

# IDENTIFICATION OF AEROELASTIC QUASI-STEADY EFFECTS THROUGH FLEX FACTOR

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## Abstract

*Rigid body modeling for estimation of aerodynamic derivatives from the flight responses of a flexible aircraft does not yield the true stability and control parameters of the aircraft, as the estimated derivatives absorb the aeroelastic effects present in the data. The analytical expressions for computing the rigid body derivatives from the flight-estimated values require knowledge of aircraft modal mass and generalized elastic deflections which is not always available. This paper considers a simplified approach based on quasi-steady representation of aeroelastic effects to identify the rigid body parameters from the flexible aircraft responses. Results show that, by combining data at several flight conditions, system identification method can be applied to separate the rigid body derivatives and the dynamic pressure dependent quasi-steady effects caused by structural deformation.*

## Introduction

Parameter estimation from flight data, as applied to aircraft in the linear flight regime, is currently being used on routine basis with the assumption that the rigid body model is valid. Elastic degrees of freedom are, therefore, absent from the aircraft derivative model used in the estimation algorithm. However, for the newly developed highly maneuverable aircraft, and particularly for the large transport aircraft, the rigid body model will be inadequate and may yield estimates that are very different from the true derivative values.

A full order model of an aircraft with rigid body and elastic degrees of freedom is required to account for the flexibility effects in aircraft dynamics. Such a model is necessary to accurately predict the aircraft handling qualities. However, an integrated model of this kind may not be easy to identify from flight data as it will have too many parameters and the reliability of the estimates may be compromised. Ref. [1] discusses methods of applying model simplifications to aeroelastic systems. An integrated modeling approach to account for the aeroelastic effects in aircraft dynamics was suggested in Ref. [2] and its use for a highly elastic aircraft demonstrated. The aeroelastic model of Ref. [2] was simplified by Ghosh and Raisinghani in Ref. [3] and the resulting model, with reduced number of unknown terms, was used in the iden-

tification of the aeroelastic aircraft. It was shown that the use of rigid body model in the estimation algorithm provides derivative values that absorb the flexibility effects present in the aircraft response and, therefore, differ from the true derivative values of the aircraft. These estimated derivatives were referred to as "equivalent derivatives". An analytical expression for the equivalent derivatives was obtained by summing the rigid body derivative and the terms related to the first elastic mode of the aircraft. Knowing the aircraft modal mass and generalized elastic deflections, one could use the analytical expression to compute the rigid body values from the estimated equivalent derivatives. The drawback of the approach, however, is that the information on the modal mass, in-vacuo frequencies and aircraft elastic deflections required in the analytical expression is not easily available. The identification of true rigid body derivatives from the flight data of a flexible aircraft is, therefore, of great interest.

One possibility is to use the generalized equations of motion accounting for the elastic degrees of freedom in parameter identification [2,4]. However, such models are very complex and generally consist of a large number of parameters which may lead to identifiability problems. A more practical approach is to model the elastic effects as quasi-steady influence on the aircraft derivatives. Assuming the rigid and structural frequencies to be well sepa-

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rated, the flexibility effects in the equivalent derivative can be lumped into a single factor known as flex factor. Flexibility effects accounted in this manner can be identified by combining flight maneuvers performed under different dynamic pressure conditions [5,6].

Separate identification of the rigid body derivatives and flexibility effects can help to better understand the structural influences on the aerodynamic derivatives. If the wind tunnel derivatives have no aeroelastic corrections imposed on them, it would be more appropriate to compare the estimated rigid body values instead of the equivalent derivatives with the wind tunnel predictions. On the other hand, if aeroelastic corrections are made to the wind tunnel results, these can be verified by identifying the flex factors from the flight data.

In the present investigations, using the approach suggested in Ref. [5], the work reported in Ref. [3] is extended by postulating linear model form for the aircraft derivatives to separately estimate the rigid body derivatives and the flex factors representing the elastic effects. In the absence of real flight data, simulated data generated for the example aircraft described in Ref. [2] is used for analysis. Output error method in time domain [7] is first applied to identify the derivatives from rigid body model. It is shown that the estimated derivatives are considerably different from the true rigid body derivative values. Next, estimation of the rigid body derivatives and the flex factors is carried out by combining aircraft data from four different flight conditions. The estimated rigid body derivatives are found to be in excellent agreement with the true rigid body parameter values. The flex factors also seem to be well estimated as indicated by the close match between the equivalent derivatives computed using the linear model postulates and the analytical form of Ref. [3].

### Mathematical Formulation

Longitudinal short period motion of a flexible aircraft can be approximated by the following set of differential equations [2]:

$$\dot{\alpha} - q = \frac{\rho u S}{2m} C_z \quad (1)$$

$$\dot{q} = \frac{\rho u^2 S c}{2I_y} C_m \quad (2)$$

where

$$C_z = C_{z_\alpha} \alpha + C_{z_q} \frac{qc}{2u} + C_{z_\delta} \delta + \sum_{i=1}^n \left( C_{z_{\eta_i}} \eta_i + C_{z_{\dot{\eta}_i}} \frac{\dot{\eta}_i c}{2u} \right) \quad (3)$$

$$C_m = C_{m_\alpha} \alpha + C_{m_q} \frac{qc}{2u} + C_{m_\delta} \delta + \sum_{i=1}^n \left( C_{m_{\eta_i}} \eta_i + C_{m_{\dot{\eta}_i}} \frac{\dot{\eta}_i c}{2u} \right) \quad (4)$$

Eqns. (3) and (4) represent the aerodynamic models for  $C_z$ , the normal force coefficient and  $C_m$ , the pitching moment coefficient. The aerodynamic models include the rigid body as well as the elastic derivatives ( $C_{z_\alpha}$ ,  $C_{m_\delta}$ , ... as defined in Ref. [2]).  $\eta_i$  and  $\dot{\eta}_i$  are the generalized elastic deflections and their time derivatives. The angle of attack ( $\alpha$ ), pitch rate ( $q$ ) and control input ( $\delta$ ) represent small perturbation motion variables from the chosen reference flight conditions. The other parameters in the above equations include, air density ( $\rho$ ), total inertial velocity ( $u$ ), wing area ( $S$ ), wing chord ( $c$ ), aircraft mass ( $m$ ) and the moment of inertia about  $Y$ -axis ( $I_y$ ).

The generalized coordinates satisfy the following equation [3] :

$$\begin{aligned} & \ddot{\eta}_i + 2\xi_i \omega_i \dot{\eta}_i + \omega_i^2 \eta_i \\ & = \frac{\rho u^2 S c}{2M_i} \left[ C_\alpha^{\eta_i} \alpha + C_q^{\eta_i} \frac{qc}{2u} + C_\delta^{\eta_i} \delta + \sum_{j=1}^n \left( C_{\eta_j}^{\eta_i} \eta_j + C_{\dot{\eta}_j}^{\eta_i} \frac{\dot{\eta}_j c}{2u} \right) \right] \end{aligned} \quad (5)$$

where  $\omega_i$ ,  $\xi_i$  and  $M_i$  are the in-vacuo frequency, modal damping and modal generalized mass of the  $i^{\text{th}}$  mode, respectively.

Assuming the contributions of the generalized elastic deflections  $\eta_i$  to be instantaneous, the terms containing the time derivatives of elastic deflections in Eq.(5) can be neglected to give the steady state equation for  $(\eta_i)_{ss}$  :

$$(\eta_i)_{ss} = \frac{\rho u^2 S c}{2M_i \omega_i^2} \left[ C_\alpha^{\eta_i} \alpha + C_q^{\eta_i} \frac{qc}{2u} + C_\delta^{\eta_i} \delta + \sum_{j=1}^n C_{\eta_j}^{\eta_i} \eta_j \right] \quad (6)$$

Assuming instantaneous elastic deflections helps to define analytical expressions for computing equivalent derivative values [3].

The matrix form of Eq. (6) given below is used to compute the aircraft elastic modes

$$\begin{bmatrix} \frac{1}{a_1} - C_{\eta_1}^{\eta_1} & -C_{\eta_2}^{\eta_1} & -C_{\eta_3}^{\eta_1} & -C_{\eta_4}^{\eta_1} \\ -C_{\eta_1}^{\eta_2} & \frac{1}{a_2} - C_{\eta_2}^{\eta_2} & -C_{\eta_3}^{\eta_2} & -C_{\eta_4}^{\eta_2} \\ -C_{\eta_1}^{\eta_3} & -C_{\eta_2}^{\eta_3} & \frac{1}{a_3} - C_{\eta_3}^{\eta_3} & -C_{\eta_4}^{\eta_3} \\ -C_{\eta_1}^{\eta_4} & -C_{\eta_2}^{\eta_4} & -C_{\eta_3}^{\eta_4} & \frac{1}{a_4} - C_{\eta_4}^{\eta_4} \end{bmatrix} \begin{Bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{Bmatrix} = \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} \quad (7)$$

where

$$A_i = C_{\alpha}^{\eta_i} \cdot \alpha + C_q^{\eta_i} \cdot \frac{qc}{2u} + C_{\delta}^{\eta_i} \cdot \delta \quad (8)$$

and

$$a_i = \frac{\bar{q} S c}{M_i \omega_i^2} \quad (\text{for, } i = 1, 2, 3, 4) ; \quad \bar{q} = \frac{1}{2} \rho u^2 \quad (9)$$

The steady state value of the first elastic mode  $(\eta_1)_{ss}$  from Eq. (7) is given by

$$(\eta_1)_{ss} = \frac{1}{\frac{M_1 \omega_1^2}{\bar{q} S c} - C_{\eta_1}^{\eta_1}} \cdot \left( C_{\alpha}^{\eta_1} \cdot \alpha + C_q^{\eta_1} \cdot \frac{qc}{2u} + C_{\delta}^{\eta_1} \cdot \delta \right) \quad (10)$$

In Ref. [3], an analytical expression for the equivalent derivatives was formulated considering only the first elastic mode of the aircraft. Using the above expression for  $(\eta_1)_{ss}$  into Eqs.(3) and (4), and neglecting the time derivative terms for the elastic modes, the coefficients of  $\alpha$ ,  $q$  and  $\delta$  were collected to express the equivalent parameters in the following form [3]:

$$C'_{Zx} = C_{Zx} + F C_{Z\eta_1} C_x^{\eta_1} ;$$

$$C'_{m_x} = C_{m_x} + F C_{m_{\eta_1}} C_x^{\eta_1} \quad (11)$$

where  $x = \alpha, q$  or  $\delta$  and

$$F = \left( \frac{M_1 \omega_1^2}{\bar{q} S c} - C_{\eta_1}^{\eta_1} \right)^{-1} \quad (12)$$

A more accurate representation of the equivalent derivatives, however, would be to include all the four elastic modes in Eq. (11). Rewriting Eq.(7) to compute the aircraft elastic modes, we get

$$\begin{Bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{Bmatrix} = [B]_{4 \times 4} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} \quad (13)$$

Using the values of  $(\eta_i)_{ss}$  from above into Eqs. (3) and (4), the analytical form for the equivalent derivatives can be formulated as:

$$C'_{Zx} = C_{Zx} + \sum_{i=1}^4 \sum_{j=1}^4 B_{ij} C_{Z\eta_i} C_x^{\eta_j}$$

$$C'_{m_x} = C_{m_x} + \sum_{i=1}^4 \sum_{j=1}^4 B_{ij} C_{m_{\eta_i}} C_x^{\eta_j} \quad (14)$$

The analytical form of Eq.(14) is more accurate than that given in Ref. [3]. However, to compute the equivalent or the rigid body derivatives from Eq.(11) or Eq.(14) would require information about the aircraft elastic derivatives  $C_{Z\eta_i}$ ,  $C_{m_{\eta_i}}$ ,  $C_{\eta_i}^{\eta_j}$  and  $C_x^{\eta_j}$ .

### Data Simulation

Flight data for estimation of stability and control derivatives of an aeroelastic aircraft can be generated from a variety of maneuvers, e.g., pitch stick doublets, rudder doublets, steady turns and frequency sweeps [8]. The aim should be to sufficiently excite the aircraft dynamic be-

havior in longitudinal and lateral axis within the constraints placed by safety and operational considerations.

For the current analysis, due to the non-availability of real flight data from a flexible aircraft, simulated data for an example aircraft described in Ref. [2] were generated

|                          |   |
|--------------------------|---|
| Geometry                 | $\bar{c} = 4.664 \text{ m}$ (mean chord)<br>$b = 21.336 \text{ m}$ (wing span)<br>$S = 180.79 \text{ m}^2$ (planform area)<br>$\Lambda = 65 \text{ deg}$ (sweep angle)          |
| Weight                   | $W = 130,642.3 \text{ Kg}$ (net weight)   |
| Inertia                  | $I_{xx} = 1,288,066 \text{ Kg-m}^2$<br>$I_{yy} = 8,677,503 \text{ Kg-m}^2$<br>$I_{zz} = 9,626,605 \text{ Kg-m}^2$<br>$I_{xz} = -71,453 \text{ Kg-m}^2$<br>$I_{xy} = I_{yz} = 0$ |
| Modal generalized masses | $M_1 = 248.94 \text{ Kg-m}^2$<br>$M_2 = 12998.0 \text{ Kg-m}^2$<br>$M_3 = 1809.3 \text{ Kg-m}^2$<br>$M_4 = 59111.3 \text{ Kg-m}^2$  |

at  $M=0.6$  and  $H=1.5 \text{ Km}$ . Details of the aircraft mass, moment of inertia, geometric characteristics, stability and control derivatives, and the first four elastic modes for the baseline configuration C2 and for the more flexible configuration C3, are obtained from Ref. 2 and provided in Tables-1 to 3 for ready reference. A multistep 3-2-1-1 elevator input with amplitude of 0.05 rad and a step size of 1 sec was used to generate the data. The simulated angle-of-attack and pitch rate responses were generated using Eqs.(1-4) with the elastic modes  $\eta_i$  computed from Eq.(13). Fig.1 shows the rigid body and aeroelastic responses for the C3 configuration at Mach 0.6 and altitude 1.5 Km.

To separately estimate the rigid body derivatives and the flex factor representing the elastic effects, simulated data at three additional dynamic pressure conditions were generated by varying the altitude while maintaining a constant Mach (Table-4). The rigid body derivative values

| Configuration | Mode 1 | Mode 2 | Mode 3 | Mode 4 |
|---------------|--------|--------|--------|--------|
| C1 (rigid)    | ---    | ---    | ---    | ---    |
| C2 (baseline) | 12.57  | 14.07  | 21.17  | 22.05  |
| C3            | 6.29   | 7.04   | 10.59  | 11.03  |

| $C_z$            |         | $C_m$            |         | $C_{\eta i}$          | Elastic Mode, i |          |          |          |
|------------------|---------|------------------|---------|-----------------------|-----------------|----------|----------|----------|
|                  |         |                  |         |                       | 1               | 2        | 3        | 4        |
| $C_{z_0}$        | -0.340  | $C_{m_0}$        | -0.252  | $C_0^{\eta i}$        | 0.00            | 0.00     | 0.00     | 0.00     |
| $C_{z_\alpha}$   | -2.922  | $C_{m_\alpha}$   | -1.660  | $C_\alpha^{\eta i}$   | -1.49e-2        | 2.58e-2  | 1.49e-2  | 3.35e-2  |
| $C_{z_q}$        | 14.700  | $C_{m_q}$        | -34.750 | $C_q^{\eta i}$        | -9.49e2         | 1.16e-2  | 3.97e-2  | 2.83e-5  |
| $C_{z_\delta}$   | -0.435  | $C_{m_\delta}$   | -2.578  | $C_\delta^{\eta i}$   | -1.28e-2        | -6.42e-2 | 2.56e-2  | 1.47e-4  |
| $C_{z_{\eta 1}}$ | -0.0288 | $C_{m_{\eta 1}}$ | -0.0321 | $C_{\eta 1}^{\eta i}$ | 5.85e-5         | 4.21e-3  | 2.91e-4  | 2.21e-5  |
| $C_{z_{\eta 2}}$ | 0.306   | $C_{m_{\eta 2}}$ | -0.025  | $C_{\eta 2}^{\eta i}$ | -9.0e-5         | -9.22e-2 | 1.44e-3  | -1.32e-4 |
| $C_{z_{\eta 3}}$ | 0.0148  | $C_{m_{\eta 3}}$ | 0.0414  | $C_{\eta 3}^{\eta i}$ | 3.55e-4         | 1.97e-3  | -3.46e-4 | 9.68e-6  |
| $C_{z_{\eta 4}}$ | -0.0140 | $C_{m_{\eta 4}}$ | -0.0183 | $C_{\eta 4}^{\eta i}$ | 1.2e-4          | 3.37e-3  | 1.44e-4  | 1.77e-3  |

| Table-4 : Flight conditions for data generation |                        |                          |
|---|------------------------|--------------------------|
| Mach = 0.6                                      |                        |                          |
| Altitude (H)                                    | Density ( $\rho^*$ )   | Dynamic Pressure ( $q$ ) |
| 1.5 Km  | 1.0 Kg/m <sup>3</sup>  | 21455 N/m <sup>2</sup>   |
| 3 Km  | 0.88 Kg/m <sup>3</sup> | 18013 N/m <sup>2</sup>   |
| 5 Km  | 0.72 Kg/m <sup>3</sup> | 14093 N/m <sup>2</sup>   |
| 7.5 Km  | 0.55 Kg/m <sup>3</sup> | 10205 N/m <sup>2</sup>   |

\* ISA tables used

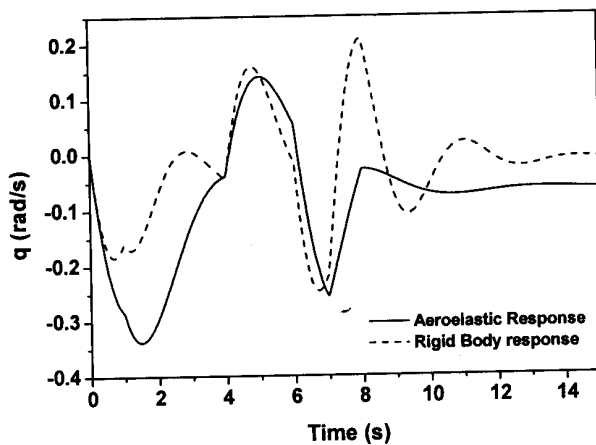
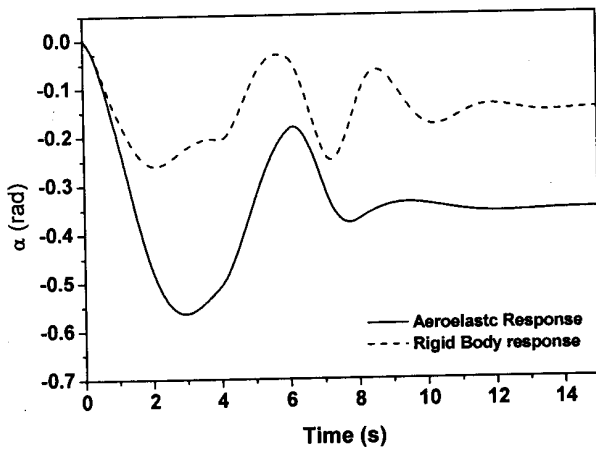


Fig.1 Aeroelastic and rigid body responses from simulation [C3 configuration]

were kept unchanged during data simulation at different altitudes.

### Rigid Body Model Identification

Output error method in time domain was applied to the simulated data for C2 and C3 configurations at M=0.6 and H=1.5 Km. Analysis was carried out with varying degrees of flexibility in the estimation model, e.g., by neglecting all the four elastic modes (rigid body modeling), by including only the first mode, by including the first and second modes, and by including all the modes. For brevity, results are presented only for the more flexible C3 configuration. From Table-5, it is observed that when all the modes are neglected (rigid body model identification), the estimated derivatives in column 3 show noticeable change from the true values listed in column 2. With the inclusion of the first elastic mode in the estimation model (column 4), there is a marked improvement in the match between the estimated and the true values.  $C_{Z_q}$  and  $C_{Z_{\delta}}$  show some differences which disappear when the second elastic mode is also included (column 5). It is evident that the first elastic mode contributes the maximum to the overall aeroelastic effects. With all modes included in column 6, the estimated values match exactly with the true values of the aircraft derivatives.

The theoretical values of the equivalent derivatives in column 7 and 8 are computed from Eqs.(11) and (14), respectively. Eq.(14) being more accurate, the values listed in column 8, particularly  $C_{Z_{\delta}}$ , show excellent match with the estimated equivalent derivatives listed in column 3.

From the results presented in Table-5, it is clear that with all the modes neglected in the estimation model, the estimated derivatives absorb the flexibility effects present in the data. The estimation thus does not give the true derivative values of the aircraft.

A simplified procedure for identifying the rigid body aircraft derivatives from the measured response of an aeroelastic aircraft is discussed next. The advantage of this approach is that, unlike the analytical expression given in Ref.[3] and Eqs.(11) and (14), it does not require any a priori information on the aircraft modal mass or elastic derivatives.

**Table-5 : Estimated derivative values from models with varying degrees of flexibility  
[C3 configuration ; M = 0.6, H = 1.5 Km]**

| Parameters    | True values | All modes neglected (estimated equivalent derivatives) | First mode included | First and second modes included | All modes included | Theoretical values of equivalent derivatives Eq. (11) | Theoretical values of equivalent derivatives Eq.(14) |
|---------------|-------------|--|---------------------|---------------------------------|--------------------|---|--|
| (1)           | (2)         | (3)  | (4)                 | (5)                             | (6)                | (7)   | (8)  |
| $C_{z\alpha}$ | -2.922      | -2.2866<br>(21.745)*                                   | -3.1562<br>(8.015)  | -2.9156<br>(0.219)              | -2.922<br>(0)      | -2.0391   | -2.2865  |
| $C_{zq}$      | 14.700      | 18.3482<br>(24.818)                                    | 12.7098<br>(13.539) | 14.6662<br>(0.230)              | 14.700<br>(0)      | 20.3240   | 18.3481  |
| $C_{z\delta}$ | -0.435      | -0.0905<br>(79.195)                                    | -0.8144<br>(87.21)  | -0.4141<br>(4.805)              | -0.435<br>(0)      | 0.3256  | -0.0904  |
| $C_{m\alpha}$ | -1.660      | -0.6532<br>(60.651)                                    | -1.6225<br>(2.259)  | -1.6422<br>(1.072)              | -1.660<br>(0)      | -0.6759   | -0.6532  |
| $C_{mq}$      | -34.750     | -28.4003<br>(18.275)                                   | -34.6848<br>(0.187) | -34.8446<br>(0.272)             | -34.750<br>(0)     | -28.4815  | -28.4004   |
| $C_{m\delta}$ | -2.578      | -1.6799<br>(34.84)                                     | -2.4867<br>(3.541)  | -2.5194<br>(2.273)              | -2.578<br>(0)      | -1.7302   | -1.6799  |

\* values in parentheses represent the percentage change with respect to true values

### Aircraft Derivatives with Flex Factors

As observed from the results of Table-5, the aeroelastic effects influencing the aerodynamic derivatives get absorbed in the estimated values when a rigid body model is used for identification. As such, what we get from estimation is not the rigid body values but the equivalent derivatives. However, the rigid body and the aeroelastic effects can be estimated separately by expressing each of the equivalent derivatives  $C_i$  in a linear form, as function of dynamic pressure  $\bar{q}$  and flex factor  $k_i$ .

$$C_i(\bar{q}) = C_i(1 + k_i \bar{q}) = C_i + C_i k_i \bar{q} \quad (15)$$

The first term on the right hand side is the rigid body derivative and the second term is the dynamic pressure dependent quasi-steady effect due to structural deformation [4,5]. The flex factor is usually a function of aircraft in-vacuo frequencies and can be estimated from flight data of a flexible aircraft using system identification techniques. The complete equations for  $C_Z$  and  $C_m$  now become:

$$C_z = C_{z_0} + C_{z_\alpha} (1 + k_{z_\alpha} \bar{q}) \cdot \alpha + C_{z_q} (1 + k_{z_q} \bar{q}) \cdot q + C_{z_\delta} (1 + k_{z_\delta} \bar{q}) \cdot \delta \quad (16)$$

$$C_m = C_{m_0} + C_{m_\alpha} (1 + k_{m_\alpha} \bar{q}) \cdot \alpha + C_{m_q} (1 + k_{m_q} \bar{q}) \cdot q + C_{m_\delta} (1 + k_{m_\delta} \bar{q}) \cdot \delta \quad (17)$$

### Identification of Aeroelastic Effects

Output error estimation is applied to the simulated data of the aeroelastic aircraft at four different flight conditions to separate the dynamic pressure dependent flexibility effects from the rigid body derivatives. The variations in the dynamic pressure at different altitudes provide enough information to estimate the flex factor representing the quasi steady effects caused by structural deformation. Results are presented in Table-6 for the more flexible C3 configuration for altitudes 7.5 and 1.5 km. The estimated rigid body derivatives listed in column 3 are seen to be in close agreement with the true parameter values in column 2. It is also noticed that, for the estimated flex factor values in column 4, the equivalent derivatives computed from Eq.(15) compare very well with the theoretical estimates from eqn.(14).

**Table-6 : Estimated rigid body and flex factor values for C3 configuration**

| Parameters    | True Values | Estimated Values        |                                  | Equivalent Derivatives              |                              |                                    |                              |
|---------------|-------------|-------------------------|----------------------------------|-------------------------------------|------------------------------|------------------------------------|------------------------------|
|               |             | Rigid body values $C_i$ | Flex factor $k_i \times 10^{-5}$ | H = 7.5 Km, $\bar{q} = 10205$       |                              | H = 1.5 Km, $\bar{q} = 21455$      |                              |
|               |             |                         |                                  | $C_i (1 + k_i \bar{q})$<br>Eq. (15) | Theoretical value<br>Eq.(14) | $C_i (1 + k_i \bar{q})$<br>Eq.(15) | Theoretical value<br>Eq.(14) |
| (1)           | (2)         | (3)                     | (4)                              | (5)                                 | (6)                          | (7)                                | (8)                          |
| $C_{z\alpha}$ | -2.922      | -2.8365                 | -0.90                            | -2.5760                             | -2.5807                      | -2.2888                            | -2.2865                      |
| $C_{zq}$      | 14.700      | 14.5980                 | 1.21                             | 16.4006                             | 16.5655                      | 18.3878                            | 18.3481                      |
| $C_{z\delta}$ | -0.435      | -0.4604                 | -3.65                            | -0.2889                             | -0.2942                      | -0.0999                            | -0.0904                      |
| $C_{m\alpha}$ | -1.660      | -1.7078                 | -2.88                            | -1.2059                             | -1.2018                      | -0.6525                            | -0.6532                      |
| $C_{mq}$      | -34.750     | -35.388                 | -0.93                            | -32.030                             | -31.8834                     | -28.327                            | -28.4004                     |
| $C_{m\delta}$ | -2.578      | -2.6026                 | -1.66                            | -2.1617                             | -2.1564                      | -1.6757                            | -1.6799                      |

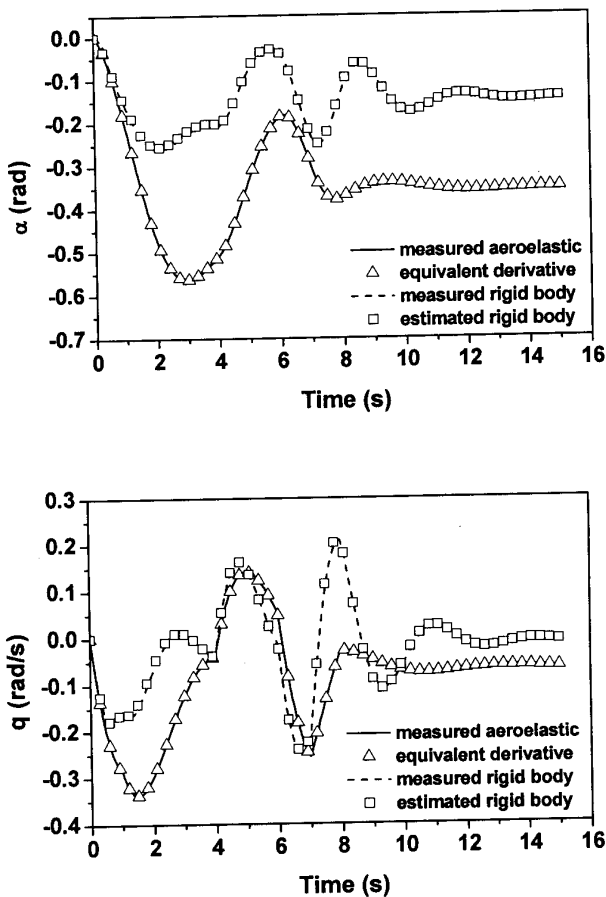


Fig.2 Measured and estimated responses with and without aeroelastic effects [ $M = 0.6$ , altitude = 1.5 Km]

In Fig. 2, the angle-of-attack and pitch rate responses generated from the model Eqs.(3) and (4) with estimated rigid body derivatives closely match the measured (simulated) rigid body responses. Also, the model responses generated with the computed derivatives listed in column 8 of Table-6 show excellent fit to the measured data with aeroelastic effects.

**Concluding Remarks**

Estimation of the rigid body derivatives from measured data of a flexible aircraft is carried out by defining the aerodynamic derivatives in a linear form with the quasi-steady aeroelastic effects described through flex factor. Output error estimation technique in time domain is applied to combined simulated data at four different flight conditions. Results show that the changes in dynamic pressure in the simulated data provide sufficient information to separately estimate the rigid body derivatives and the flex factor. The suggested methodology could be effectively applied to the flight data from large transport aircraft where the flexibility effects are likely to dominate. The present analysis assumes the rigid body derivatives at the selected flight conditions to be the same. Further work should focus on modeling the dependency of the rigid body derivatives on Mach and angle of attack.

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