# THE DYNAMICS OF BOX BEAMS

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**Abstract**

*Box beams bend essentially with top and bottom skins providing bending rigidity and shear webs giving shear flexibility. When the width of the beam is comparable to the length of the beam and considerably larger than the depth of the shear webs, there is an added complication called the shear lag effect. The free bending vibrations of box beams can be determined analytically by incorporating shear web assumption selectively into the Timoshenko equations so that areas active in inertia, bending deformation and shear deformation are carefully identified. This will account for both shear flexibility and rotary inertia, factors that are omitted in classical Euler- Bernoulli beam descriptions. Frequencies from these modified Timoshenko type equations are calculated for three types of end conditions namely simply supported, clamped-free and clamped-clamped. However, no single analytical treatment is possible to account for the shear lag effects in the cover sheets of the box beam. Here, the finite element method allows a computational treatment of the problem. Frequencies are therefore obtained from finite element models of wing type box beam structures. The finite element models can now include the shear lag effects, which are not sensed by the Timoshenko beam model. Comparisons show how the box beam model can serve as a bench mark for evaluating finite element dynamic modeling and the relative influences of shear lag and shear flexibility coupled with rotatory inertia can be identified.*

## Nomenclature

$L$ = length of the beam
$b$ = width of the beam
$h$ = height of the beam
$E$ = Young's modulus of elasticity
$k$ = shear correction factor
$G$ = shear modulus of cover sheet of box beam
$\rho$ = density
$U$ = strain energy
$T$ = kinetic energy
$I$ = bending moment of inertia of beam cross section
$\omega$ = circular frequency of natural mode of vibration, radians per second
$q$ = frequency of natural mode of vibration, cycles per second
$n$ = mode number
$w$ = transverse deflection
$\theta$ = cross sectional rotation
$A_s$ = effective area in shear
$A_T$ = area for translatory motion

## Introduction

Wing type aircraft structures are essentially designed as box beams based on stressed skin approaches using very thin metal or even thinner composite laminate skins (closely-spaced rib- spar-skin construction). These are highly indeterminate and cannot be solved easily through analytical approaches derived from mathematical models. The finite element approach is one simple way to examine the static and dynamic behavior of such structures [1]. It will be useful to develop analytical benchmarks, even for a single bay box beam, against which the finite element results can be compared. In this present analysis, we formulate the Timoshenko equations for a box beam based on the shear web assumption, that the top and bottom skins are effective in compression and tension while the webs act only in shear and solve for the free vibration dynamics under various boundary conditions. The present analysis carefully identifies the areas giving rise to shear and bending deformation contributing to the translatory motion and the solution is obtained assuming the Timoshenko theory where the shear deformation and rotatory inertia effects are included. Classical Euler-Bernoulli beam theo-

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ries ignore both shear flexibility and rotatory inertia effects. Analytical frequencies for bending (torsion and extensional modes are not investigated) are compared with numerical frequencies obtained by finite element models. These finite element models can be refined to include shear lag effects as well, which are not easy to incorporate in the present analytical model. The present study therefore highlights the relative influences of all these complicating effects.

### Shear Web Assumptions of Box Beam

The box beam analysed here is a structure with parallel shear webs and top and bottom covers (Fig.la). The rectangular sections are symmetrical about a vertical plane. The top and bottom covers are influenced by bending deformation and the webs are dominated by shear deformation (Fig.1b). The finite element models of the box beam are shown in Figs.2a and 2b. It is assumed that the shearing stress acting at the sides of the beam is uniformly distributed over the web thickness $t$. In the case of rectangular cross section let $b$, $h$ be the width and depth. The effective area on which the shear stress acts is given by $A_s = 2ht$ (Fig.la) and the moment of inertia is given by $I = (b_1h_1^3 - b_2h_2^3)/12$ (Fig.1c), where $b_1 = b+t$; $b_2 = b-t$; $h_1 = h+t$; $h_2 = h-t$. Under such assumptions, the Timoshenko beam theory can be used to model the behaviour, where shear lag effects are not expected to be significant.

### Shear Lag Effects on Vibrations of Box Beam

In the elementary theory of beams, the influence of shear strains on any cross section are small and have negligible effect on stress distribution. In the case of box beams with wide and thin covers, the shear strains significantly influence the stress distribution in the top and bottom skin covers (Fig. 1a, 1b, 1c), and the normal stresses are now larger at the side webs and smaller near the centre of the cover, i.e. stresses at the centre "lag behind" that at the covers. This is called the shear lag problem. For such beams in which the shear deformation is significant, larger deflections than predicted by elementary beam theory are found, and these beams are less stiff than those without shear lag. Therefore in shear lag problems the usual stiffness needs to be replaced by effective stiffness, which takes account of shear-lag strains present in the beam. Under the inertia loading conditions, this effective stiffness changes the vibration characteristics. This phenomenon is known as the shear lag effect. Theoretical and experimental investigations to determine the magnitude of this effect are available in literature [2-3] but are not simple to implement. Experimental studies and Finite element analysis on shear lag effect are conducted by Luo et al. [4], for box girders with varying depth in cross section and for box girders under simultaneous axial and lateral loads to address the beam-column action and the effect of varying depth upon the shear lag of box girders. More recently, a systematic approach to the shear lag analysis of structures that are subjected to simultaneous bending and axial forces is presented by Luo et al. [5]. Based on the principle of minimum potential energy the shear lag effects in beam action and column action are

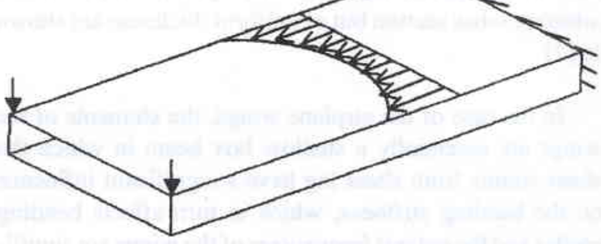

*Fig. 1a Bending stress distribution in a cantilevered box beam*

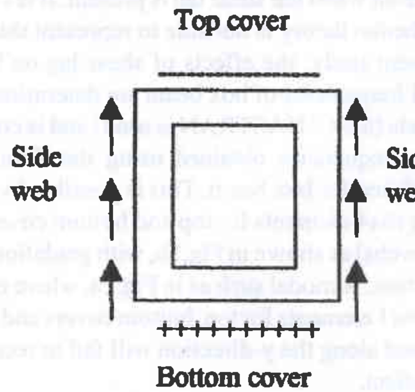


*Fig. 1b Rectangular section of a box beam*

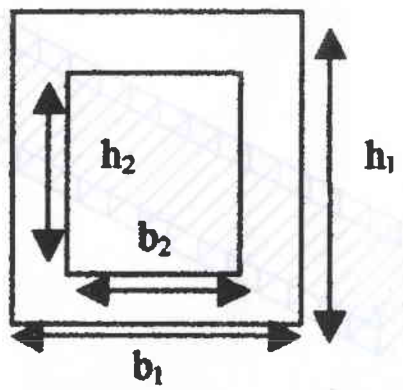


*Fig. 1c Effective area in shear $A_s$=2ht Effective area for translatory motion $A_t = b_1h_1 - b_2h_2$ ; $I = (b_1h_1^3 - b_2h_2^3)/12$ ; $b_1 = b + t$ ; $h_1 = h + t$ ; $b_2 = b - t$ ; $h_2 = h - t$*

considered separately in analogy to the stress calculation of beam-columns, using box girder as an illustrating example [5]. Kuhn and Chiarito [6] present the methods of shear-lag analysis suitable for practical use and describes strain-gage tests to verify the theory. Also the report [6] gives numerical examples illustrating the methods of analysis. Using the variational principle, Bernard et.al [7] presented the analysis of transverse vibrations of hollow thin-walled cylindrical beams. The combined influence of the secondary effects of transverse shear deformation, shear lag, and secondary effects of transverse shear deformation, shear lag and longitudinal inertia on the frequencies of cantilever and free-free cylindrical beams of arbitrary cross section but of uniform thickness are shown in [7].

In the case of the airplane wings, the elements of the wings are essentially a shallow box beam in which the shear strains from shear lag have a significant influence on the bending stiffness, which in turn affects bending modes and the natural frequencies of the wings are significantly reduced when the shear lag is present. It is clear that the Timoshenko theory is not able to represent this effect. In the present study, the effects of shear lag on bending modes and frequencies of box beam are determined using FEM models (MSC/ NASTRAN is used) and is compared with those frequencies obtained using the Timoshenko theory modified for box beam. This is possible by using a fine mesh (16x4 elements for top and bottom covers, 16xl at the side webs) as shown in Fig.2b, with gradations along the x-direction. A model such as in Fig.2a, where only one element (16x1 elements for top, bottom covers and the side webs) is used along the y-direction will fail to recover the shear lag effect.

**Frequencies from Analytical Approach**

We derive the closed form expressions for frequencies as follows:

The strain energy U and the kinetic energy T of a box beam are given by

$$U=\frac{1}{2}\int_0^L EI\theta_{,x}^2\,dx+\frac{1}{2}\int_0^L kGA_S(\theta-w_{,x})^2\,dx \qquad (1)$$

$$T=\frac{1}{2}\int_0^L \rho I\theta_{,t}^2\,dx+\int_0^L \rho A_T w_{,t}^2\,dx \qquad (2)$$

Note that unlike the familiar Timoshenko equations for solid section beams, here we make the distinction where the area effective in shear $A_s$, is from the shear webs, and the areas effective for the inertia of translatory motion $A_T$ takes into account both the shear webs and top and bottom skin areas. $I$ is the moment of inertia of the section and this appear both in the bending strain energy and the rotatory inertia term. It is assumed that the flange areas and top and bottom skins as well as webs are effective in bending and contribute to $I$. The analysis is restricted to considering only bending vibration and ignores torsional and extensional effects.

On applying Hamilton's principle, we get the following Euler - Lagrange equations of motion

$$\rho A_T\ddot{w}-kGA_s(w_{,xx}-\theta_{,x})=0 \qquad (3)$$

$$\rho I\ddot{\theta}-EI\theta_{,xx}-kGA_s(w_{,x}-\theta)=0 \qquad (4)$$

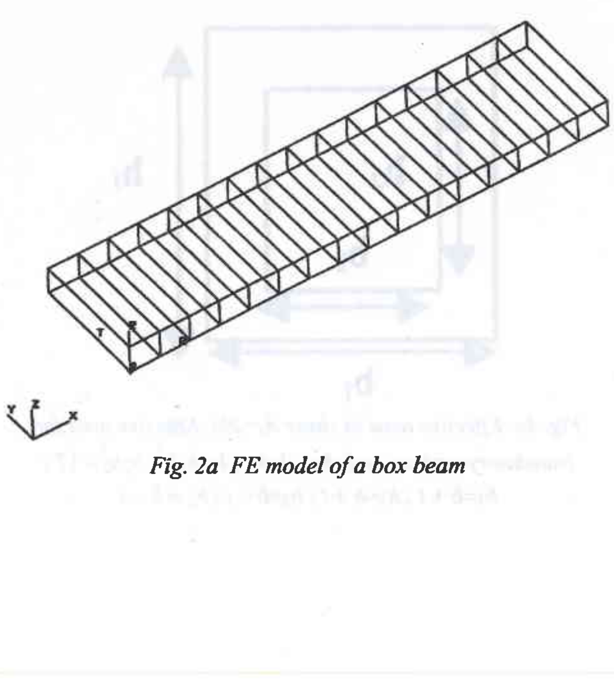

*Fig. 2a FE model of a box beam*

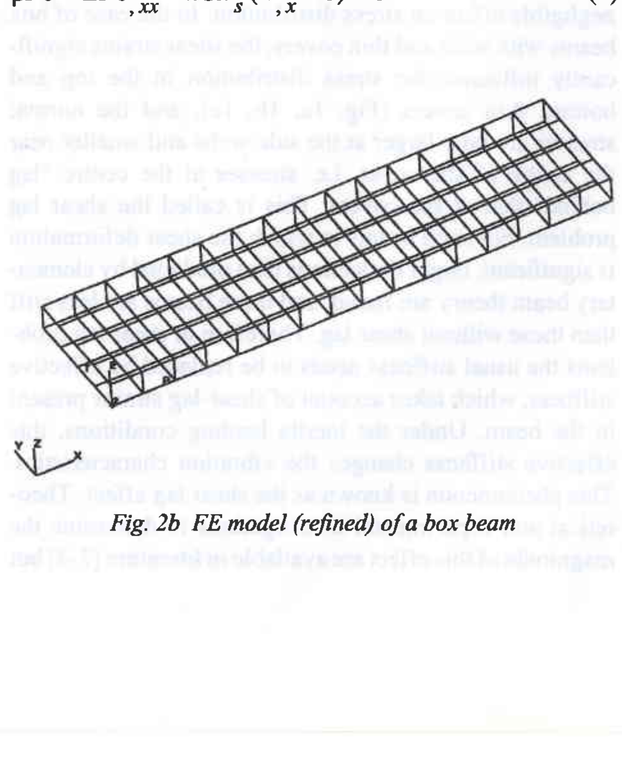

*Fig. 2b FE model (refined) of a box beam*

Uncoupling the above equations (3-4), we get the fourth order partial differential equation as

$$EI\, w_{,xxxx} + \rho A_T w_{,tt} - \rho I\,(1 + (E/kG)\,(A_T/A_S))\, w_{,ttxx}$$

$$+ \rho^2 I/kG\,(A_T/A_S)\, w_{,tttt} = 0 \quad (5)$$

If we assume $w(x,t) = W(x)\sin\omega t$, we get the function $W(x)$ with four constants whose values are determined from the boundary conditions.

$$W(x) = A\cos qx + B\sin qx + C\cosh qx + D\sinh qx \quad (6)$$

The frequency $q$ is obtained by applying the appropriate boundary conditions and the frequency $\omega$ is obtained from the following fourth degree polynomial equation

$$(\rho^2 L^4/EkG)\,(A_T/A_S)\,\omega^4 + [\rho A_T tL^4/EI$$

$$+ (\rho L^2/E)\,(1 + (E/kG)\,(A_T/A_S)\,q^2]\,\omega^2 - q^4 = 0 \quad (7)$$

In the case of simply supported beam the frequencies are obtained from

$$\sin qL = 0 \quad (8)$$

and hence $q = n\pi/L$, where $n$ is the mode number.

In the case of clamped-free conditions the frequency equation is of the form

$$\cos qL \cosh qL = -1 \quad (9)$$

In the case of clamped-clamped conditions the frequency equation is given by

$$\cos qL \cosh qL = 1 \quad (10)$$

The above equations (8-10) are solved numerically for the values of $q$ and the natural frequencies are obtained by substituting the values of $q$ in the fourth degree equation (7) in $\omega$. The frequencies of the box beam for bending modes thus obtained are used as a standard result for comparison of the frequencies obtained computationally from finite element models, where there is no shear lag effect.

**Frequencies from Computational Approach**

Natural frequencies of the box beam are calculated using NASTRAN finite element models. The geometry is modelled using CQUAD4, isoparametric quadrilateral plate element. Bending and transverse shear deformation properties of this element are input through PSHELL property entry. In one model (Fig. 2a) the webs, top and bottom surfaces are discretised using 64 elements in all, with each face represented by 16 elements along the length of the beam with the element thickness of 2.0 mm. The dimensions are $L$ = 900mm along x, $b$ = 300mm along y and $h$ = 75mm along z ($L/b$ = 3.0) with the material property $E/kG$=2.0, where E = 7000 kg/mm$^2$ is the Young's modulus, $k$ the shear correction factor and the shear modulus $G$. The density of the material is given by $\rho$ = 2.8E-6 Kg/mm$^2$. The natural frequencies are obtained as the solutions of eigen value problem by considering six (three translational and three rotational) degrees of freedom at each unrestrained node. The frequencies for different boundary conditions of the beam are obtained by imposing the free or fixed conditions on the nodal degrees of freedom. Note that this model will not allow for the stress diffusion effects on the top and bottom skin covers, which are indicative of the shear lag effects. To include this factor, the computations are repeated with a refined model (Fig. 2b), which will now sense shear lag effects.

**Results And Discussions from Numerical Experiments**

In the present investigation, the frequencies of the box beam under three different boundary conditions namely (i) simply-supported, (ii) clamped-clamped, (iii) clamped-free conditions were calculated using FEM package NASTRAN, for first few modes of vibrations where the length to width ratio of the beam is $L/b$= 3 with $E/kG$ = 2.0. Frequencies obtained using finite element model (Fig. 2a) were compared with those frequencies obtained from Timoshenko beam theory modified for box beam with appropriate areas for shear deformation and translatory motion in Tables 1-3. It is observed that the frequencies predicted from Timoshenko theory differ slightly from the frequencies obtained using FEM. for all the three boundary conditions when shear lag effects are ignored. The mode shapes corresponding to first four modes were calculated both from Timoshenko theory and finite element model and presented in graphical forms. Figs. 3-6 give the mode shapes of cantilever beam for mode number n = 1 to n = 4. Figs.7-10 give the mode shapes of simply-supported beam for n = 1 to n = 4.

Uncoupling the above equations (3-4), we get the fourth order partial differential equation as

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**Table-1 : Frequencies (Cycles/sec) for Simply-Supported Box Beam (no shear lag)**

| Mode No. | NASTRAN | Theory |
|---|---|---|
| 1 | 7.617 | 7.199 |
| 2 | 28.019 | 26.505 |
| 3 | 56.3 | 53.279 |
| 4 | 93.07 | 83.684 |

**Table-2 : Frequencies (Cycles/sec) for Clamped-clamped Box Beam (no shear lag)**

| Mode No. | NASTRAN | Theory |
|---|---|---|
| 1 | 14.92 | 15.73 |
| 2 | 36.61 | 39.23 |
| 3 | 64.14 | 68.204 |
| 4 | 95.49 | 99.507 |
| 5 | 129.84 | 131.63 |
| 6 | 166.70 | 163.86 |

**Table-3 : Frequencies (Cycles/sec) for Clamped-free Box Beam (no shear lag)**

| Mode No. | NASTRAN | Theory |
|---|---|---|
| 1 | 2.62 | 2.62 |
| 2 | 15.26 | 15.51 |
| 3 | 38.91 | 39.25 |
| 4 | 68.60 | 68.20 |

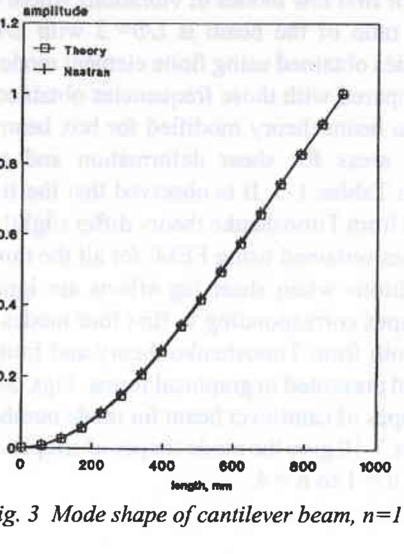


*Fig. 3 Mode shape of cantilever beam, n=1*

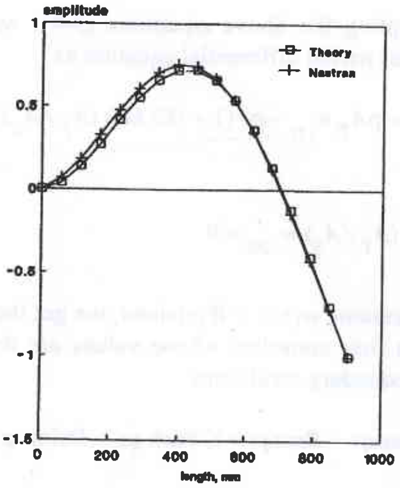


*Fig. 4 Mode shape of cantilever beam, n=2*

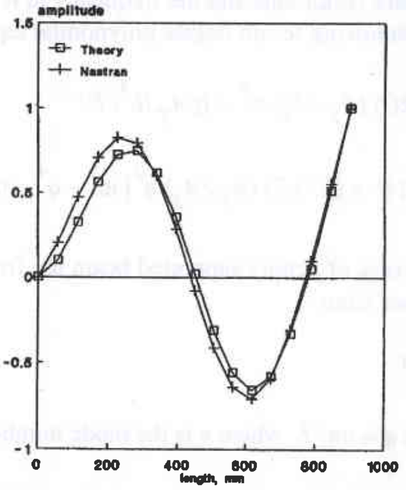


*Fig. 5 Mode shape of cantilever beam, n=3*

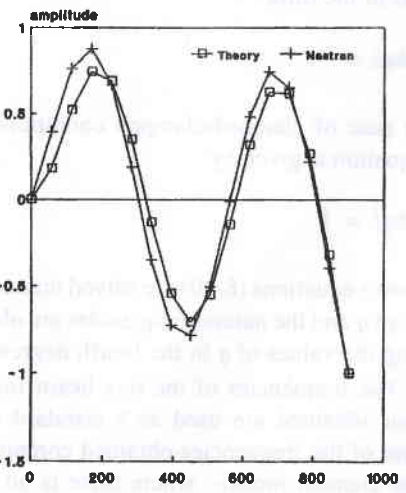


*Fig. 6 Mode shape of cantilever beam, n=4*

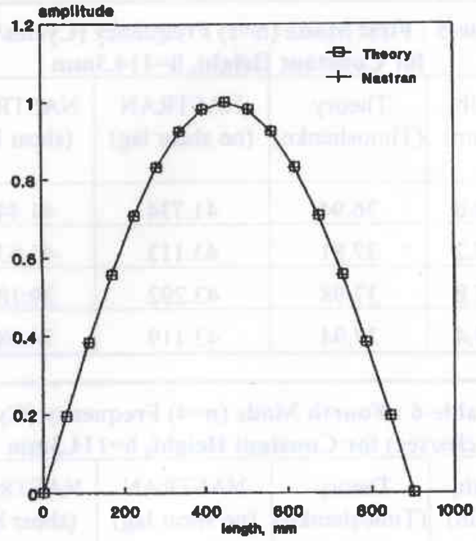


*Fig. 7 Mode shape of simply-supported beam, n=1*

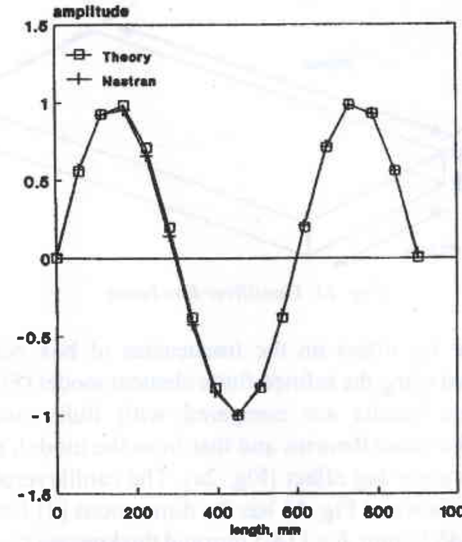


*Fig. 9 Mode shape of simply-supported beam, n=3*

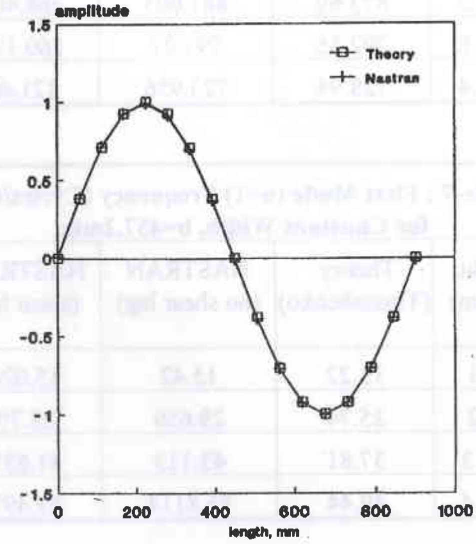


*Fig. 8 Mode shape of simply-supported beam, n=2*

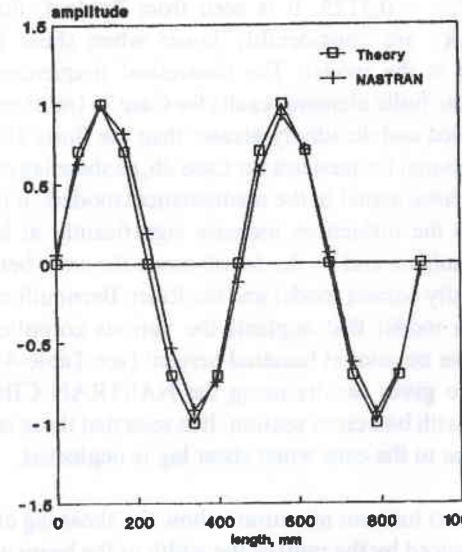


*Fig. 10 Mode shape of simply-supported beam, n=4*

**Table-4 : Frequencies (Cycles/sec) for Cantilever Box Beam**

| Mode No. | Theory (Timoshenko) | Euler Theory | NASTRAN CBEAM Model | NASTRAN Plate Model (no shear lag) | NASTRAN Plate Model (shear lag) |
|---|---|---|---|---|---|
| 1 | 37.81 | 38.5 | 37.71 | 43.133 | 41.64 |
| 2 | 216.71 | 239.813 | 205.06 | 229.61 | 163.73 |
| 3 | 523.269 | 662.929 | 489.169 | 539.10 | 228.67 |
| 4 | 873.695 | 1275.205 | 812.85 | 881.00 | 268.46 |

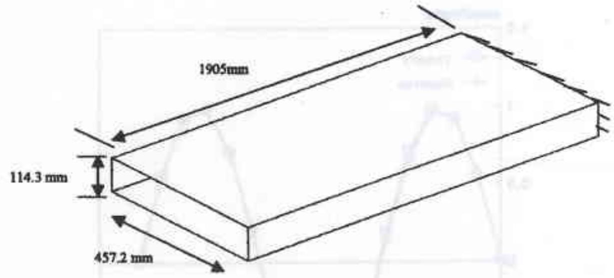


*Fig. 11 Cantiliver box beam*

Shear lag effect on the frequencies of box beam is calculated using the refined finite element model (Fig. 2b) and these results are compared with Euler and Timoshenko beam theories and that from the model, which removes shear lag effect (Fig. 2a). The cantilevered box beam as shown in Fig. 11 has the dimensions [2] $L$= 1905 mm, $b$=457.2mm, $h$=114.3 mm and thicknesses $t$=10.16 mm for the covers and $t$ = 8.12 mm for the webs. In this case the material properties are given by $E$=7239.5 Kg/mm$^2$, $\nu$ = 0.3125. It is seen from Table-4, that the frequencies are considerably lower when shear lag is included in the model. The theoretical frequencies are close to the finite element results for Case 2a (no shear lag) as expected and decidedly greater than the finite element computational frequencies for Case 2b, as shear lag effects were not anticipated in the mathematical models. It is also seen that the influences increase significantly at higher mode numbers, and by the fourth mode the error between a physically correct model and the Euler-Bernoulli classical beam model that neglects the various complicating effects can be several hundred percent (see Table-4). Table-4 also gives results using the NASTRAN CBEAM element with box cross section. It is seen that these results are similar to the case when shear lag is neglected.

We next turn our attention to how the shear lag effects are influenced by the ratio of the width of the beam and as the depth of the beam varies. For this purpose we keep the length $L$ at 1905 mm and vary $b$ and $h$ respectively. Table-5 shows how the frequencies of the first fundamental mode change considerably as the shear lag effect increases as the width of the beam is increased. This is to be expected as now the stress diffusion in the cover sheets will depart considerably from that where there is no shear lag effect and the effective stiffness will change. In Table-6, we see that at larger 'mode numbers (here, $n$ = 4), the shear lag influence is more critical and for widths $b$ which are of the order of magnitude of the length $L$, the frequencies can drop to a fifth of that produced using the assumptions of shear deformation in the webs only (Timoshenko theory).

**Table-5 : First Mode (n=1) Frequency (Cycles/sec) for Constant Height, h=114.3mm**

| Width, b (mm) | Theory (Timoshenko) | NASTRAN (no shear lag) | NASTRAN (shear lag) |
|---|---|---|---|
| 228.6 | 36.94 | 41.734 | 41.449 |
| 457.2 | 37.81 | 43.113 | 41.637 |
| 685.8 | 37.98 | 43.292 | 39.187 |
| 914.4 | 37.94 | 43.119 | 34.683 |

**Table-6 : Fourth Mode (n=4) Frequency (Cycles/sec) for Constant Height, h=114.3mm**

| Width, b (mm) | Theory (Timoshenko) | NASTRAN (no shear lag) | NASTRAN (shear lag) |
|---|---|---|---|
| 228.6 | 974.12 | 999.804 | 685.348 |
| 457.2 | 873.69 | 881.003 | 268.461 |
| 685.8 | 792.55 | 791.27 | 160.177 |
| 914.4 | 728.94 | 723.936 | 121.40 |

**Table-7 : First Mode (n=1) Frequency (Cycles/sec) for Constant Width, b=457.2mm**

| Height, h (mm) | Theory (Timoshenko) | NASTRAN (no shear lag) | NASTRAN (shear lag) |
|---|---|---|---|
| 38.1 | 13.22 | 15.42 | 15.026 |
| 76.2 | 25.74 | 29.659 | 28.79 |
| 114.3 | 37.81 | 43.113 | 41.637 |
| 152.4 | 49.44 | 55.8114 | 53.497 |

**Table-8 : Fourth Mode (n=4) Frequency (Cycles/sec) for Constant Width, b=457.2mm**

| Height, h (mm) | Theory (Timoshenko) | NASTRAN (no shear lag) | NASTRAN (shear lag) |
|---|---|---|---|
| 38.1 | 385.33 | 421.548 | 233.488 |
| 76.2 | 660.67 | 685.537 | 262.109 |
| 114.3 | 873.69 | 881.0028 | 268.4613 |
| 152.4 | 1045.16 | 1034.227 | 272.3457 |

We shall next investigate how the depth of the box beam influences the shear lag effects on the frequencies. In this case, the width b is kept constant at 457.2 mm; the length $L$ at 1905 mm and the depth is varied. In Table-7, we observe that the frequencies of the first mode increases as the depth of the beam is increased in all the three cases namely in modified Timoshenko theory, no shear lag and shear lag. This is expected as the moment of inertia increases with $h^2$ for a box beam. Correspondingly, the shear deformation and rotatory inertia effects also become prominent. For the first mode $n=1$, these effects are not large but they become evident for example when the fourth mode $n=4$ is examined as shown in Table-8. The frequencies can become as small as one fifth of that produced using modified Timoshenko theory and the case where there is no shear lag present.

## Concluding Remarks

By modifying the Timoshenko beam model to account separately for area effective in shear, bending and inertia, it is possible to offer an analytical model for the dynamic behavior of aircraft wing type box beams which do not have significant shear lag effects. These can be used as benchmarks against which dynamics emerging from finite element models of box beams can be validated. However, it must be understood that where shear lag effects are significant, it is difficult to obtain comprehensive analytical models and finite element modelling is the most practical approach in such cases. It is also seen that shear flexibility, rotatory inertia and shear lag effects significantly influence the flexural dynamics of box beams, and these increase as mode number increases.

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