PARAMETRIC INSTABILITY OF STIFFENED PLATES WITH CUTOUTS

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Abstract

The dynamic instability behaviour of stiffened plates with cutout is of great technical importance for understanding the behaviour of the system. In the present study, dynamic instability behaviour of stiffened plates with cutouts are investigated by using the finite element method. The method of Hill's infinite determinants is applied to analyze the dynamic instability regions. Rectangular stiffened plates with cutouts possessing different boundary conditions, aspect ratios, varying mass and stiffness properties and varying number of stiffeners have been analyzed for dynamic instability. The influence of various parameters like effects of size of cutout, load position, aspect ratios, and boundary condition, other parameters of stiffened plates have been studied. The method highlights interesting effects when stiffened plate with cutout is subjected to uniform in-plane edge load. In the structural modelling, the plate and the stiffener are treated as separate elements where the compatibility between these two types of elements are maintained. The present approach is more flexible than any other finite element modelling in that the mesh division is independent of the location of the stiffeners.

Notations

a,b = dimensions of the plate

g,d = dimensions of cutout in plate

E,G = modulus of elasticity and shear modulus of rigidity

t = thickness of the plate

u,v = in-plane displacements

w = out of plane displacement

q = load intensity

b_s, d_s = web thickness and depth of a stiffner

P_{cr} = critical buckling load

U = upper excitation frequency parameter

L = lower excitation frequency parameter

α ,β = static and dynamic load factor

Introduction

Cutouts are inevitable in aerospace, civil, mechanical and marine structures mainly for. practical considerations. In aerospace structures, cutouts are commonly found as access ports for mechanical and electrical systems, or simply to reduce weight. Cutouts are also made to lighten the loads, provide ventilation and for modifying the resonant frequency of the structures. However, when interior holes are cut from a plate structure, the mechanical behaviour of the structure is changed. The instability effects are improved with the provision of stiffeners. The studies of static and dynamic behavior of stiffened plate elements with cutouts subjected to uniform in-plane stresses are of considerable importance. Structural elements subjected to in-plane periodic forces may induce transverse vibration, which may be resonant for certain combinations of natural frequency of transverse vibration, the frequency of the in-plane forcing function and the magnitude of the in-plane load. The spectrum of values of parameters causing unstable motion is referred to as the regions of dynamic instability or parametric resonance.

Numerical results obtained by the finite element method have been reported by Ali and Atwal [1], Shastry and Rao [2], Reddy [3] and Laura et al. [4]. Ali and Atwal [1] studied the natural frequencies of simply supported rectangular plates and rectangular cutouts using the Rayleigh-Ritz method. Paramsivam [5] used a finite difference approach in analyzing the effects of openings on the fundamental frequencies of plates with simply supported and clamped boundary conditions. A finite element analysis of clamped thin plates with different cutout sizes,

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along with experiments was carried by Monahan et al. [6]. Mundkur et al. [7] studied the vibration of square plates with square cutouts by using boundary characteristics orthogonal polynomials satisfying the boundary conditions. Chang and Chiang [8] studied the vibration of the rectangular plate with an interior cutout by using finite element method. Lee et. al [9] predicted the natural frequencies of rectangular plates with an arbitrarily located rectangular cutout. Ritchie and Rhodes [10] have investigated theoretically and experimentally the behaviour of simply supported uniformly compressed rectangular plates with central holes, using a combination of Raleigh-Ritz and finite element methods.

The studies on vibration characteristics of stiffened plates with cutout are scanty in literature. The free vibration characteristics of unstiffened and longitudinally stiffened square panels with symmetrically square cutout are investigated by Sivasubramonian et. al [11] using the finite element method. A rectangular shell element with seven degrees of freedom per node together with a beam element of seven degree of freedom per node is used for the analysis. Lam and Hung [12] studied the vibrations of plates with stiffened openings using orthogonal polynomials and partitioning method. The method involves partitioning of the plate domain into appropriate rectangular segments in order to approximate the deflection function of each segment by sets of orthogonal polynomials. Natural frequencies of simply supported and fully clamped plates with stiffened openings are presented, Paramsivam and Sridhar Rao [13] modified the grid framework model suitably to obtain the natural frequencies of square plate with stiffened square openings. This investigation provides a general feeling about the changes in fundamental frequencies that occur when stiffeners are introduced.

The present paper deals with the effects of various parameters such as size and location of cutouts, aspect ratios, different boundary conditions and stiffener parameters on dynamic stability characteristics of rectangular stiffened plates with cutouts

The finite element is applied to analyze vibration and dynamic stability behaviour of stiffened plates subjected to uniform edge loading. In the present analysis, the element is nine-nodded isoparametric quadratic, which has advantages such as accommodating irregular boundaries, laminated materials and accounting for shear deformation and rotary inertia. The stiffeners can be positioned anywhere within the plate element and need not necessarily be placed on the nodal lines.

Governing Equation

The equation of equilibrium for vibration of a structure subjected to in-plane loads can be written in matrix form as

$$[M]\{\ddot{q}\}+\left[[K_b]-P\,[K_G]\right]\{q\} = 0 \tag{1}$$

The in-plane load *P(t)* may be periodic and can be expressed in the form

$$P(t) = P_S + P_t \cos \overline{\Omega}\, t \tag{2}$$

$$P_S = \alpha P_{cr},\ P_t = \beta P_{cr} \tag{3}$$

where P_s is the static portion of P. P_t is the amplitude of the dynamic portion of P and $\overline{\Omega}$ is the frequency of excitation. α and β are termed as static and dynamic load factors respectively. The equation of motion is reduced to:

$$[M]\{\ddot{q}\}+\left[K_b]-\alpha P_{cr}\,[K_G]-\beta P_{cr}\,[K_G]\cos\overline{\Omega}\,t\right]\{q\}=0 \tag{4}$$

Equation 4 represents a system of second order differential equations with periodic coefficients of Mathieu-Hill type. The development of regions of instability arises from Floquet's theory, which establishes the existence of periodic solutions. The boundaries of dynamic instability are formed by the periodic solution of period T and 2T, where $T = 2\pi/\overline{\Omega}$.. The boundaries of the primary instability regions with period 2T are of practical importance and the solution can be achieved in the form of trigonometric series. Principal instability region, which is of practical importance leads to

$$\left[[K_b]-\alpha P_{cr}\,[K_G]\pm\frac{1}{2}\beta P_{cr}\,[K_G]-\frac{\overline{\Omega}^2}{4}[M]\right]\{q\}=0 \tag{5}$$

Equation 5 represents an eigenvalue problem for known values of α, β and P_{cr}. The two conditions under a plus and minus sign correspond to two boundaries of the dynamic instability region. The eigenvalues are $\overline{\Omega}$, which give the boundary frequencies of the instability regions for given values of α and β.

Finite Element Formulation

The formulation is based on Mindlin's plate theory, which will allow for the incorporation of shear deformation. The plate skin and the stiffeners are modelled as separate elements but the compatibility between them is maintained. The element matrices of the stiffened plate element consist of the contribution of the plate and that of the stiffener. This is similar to the concept proposed by Mukherjee and Mukhopadhyay [14]. The nine noded isoparametric quadratic element with five degrees of freedom (u, v, w, θ_x, and θ_y) per node is employed in the present analysis.

The elastic stiffness matrix $[K_p]$, geometric stiffness matrix $[K_{Gp}]$ and mass matrix $[M_p]$ of the plate element may be expressed as follows:

$$[K_p] = \int_{-1}^{+1}\int_{-1}^{+1} [B_p]^T [D_p] [B_p] |J_p| \, d\xi \, d\eta \tag{6}$$

$$[K_{Gp}] = \int_{-1}^{+1}\int_{-1}^{+1} [B_{Gp}]^T [\sigma_p] [B_{Gp}] |J_p| \, d\xi \, d\eta \tag{7}$$

$$[M_p] = \int_{-1}^{+1}\int_{-1}^{+1} [N]^T [m_p] [N] |J_p| \, d\xi \, d\eta \tag{8}$$

where

$$[B_p] = \left[[B_p]_1 \;\; [B_p]_2 \;\; \cdots \;\; [B_p]_r \;\; \cdots \;\; [B_p]_9 \right] \tag{9}$$

$$[B_{GP}] = \left[[B_{GP}]_1 \;\; [B_{GP}]_2 \;\; \cdots \;\; [B_{GP}]_r \;\; \cdots \;\; [B_{GP}]_9 \right] \tag{10}$$

The elastic stiffness matrix $[K_s]$, geometric stiffness matrix $[K_{Gs}]$ and mass matrix $[M_s]$ of a stiffener element placed anywhere within a plate element and oriented in the direction of x may be expressed, in a manner similar to those of the plate element as follows:

$$[K_S] = \int_{-1}^{+1} [B_S]^T \, [D_S] [B_S] \, |J_S| \, d\xi , \tag{11}$$

$$[K_{GS}] = \int_{-1}^{+1} [B_{GS}]^T \, [\sigma_S] [B_{GS}] \, |J_S| \, d\xi , \tag{12}$$

$$[M_S] = \int_{-1}^{+1} [N]^T \, [m_S] [N] \, |J_S| \, d\xi , \tag{13}$$

and $|J_s|$ is the Jacobian of the stiffener, which is one-half of its actual length within an element.

The overall elastic stiffness matrix, geometric stiffness matrix and mass matrix are generated from the assembly of those element matrices and stored in a single array where the variable bandwidth profile storage scheme is used. The solution of eigenvalues is performed by the simultaneous iteration technique proposed by Corr and Jennings [15].

Numerical Result and Discussion

The problem considered here consists of a rectangular plate (a x b) with stiffeners having a rectangular cutout of size (g x d) at the center as shown in Fig.1. The plate with stiffener subjected to in-plane uniform edge loading at the plate boundary and stiffener cross-section are shown in Fig.2. All the boundaries of the plates are simply supported unless otherwise stated. In the discussion that follows, S, C denote simply supported, clamped respectively. The notation SCSC identifies a plate with the edges: x = 0, x = a, y = 0, y = b. For comparison problems, the boundary conditions are considered as reported in the respective studies.

The non-dimensionalisation of different parameters are taken as given:

Natural frequency parameter $(\omega) = \bar{\omega} \, b^2 \sqrt{\rho t / D}$, excitation frequency parameter $(\Omega = \bar{\Omega} \, b^2 \sqrt{\rho t / D})$ where D is the plate flexural rigidity, ρ is the density of the plate material and t is the plate thickness. The dynamic instability regions are plotted for rectangular stiffened plates with

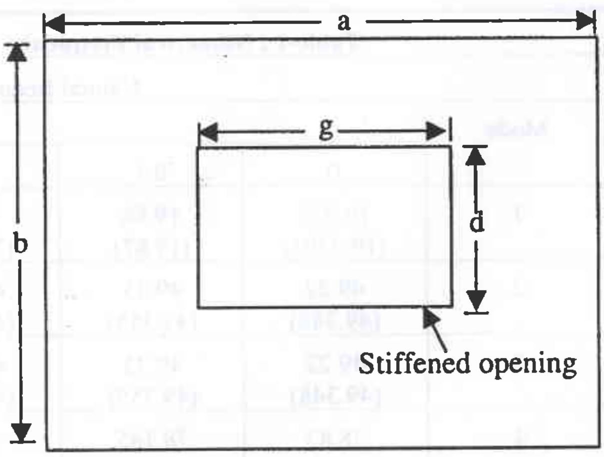
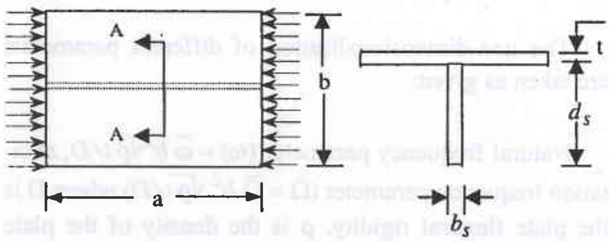


Fig. 1 Rectangular plate with stiffened opening

cutouts subjected to in-plane uni-axial uniform edge loading along stiffener direction as shown in (Fig.2) to consider the effect of static load factor, aspect ratio, boundary condition, effect of cutouts etc on dynamic instability regions. The results are presented in graphical form where the instability region is shown by the upper and lower values of excitation frequency parameter ($\Omega = \bar{\Omega}\, b^2 \sqrt{\rho\, t / D}$). In all these cases, the frequency parameter is plotted against dynamic load factor (β) for different values of the parameters as mentioned above. Here Ω and ω are non-dimensionlized terms, which is used as dynamic excitation frequency parameter and natural frequency parameter respectively. The terms $\bar{\Omega}$ and $\bar{\omega}$ are excitation frequency and natural frequency in rad/s.

In order to validate the results, linear fundamental frequencies obtained in the study can be compared with those available in the literature. Table-1 contains non-dimensionlized natural frequency of a simply supported square plate with a square cutout for various values of plate side to cutout side ratio. The poison's ratio is taken as (ν = 0.3).

An initial decreasing trend with increased cutout size is observed from the Table-1. The results are well compared with Mundkur et at. [7] given in bracket in the Table-1.



Section A A

Fig. 2 Stiffened plate cross-section

Dynamic Stability Studies of Stiffened Plate

The dynamic instability studies are made for rectangular stiffened plate with central square cutout, subjected to uniform compressive edge loading throughout the investigation. The cross - section of the stiffened plate is as : a = 600mm, b = 600mm, t = 1mm, b_s = 12.7mm and d_s = 22.2mm.

Effect of Static Component of Load on the Dynamic Stability of Stiffened Plates

The effect of the static component of load (α = 0.2, 0.4, 0.5, 0.6) on the instability region is shown in Fig.3. The study is done for cutout ratio g/a = 0.4. It is observed that due to an increase in the static component, instability

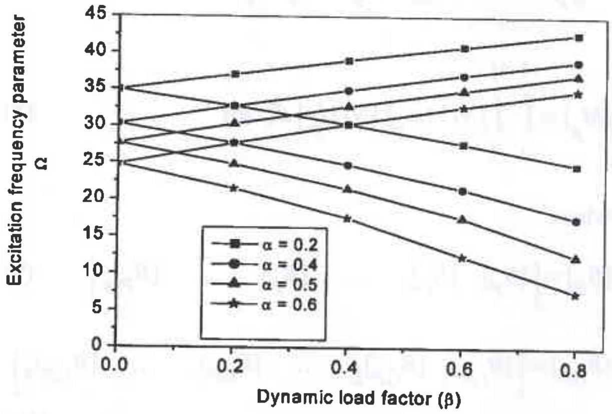


Fig. 3 Effect of static load factor on dynamic stability region for stiffened square plate with cutout having one central stiffener subjected to uniform edge loading. g/a=0.4

Table-1 : Numerical Frequency Coefficient of Simply Supported Plate

Mode	Natural frequency parameter (ω)					
	Cut out ratio (g/a)					
	0	0.1	0.2	0.3	0.5	0.8
1	19.735 (19.7392)	19.86 (19.87)	20.19 (20.193)	20.87 (20.6883)	23.45 (24.243)	58.91 (58.358
2	49.22 (49.348)	49.35 (49.355)	49.492 (49.416)	49.514 (49.512)	48.40 (49.409)	73.83 (76.012)
3	49.22 (49.348)	49.35 (49.355)	49.492 (49.416)	49.514 (49.512)	48.40 (48.409)	73.83 (76.012)
4	78.83 (78.956)	78.145 (78.427)	78.04 (77.152)	77.12 (76.052)	73.67 (76.58)	121.92 (126.71)

regions tend to shift to lower frequencies and become wider. All further studies were made with a static load factor of 0.2 (unless otherwise mentioned).

Effect of Boundary Conditions on the Dynamic Stability of Stiffened Plates

The effect of boundary conditions on the instability region is shown in Fig. 4. The study is done for cutout ratio g/a = 0.4. From Fig.4 it is seen that the onset of instability occurs later with narrow zones of instability and with the addition of restraint at the edges. So for the same aspect ratio, the excitation frequency of stiffened plate with all edges clamped will be more than the other boundary conditions.

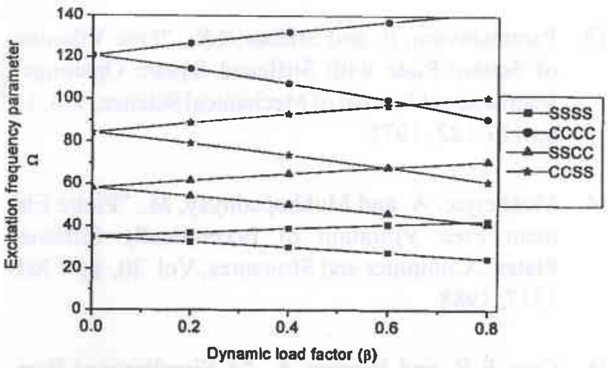


Fig. 4 Effect of boundary condition on dynamic stability region for stiffened square plate with cutout having one central stiffener subjected to uniform edge loading. α=0.2, g/a=0.4

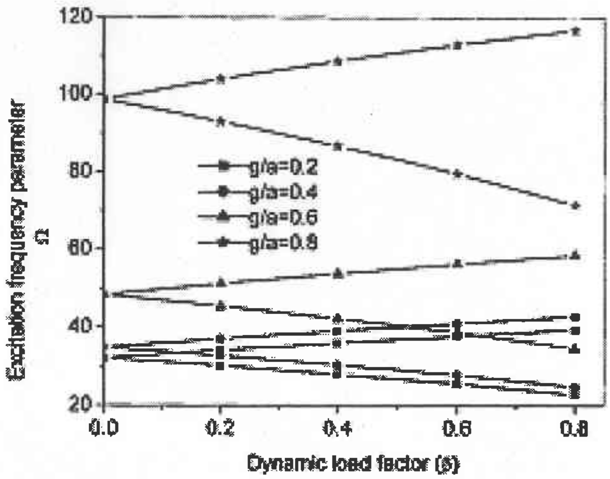


Fig. 5 Effect of cutout size (g/a) on instability region for simply supported stiffened square plate with cutout having one central stiffener subjected to uniform edge loading. α=0.2

Effect of Cutout Size (g/a) on the dynamic Stability of Stiffened Isotropic Plates

The effect of cutout size (g/a = 0.2, 0.4, 0.6, 0.8.) on the instability region of simply supported stiffened plate is studied in Fig.5. It can be observed that the onset of instability occurs with lower excitation frequencies for small cutout. With increase of cutout size, the onset of excitation frequencies increases along with wider dynamic instability regions.

Conclusion

The results from a study of the instability behavior of isotropic stiffened plates subjected to uniform periodic in-plane compressive loading can be summarized as follows:

The dynamic stability characteristics of stiffened plats with cutout is significantly affected by static components of load. The dynamic instability behaviour of stiffened plates with cutout is more pronounced in comparison to the unstiffened plates.

Due to static components, the instability regions tend to shift to lower frequencies, showing a destabilizing effect on the dynamic stability behavior of the stiffened plate with cutout.

The onset of instability occurs later with narrow zones of instability with addition of restraint at the edges for stiffened plates with cutouts.

The onset of instability occurs with lower excitation frequencies for small cutout. With increase of cutout size, the onset of excitation frequency increases along higher frequency axis with wider instability region.

References

1. Ali, R. and Atwal, S.J., "Prediction of Natural Frequencies of Vibration of Rectangular Plates with Rectangular Cutouts". Computer and Structures, Vol.12, pp.819-823, 1987.

2. Shastry, B. P. and Rao, G. V, "Vibration of Thin Rectangular Plates with Arbitrary Oriented Stiffeners". Computer and Structures, Vol. 7, pp. 627-629, 1977.

3. Reddy, J. N., "Large Amplitude Flexural Vibration of Layered Composite Plates with Cutouts", Journal of Sound and Vibration, Vol.83, No.1, pp.1-10, 1982.

4. Laura, P. A. A., Utjes, J. C. and Palluzzi, V. H., "On the Effect of Free, Rectangular Cutouts Along the Edge on the Transverse Vibrations of Rectangular Plates", Applied Mechanics, Vol.19, pp.139-151, 1986.

5. Paramasivam, P., "Free Vibration of Square Plate with Square Openings", Journal of Sound and Vibration, Vol. 30, No.2, pp.173-178, 1973.

6. Monahan, I. J., Nemergut, P. J. and Maddux, G. E., "Natural Frequencies and Mode Shapes of Plates with Interior Cutouts", The Shock and Vibration Bulletin, Vol. 41, pp. 37-49, 1970.

7. Mundkur, G., Bhat, R. B. and Neria, S., "Vibration of Plates with Cutouts Using Boundary Characteristics Orthogonal Polynomial Functions in the Raleigh-Ritz Method". Journal of Sound and Vibration, Vol. 176, No.1, pp.136-144, 1994.

8. Chang, C.D. and Chiang, F.K., "Vibration Analysis of a Thick Plate with an Interior Cutout by a Finite Element Method." Journal of Sound and Vibration, Vol.125, No.3, pp.477-486, 1988.

9. Lee, H.P., Lim, S.P. and Chow, S. T., "Prediction of Natural Frequencies of Rectangular Plates with Rectangular Cutouts", Computer and Structures, Vol. 36, No. 5, pp. 861-869, 1990.

10. Ritchie, D. and Rhodes., "Buckling and Post-buckling Behaviour of Plates with Holes", Aeronautical Quarterly, Vol. 24, pp. 281-296, 1975.

11. Sivasubramonium, B., Rao, G. V. and Krishnan, A., "Free Vibration of Longitudinally Stiffened Curved Panels with Cutout". Journal of Sound and Vibration, Vol. 226, No.1, pp. 41-55, 1999.

12. Lam, K. Y. and Hung, K. C., "Vibration Study on Plates with Stiffened Openings Using Orthogonal Polynomials and Partitioning Method", Computer and Structures, Vol. 33, No.3, pp. 295-301, 1990.

13. Paramshivam, P. and Sridhar, J.K., "Free Vibration of Square Plate with Stiffened Square Openings", International Journal of Mechanical Science, Vol. 15, pp.117-122, 1973.

14. Mukherjee, A. and Mukhopadhyay, M., "Finite Element Free Vibration of Eccentrically Stiffened Plates", Computer and Structures, Vol. 30, pp.1303-1317, 1988.

15. Corr, R.B. and Jenning. A., "A Simultaneous Iteration Algorithms for Symmetric Eigenvalue Problem.", International Journal of Numerical Method Engineering, Vol.10, pp. 647-663, 1976.