour C_1 ignores the unusable E_x field which is replaced by the zero-valued tangential electrical field along the surface of the perfect conductor. l_i are the lengths as shown in Fig.1. In contour C_2 the contour is again distorted to include the zero-valued tangential electric field, ignoring the unusable E_x component. The unusable E_y component is replaced by its collinear neighbour for the portion of the contour represented by length l_3 and so on.

In contrast to the uniform FDTD scheme, the magnetic field update equation above requires the *geometrical information* pertaining to the intersection of the object surface with the Cartesian grid [4]. This is done in the present paper at the mesh stage itself and is obtained from the geometry and the intersecting grid as described in the next section. The interface information in then stored in separate media parameter arrays and used with the FDTD update equations directly instead of calculating the individual cell areas separately at the time of time stepping.

Curved Dielectric Surfaces: The CPFDTD has been generalized in [6] to improve the simulation accuracies of

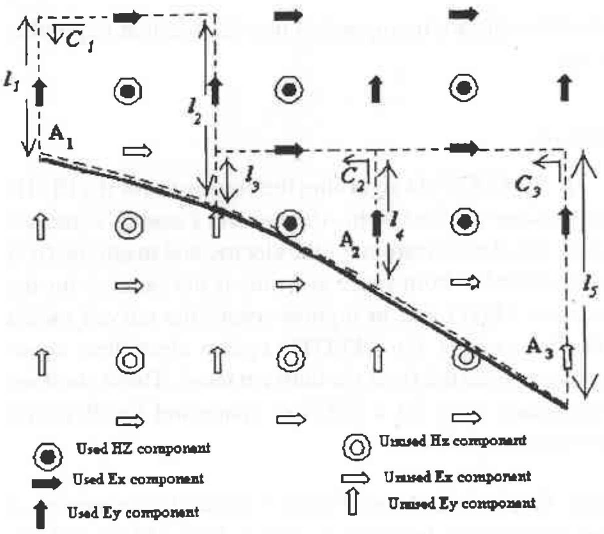


Fig. 1 Two-D configuration of intersection between a curved PEC and Cartesian mesh

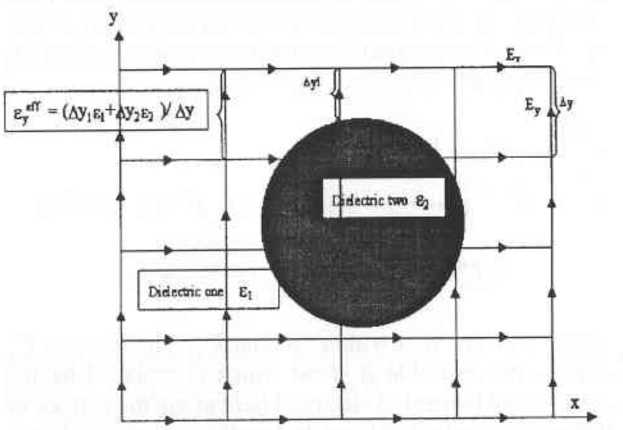


Fig. 2 Two-D configuration of intersection between a curved and Cartesian mesh

dielectric interface geometries (Fig.2). The method is similar to PEC interfaces requires the mesh truncation information of dielectric objects to calculate the effective dielectric constant. Unlike existing conformal techniques for handling dielectric we use the method of Yu and Mittra [6] to calculate and effective dielectric constant (instead of a weighted average procedure) for the electric update equations. The corresponding interface nodes for implementing the appropriate CPFDTD update equations on both sides of the dielectric interface are obtained from the mesh generator directly as described in the next sections.

The Implementation of the Mesh

The mesh is obtained in a five-stage process. Each stage uses a set of routines that constitutes the tools required for the straightforward implementation of the code and serves as an input for the successive stage.

In the first stage the object(s) and the surrounding region(s) are first described by a set of co-ordinates with a prescribed syntax using a text file editor. A Cartesian/Rectangular Co-ordinate system is assumed and the co-ordinates dimensioned are in MKS units. Any number of materials can be described and the material in a region is described by its permeability, permittivity, and conductivity. The cross section area of each homogeneous material defines a *region* and the edge of a region defines a *region boundary. Region boundaries* are described as closed polygons composed of segments: lines, circular and elliptic areas. A region can contain discontinuities whose characteristics in a homogeneous region can be different from its surrounding region. A region can have single or multiple boundaries but are required not to cross or intersect each other. The problem domain size, the discretization size is directly given as inputs in the text file itself.

In the second stage the entire domain is discretized. A classification and identification of cells in a particular region (using separate identifiers), its location on the problem boundary or region boundary, its position on or off the grid axis is found.

A structure MeSHstruc containing details about mesh internals namely points, segments, contours, cells, media types is used for storing mesh information and is generated following the rules given in [4] and [6].

```
1. typedef struct meshinternals {/*contains private mesh data */
PPstruc     problem;    /* problem connectivity data */
RPLstruc  *regplem;  /* region connectivity data */
int      **regorient;  /*region boundary orientations */
PTstruc  **pointtlist;     /*list of point data objects */
SGstruc  **seglist;         /*list of segment data objects */
int          *media_ex;     /* media type array for Ex */
int          *media_ey;     /* media type array for Ey */
int          *media_ez;     /* media type array for Ez */
int          *media_hx;     /* media type array for Hx */
int          *media_hy;     /* media type array for Hy */
int          *media_hz;     /* media type array for Hz */
GGPop      ggop;            /* mesh generator data */
} MeSHstruc
```

To check whether a testing point lies on a vertex of a grid we calculate the distance between the testing point and a grid point by

$$ds = \sqrt{(v_x - t_x)^2 + (v_y - t_y)^2 + (v_z - t_z)^2}$$

where $v_{x,y,x}$ are the x, y and z co-ordinates of the vertex, $t_{x,y,z}$ are the x, y and z co-ordinates of the testing point and ds is the distance between the testing point and the grid.

A small tolerance is set for the distance comparison of the two points. If the distance ds is less than the tolerance, the two points can be considered to be a single point on the grid. The tolerance is chosen to be much smaller than the uniform cell size. All repetitive points are eliminated using simple algebraic checks. For a particular field component (Hz, Ex, Ey, Hx, Hy, Ez) depending on the boundary type (**E** or **H**) termination separate media type arrays with corresponding dimensions and media identifiers are created and stored as separate files. Corresponding to these media types a separate function program in the third stage creates a stepped postscript file for plotting and viewing the *stepped* boundary approximation. The media array data serves as the solver input for the contour path mesh algorithm.

In the fourth stage of the pre-processor development the stepped array data is reconstructed for the contour path approximations. The pertinent information to be obtained are the media parameter arrays in terms of the equivalent cell size for a PEC object or the equivalent material constants for the dielectric ones. These can be determined if we know the intersections of the grid lines with the object surface and can uniquely distinguish between the interior and exterior of the object. The intersection points are determined relatively using the aforementioned approach easily since the geometrical description of the object is available. In the program we find the intersections between a grid line and the edges of the object, sort the intersections by their co-ordinates, pair them and find the grid points located between each pair of intersections. To model thin layers smaller than the spatial step size a local sub girding approach based on the work of Taflove et.al [7] that allows fine geometries to be modeled is used. For this a study of the accuracy of the Faraday's Law contour path model for a narrow slot having a sub cell air gap was repeated following the work of [7] and good co-relation was found with the reported results.

The output of the *Mesh* data is a text file describing the mesh of the problem to be solved. It is often necessary to visualize the mesh described by this output file in order to check its suitability to the problem under examination. Since the package developed in this project is designed with flexibility and portability as main issues, developing graphical routine would bind the user to a specific complier and machine. To overcome this problem, a subroutine that converts the mesh contained in the output mesh description file in a *PostScript* file format is provided. *PostScript* language is a well-known standard common to every platform and can be directly managed by *PostScript* printers. Moreover, the freeware package *GhostView* available for most platforms, provides a simple means of visualizing the *PostScript* files and allows the user to print them even on non-*PostScript* printers. Therefore conversion of the mesh file visualization is done in stage five.

Results and Validation

Figure 3 shows the stepped and contour path mesh obtained for a circular and square cylinder in two and three dimensions. Further the mesh generator developed has been tested and validated for canonical geometries with curved surfaces. The mesh software is integrated with an in-house developed FDTD solver and used to obtain the resonance frequency for different resonator configurations. For the simulation by the FDTD method, second order Mur's boundary conditions are used. The resonance frequency of the resonators is extracted from the return loss information of the DFT transformed data of the FDTD simulations. Table-1 gives results obtained for different mesh configurations and are compared with exact theoretical results for a parallel plate dielectric rod resonator (Fig.4) studied indicating the validity of the software developed.

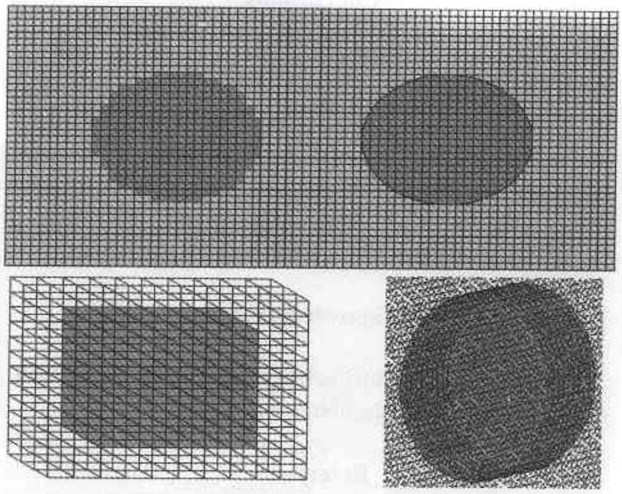


Fig. 3 Stepped and Contour path mesh configuration in Cartesian coordinates for a circle, contour path mesh for a cube and cicular cylinder

Table-1 : Resonance frequency of a parallel plate dielectric rod resonator

Mode	Resonance frequency (GHz)		
	Theoretical	Staircase	CPFDTD
HEM_{111}	6.214	6.208	6.211
HEM_{211}	7.514	7.549	7.511
HEM_{311}	9.003	8.97	8.99

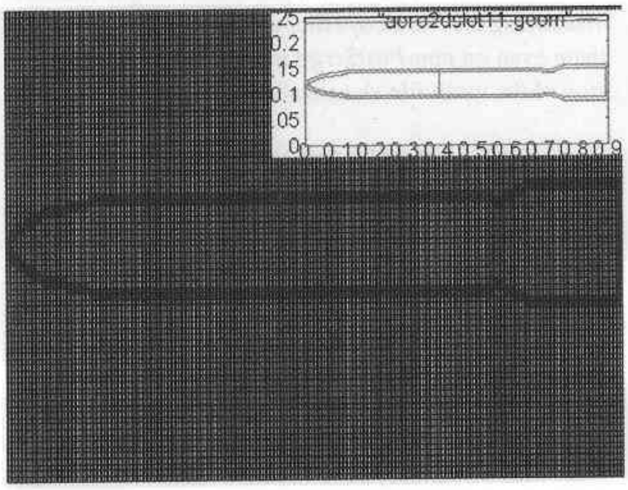
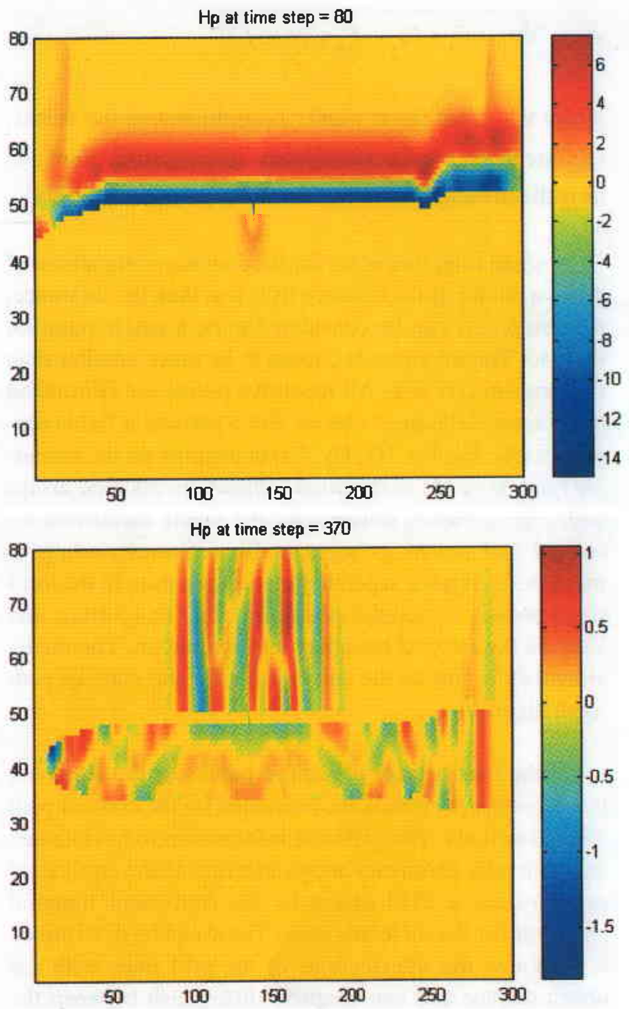


Fig. 4

Figure 5 shows a typical animation obtained at two different time-steps for a launch vehicle with a hole smaller than the discretization size with the developed code. The modal field distribution is easily seen.



Conclusions

A versatile and efficient mesh generator for the CPFDTD method has been presented for modeling bodies with curved surfaces and angles. The mesh when integrated with a CPFDTD solver can be used to solve complex and electrically large structures like aerospace vehicles for the propagating (wave guides), penetrating (EMI/EMC) and radiation (RCS, antenna) problems.

Acknowledgement

The authors gratefully acknowledge the support of AR and DB received for the above work.

References

1. Yee, K.S., "Numerical Solution of Initial Boundary Value Problems Involving Maxwell's Equations in Isotropic Media", IEEE Trans. Antennas Propagat, May 1966, AP-14, pp.302-307.

2. Schneider, J.B. and Kurt L. Shlager., "FDTD Simulations of TEM Horns and the Implications for Staircased Representations", IEEE Trans. Antennas Propagat, December 1997, AP-45, pp.1830-1838.

3. Holland, R., "Finite Difference Solution of Maxwell's Equations in Generalized Non-Orthogonal Co-ordinates", IEEE Trans. Nuclear Science, 1983, NS-30, pp.4589-4591.

4. Jurgens, T.G., Taflove, A., Umashankar, K. and Moore, T.G., "Finite Difference Time Domain Modeling of Curved Surfaces", IEEE Trans. Antennas Propagat, April 1992, AP-40, pp.357-366.

5. Harfoush, F. and Jurgens, T.G., "FDTD Conformal Modeling of 3D Wake Fields", IEEE Particle Accelerator Conference, May 1991, ERA46, San Francisco, California, pp.37-45.

6. Wenhua, Yu. And Raj Mittra, "A Conformal Finite Difference Time Domain Technique for Modeling Curved Dielectric Surfaces", IEEE Microwave and Guided Wave Letters, January 2001, Vol.11, No.1, pp.25-27.

7. Taflove, A., Umashankar, K., Beker, B., Harfoush, F. and Yee, K.S., "Detailed FDTD Analysis of Electromagnetic Fields Penetrating Narrow Slots and Lapped Joints in Thick Conducting Screens", IEEE Trans. Antennas Propagat, 1988, AP-36, pp.247-257.