TECHNICAL NOTE

= twisting moment resultants per unit length

BUCKLING OF LAMINATED COMPOSITE CYLINDRICAL PANELS UNDER TRANSVERSE LOAD

K.S. Sai Ram* and T. Sreedhar Babu⁺

Abstract

The buckling of laminated composite cylindrical shell panels subjected to transverse load is investigated. The geometrical non-linear analysis is carried out using the finite element method based on a higher-order shear deformation theory. An eight nodded degenerated isoparametric shell element with nine degrees of freedom at each node is considered. The geometric non-linear behaviour and the collapse pressures are presented for symmetrically and antisymmetrically laminated cross-ply and angle-ply cylindrical shell panels subjected to uniform normal pressure.

 S_x, S_v

Nomenclature

a,b = arc-lengths of the shell in XZ and YZ planes, respectively E_1,E_2 = Young's moduli along 1 and 2 axes of a lamina, respectively G_{12},G_{13} , = shear moduli in 1-2, 1-3 and 2-3 planes of a G_{23} lamina, respectively K_x,K_y,K_{xy} = curvatures of a shell K_x^*,K_y^* = higher-order curvatures of a shell K_x^*,K_y^* = higher-order moment resultants per unit M_x^*,M_y^* = higher-order moment resultants per unit M_x^*,M_y^* = higher-order moment resultants per unit M_x^*,M_y^* = higher-order moment resultants per unit M_x^* length I_3,m_3,n_3 = direction cosines between z and X, z and Y, z and Z axes, respectively N_i = shape function of the finite element at a node i $N_{i,x}$, w_y^* membrane forces per unit length N_x^*,N_y^* = higher-order membrane forces per unit N_{xy}^* length P_0 = intensity of normal pressure P_n = $p_0/(a/t)^4/E_2$ P_{nc} = normalized collapse pressure Q_x,Q_y^* = transverse shear forces per unit length Q_x^*,Q_y^* = higher-order transverse shear forces per unit	t = thickness of a shell u,v,w = displacement components along x,y and z axes, respectively u_o,v_o,w_o = displacements of the mid-surface along x,y and z axes, respectively u_o^*,v_o^* = higher-order displacements of the mid-surface along x and y axes, respectively U_{oi},V_{oi} = displacements of the mid-surface along W_{oi} X,Y and Z axes, respectively at a node i U_{oi}^*,V_{oi}^* = higher-order displacements of the mid- surface along X and Y axes, respectively at a node i $u_{o,x},v_{o,x}$ = derivatives of a variable with respect to a $w_{o,x}$ etc subscript W = central deflection of a panel along Z-axis W_n = W/t x,y,z = local Cartesian coordinate axes at any point on the mid-surface of a shell, x and y axes being tangential to the mid-surface X,Y,Z = global Cartesian coordinates axes γ_{xy} = shear strain in xy plane at a distance z from the mid-surface γ_{xyo} * higher-order shear strain of the mid-surface in xy plane
Q_x^*, Q_y^* = higher-order transverse shear forces per unit length R_x, R_y = radii of curvature in XZ and YZ planes, respectively	γ_{xz}, γ_{yz} = transverse shear strains at a distance z from the mid-surface

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$\gamma_{xzo}^{*},$	= higher-order transverse shear strains
γ_{yzo}^{*}	
$\varepsilon_x, \varepsilon_y$	= strains along x and y axes, respectively at a
	distance z from the mid-surface
$\epsilon_{xo}, \epsilon_{yo}$	= strains of the mid-surface along x and
	y axes, respectively
ε _{xo} *,ε _{yo}	* = higher-order strains of the mid-surface
n	= local natural coordinate of an element
θ	= fibre orientation in a lamina with reference to x-axis
$\boldsymbol{\theta}_{x}, \boldsymbol{\theta}_{y}$	= rotations of a shell about x and axes, respectively
θ_x^*, θ_v^*	= higher-order rotations of a shell about x and
	y axes, respectively
θ_{xi}, θ_{yi}	= rotations of a shell about x and y axes,
	respectively at a node i
$\theta_{x i}^{*}, \theta_{y i}^{*}$	= higher-order rotations of a shell about x and y axes, respectively at a node i
V_{12}, V_{21}	= Poisson's ratios with respect 1 and 2 axes
12 21	of a lamina
ξ	= local natural coordinate of an element
σ_{x}, σ_{y}	= normal stresses along x and y axes,
Alaxin's	respectively
$\tau_{xy}, \tau_{xz}, \tau_{yz}$	= shear stresses in xy, xz and yz planes,
	respectively
ϕ_x, ϕ_y	= shear rotations in xz and yz planes,
	respectively
ϕ_x^*, ϕ_y^*	= higher-order shear rotations in xz and
	yz planes, respectively

Introduction

Fibre reinforced plastic laminated composite shells are finding wide applications in aerospace and other industries due to their superior performance over conventional metal shells. Since these are very thin, they undergo buckling for axial and transverse loading. The buckling of shells assumes added significance because even for transverse loads the shells buckle due to their geometry. The static instability of these shells is an important engineering problem.

The study of buckling of composite shells using a geometric non-linear analysis has been considered by

some researchers. Bauld Jr. and Khot [1] investigated the buckling behaviour of axially compressed composite cylindrical panels using numerical and experimental techniques. Load verses displacement behaviour was studied for perfect and imperfect cylindrical panels. Jun and Hong [2] studied the buckling of axially loaded cylindrical panels using a non-linear finite element analysis. Buckling loads and mode shapes were presented for $(0/\pm 0/90)_s$ laminated cylindrical panels. Palazotto and Tisler [3] investigated the effect of a cutout on the collapse loads of composite cylindrical panels under axial compression. Both experimental and geometric non-linear finite element techniques were used. Goldmanis and Riekstinsh [4] studied the post-buckling of composite cylindrical panels in axial compression using a non-linear finite element analysis. The effect of the reinforcement angle on the buckling load as well as post-buckling behaviour of the panels is analysed. The non-linear and post-buckling response of curved unstiffened cylindrical panels with a circular cutout was studied by Noor et.al [5]. The panels were subjected to applied edge displacements and temperature changes. The analysis was based on the first-order shear deformation theory. Chaplin and Palazotta [6] studied the collapse of composite cylindrical panels with central cutouts subjected to axial compression using three theories, viz. Simplified large displacement moderate rotation theory, Donnel cylindrical shell theory and classical Donnel cylindrical shell theory. The effects of the size of the cutout, the transverse shear and the rotations of the shell on the collapse were investigated. The buckling of moderately thick laminated cylindrical shells was reviewed by Simitses [7]. A few results from a non-linear analysis were presented for axial compression. Kim [8] presented the buckling behaviour of composite panels under axial compression using the finite element method by performing a non-linear analysis. Results were presented for the post-buckling response of flat and curved boron/epoxy panels. Kim and Voyiadjis [9] used a nonlinear finite element formulation based on the updated Lagrangian method to study the non-linear behaviour of composite panels subjected to axial compression and transverse load. The finite element formulation was based on the first-order transverse shear deformation theory. The effect of initial geometric imperfect shape and amplitude on the non-linear response was studied. In addition to the response due to axial load, an example for the non-linear response of cylindrical panel subjected to a central transverse concentrated load was presented. Prema Kumar and Palaninathan [10] investigated the geometric non-linear response of laminated composite cylindrical panels subjected to axial compression and central concentrated load.

An eight nodded degenerated layered element with an efficient explicit through thickness integration scheme is used. Hilburger et.al [11] studied the response of compression loaded quasi-isotropic curved panels with a centrally located cutout using geometric non-linear analysis as well as experiments. The effects of cutout size, panel curvature and initial geometric imperfections on the buckling response were investigated.

From the above review, it is evident that most of the researchers considered buckling of composite cylindrical shell panels subjected to axial compression. Earlier, the authors investigated the buckling of laminated composite spherical shell panels under transverse load [12]. This paper deals with the buckling of laminated composite cylindrical shell panels subjected to transverse load. The geometric non-linear analysis is carried out using the finite element method with an eight nodded degenerated isoparametric shell element based on a higher-order transverse shear deformation theory. Nine degrees of freedom are considered at each node. A Lagrangian approach is used for this purpose. The non-linear behaviour and collapse pressures are presented for symmetrically and antisymmetrically laminated cross-ply and angle-ply cylindrical shell panels subjected to uniformly distributed normal pressure.

Governing Equations

Consider a laminated shell of uniform thickness, consisting of a number thin laminae, each of which may be arbitrarily oriented at an angle θ with reference to the x-axis of the local coordinate system (Fig.1). The displacements along the local coordinate axes x, y and z at any point in the shell are assumed as

$$u = u_0 + z\theta_y + z^2 u_0^* + z^3 \theta_y^*,$$

$$v = v_0 - z\theta_x + z^2 v_0^* - z^3 \theta_x^*,$$

$$w = w_0.$$
(1)

The strains at any point in the shell along the local coordinate axes x,y and z are expressed as

$$\varepsilon_{x} = u_{,x} = \varepsilon_{x0} + zK_{x} + z^{2} \varepsilon_{x0}^{*} + z^{3} K_{x}^{*},$$

$$\varepsilon_{y} = v_{,y} = \varepsilon_{y} + zK_{y} + z^{2} \varepsilon_{y0}^{*} + z^{3} K_{y}^{*},$$

$$\gamma_{xy} = u_{,y} + v_{,x} = \gamma_{xy0} + zK_{xy} + z^{2} \gamma_{xy0}^{*} + z^{3} K_{xy}^{*},$$

$$\gamma_{xz} = u_{,z} + w_{,x} = \phi_x + z \gamma_{xz0}^* + z^2 \phi_x^*,$$

$$\gamma_{yz} = v_{,z} + w_{,y} = \phi_y + z \gamma_{yz0}^* + z^2 \phi_y^*,$$
(2)

where

$$\begin{aligned} \varepsilon_{x0} &= u_{0,x} + (w_{0,x})^2 / 2, \ \varepsilon_{y0} &= v_{0,y} + (w_{0,x})^2 / 2, \\ \gamma_{xy0} &= u_{0,y} + v_{0,x} + w_{0,x} \ w_{0,y}, \ K_x &= \theta_{y,x}, \ K_y &= -\theta_{x,y}, \\ K_{xy} &= \theta_{y,y} - \theta_{x,x}, \ \varepsilon_{x0}^* &= u_{0,x}^*, \ \varepsilon_{y0}^* &= v_{0,y}^*, \ \gamma_{xy0}^* &= u_{0,y}^* + v_{0,x}^*, \\ K_x^* &= \theta_{y,x}^*, \ K_y^* &= -\theta_{x,y}^*, \ K_{xy}^* &= \theta_{y,y}^* - \theta_{x,x}^*, \ \phi_x &= \theta_y + w_{0,x}, \\ \phi_y &= -\theta_x + w_{0,y}, \ \gamma_{xz0}^* &= 2u_0^*, \ \gamma_{yz0}^* &= 2v_0^*, \ \phi_x^* &= 3\theta_y^*, \\ \phi_y^* &= -3\theta_x^*. \end{aligned}$$
(3)

The incremental strains of the shell along coordinate axes x,y and z axes are obtained as

$$d\varepsilon_{x0} = (du_0)_{,x} + w_{0,x} (dw_0)_{,x}, d\varepsilon_{y0} = (dv_0)_{,y} + w_{0,y} (dw_0)_{,y}, d\gamma_{xy0} = (du_0)_{,y} + (dv_0)_{,x} + w_{0,x} (dw_0)_{,y} + w_{0,y} (dw_0)_{,x}, dK_x = (d\theta_y)_{,x}, dK_y = -(d\theta_x)_{,y}, dK_{xy} = -(d\theta_y)_{,y} - (d\theta_x)_{,x}, d\varepsilon_{x0}^* = (du_0^*)_{,x}, dK_{xy} = -(d\theta_y)_{,y} - (d\theta_x)_{,x}, d\varepsilon_{x0}^* = (du_0^*)_{,x}, dK_{xy} = -(d\theta_y)_{,y} - (d\theta_y)_{,x}, d\varepsilon_{x0}^* = (du_0^*)_{,x}, d\xi_{x0} = (du_0^*)_{,x}, d\xi_{x0} = (du_0^*)_{,x}, d\xi_{x0} = (du_0^*)_{,x}, d\xi_{x0} = (d\xi_{x0})_{,x}, d$$

Fig. 1 Cylindrical shell panel

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$$d\varepsilon_{y0}^{*} = (dv_{0}^{*})_{,y}, d\gamma_{xy0}^{*} = (du_{0}^{*})_{,y} + (dv_{0}^{*})_{,x}, dK_{x}^{*} = (d\theta_{y}^{*})_{,x},$$

$$dK_{y}^{*} = -(d\theta_{x}^{*})_{,y}, dK_{xy}^{*} = (d\theta_{y}^{*})_{,y} - (d\theta_{x}^{*}), d\phi_{x} = d\theta_{y} + (dw_{0})_{,x}, d\phi_{y} = -d\theta_{x} + (dw_{0})_{,y}, d\gamma_{xz0}^{*} = 2 du_{0}^{*},$$

$$d\gamma_{yz0}^{*} = 2dv_{0}^{*}, d\phi_{x}^{*} = 3 d\theta_{y}^{*}, d\phi_{y}^{*} = -3 d\theta_{x}^{*}.$$
 (4)

The incremental constitutive equations of the shell are given by

$$\left\{ dF \right\} = [D] \left\{ d\chi \right\},\tag{5}$$

where

$$\{ dF \} = \{ dN_x, dN_y, dN_{xy}, dM_x, dM_y, dM_{xy}, dN_{xy}^*, dN_x^*, dN_y^*, dN_x^*, dN_y^*, dN_x^*, dM_y^*, dM_{xy}^*, dQ_x, dQ_y, dS_x, dS_y, dQ_x^*, dQ_y^* \}^T, \{ d\chi \} = \{ d\varepsilon_{x0}, d\varepsilon_{y0}, d\gamma_{xy0}, dK_x, dK_y, dK_{xy}, d\varepsilon_{x0}^*, d\varepsilon_{y0}^*, d\gamma_{xy0}^*, dK_x^*, dK_y^*, d\phi_x, d\phi_y, dK_x^*, d\gamma_{xy}^*, d\phi_x^*, d\phi_y^* \}^T.$$

Expressions for stress resultants N_x , N_y , N_{xy} , M_x , M_y , M_{xy} , N_x^* , N_y^* , N_{xy}^* , M_x^* , M_y^* , M_{xy}^* , Q_x , Q_y , S_x , S_y , Q_x^* , Q_y^* and various stiffnesses of the elasticity matrix [D] are available in Ref.[12].

Finite Element Formulation

An eight nodded degenerated isoparametric shell element [13,14] is considered in the present analysis. Nine degrees of freedom are considered at each node. The tangent stiffness matrix and incremental nodal load vector of the element are derived using the principle minimum potential energy. The geometry of the element is defined by the global coordinates X,Y and Z. The displacements at any point in the element are expressed in terms of the displacements at the nodes using the element shape functions and direction cosines [12].

Element Stiffness Matrix

Substituting the expressions for displacements at any point in the element in terms of the nodal displacements [12] in equations (4), the incremental strain vector of the element is represented in the form

$$\left[d\chi \right] = \left[\left[B^{L} \right] + \left[B^{NL} \right] \right] \left\{ d\delta^{e} \right\}, \tag{6}$$

where

$$\left[d\delta^{e} \right] = \left\{ dU_{01}, dV_{01}, dW_{01}, d\theta_{x1}, d\theta_{y1}, dU_{01}^{*}, dV_{01}^{*}, d\theta_{x1}^{*}, d\theta_{y1}^{*}, \dots, dU_{08}, dV_{08}, dW_{08}, d\theta_{x8}, d\theta_{y8}, d\theta_{x8}^{*}, d\theta_{y8}^{*}, dU_{08}^{*}, dV_{08}^{*}, d\theta_{x8}^{*}, d\theta_{y8}^{*}, dU_{08}^{*}, dV_{08}^{*}, d\theta_{x8}^{*}, d\theta_{y8}^{*} \right\}^{T}$$

and the non-zero elements of linear and non-linear incremental strain-displacement matrices $[B^{L}]$ and $[B^{NL}]$ are given in Ref.[12].

The linear stiffness matrix (due to small displacement) of the element is given by

$$[K^{eL}] = \int_{-1-1}^{1} \int_{-1}^{1} [B^{L}]^{T} [D] [B^{L}] |J| d\xi d\eta, \qquad (7)$$

where |J| is the determinant of the Jacobian matrix [12].

The initial displacement stiffness matrix (due to large displacement) is given by

$$[K^{eL}] = \int_{-1-1}^{1} \int_{-1-1}^{1} ([B^{L}]^{T}[D] [B^{L}] + [B^{NL}] [D] [B^{NL}] + [B^{NL}]^{T}[D] [B^{L}]) |J| d\xi d\eta.$$
(8)

Element Initial Stress Stiffness Matrix

The non-linear strains of the shell are expressed as

$$\left\{ \epsilon_{xnl}, \epsilon_{ynl}, \gamma_{xynl} \right\}^{T} = \left\{ (w_{0,x})^{2} / 2, (w_{0,y})^{2} / 2, w_{0,x} w_{0,y} \right\}^{T}$$
$$= [U] \left\{ f \right\} / 2, \tag{9}$$

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where $\{f\} = \{w_{,x}, w_{,y}\}^T$ and [U] is obvious from equations (9). $\{f\}$ is expressed as

$$\left\{f\right\} = \left[G\right]\left\{\delta^{e}\right\},$$

where

$$[G] = \sum_{i=1}^{8} \begin{bmatrix} 1_{3} N_{i,x} & m_{3} N_{i,x} & n_{3} N_{i,x} & 0 & 0 & 0 & 0 & 0 \\ 1_{3} N_{i,y} & m_{3} N_{i,y} & n_{3} N_{i,y} & 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix}, \\ \left\{\delta^{e}\right\} = \left\{U_{01}, V_{01}, W_{01}, \theta_{x1}, U_{01}^{*}, V_{01}^{*}, \theta_{x1}^{*}, \theta_{y1}^{*}, \dots, \right\}$$

$$U_{08}, V_{08}, W_{08}, \Theta_{y8}, U_{08}^*, V_{08}^*, \Theta_{x8}^*, \Theta_{y8}^* \}^T.$$

The initial stress stiffness matrix of the element is given by

$$[K_{\sigma}^{e}] = \int \int [G]^{T} [S] [G] \mathcal{V} | d\xi d\eta, \qquad (10)$$

where

$$[S] = \begin{bmatrix} N_x & N_{xy} \\ & & \\ N_{xy} & N_y \end{bmatrix}.$$

The tangent stiffness matrix of the element is obtained by adding $[K^{eL}]$, $[K^{eNL}]$ and $[K\sigma^{e}]$, i.e.

$$[K_T^e] = [K^{eL}] + [K^{eNL}] + [K_{\sigma}^e].$$
(11)

Element Incremental Load Vector

The incremental element load vector due to incremental uniform normal pressure dp_o , assuming that the load acts on the mid-surface of the shell, is given by

$$\left\{dP^{e}\right\} = \left\{\left\{dP^{1}\right\}\dots,\left\{dP^{8}\right\}\right\}^{T},$$
(12)

where
$$\{dP^i\} = \int_{-1}^{1} \int_{-1}^{-1} N_i |dq| |J| d\xi d\eta$$
, (i = 1 to 8)

in which

$$\{dq\} = \{l_3 dp_0, m_3 dp_0, n_3 dp_0, 0, 0, 0, 0, 0, 0\}^T$$

Solution Process

Equations (7), (8), and (12) are evaluated by performing numerical integration using the 2 x 2 Gauss quadrature whereas equation (10) is evaluated using 3 x 3 Gauss quadrature. The element tangent stiffness matrices $[K_T^e]$ and element incremental load vectors $\{dP^e\}$ are assembled to obtain their respective global matrices $[K_T]$ and $\{dP\}$. The incremental unknown displacements at the nodes of the shell are obtained from the incremental equilibrium condition

$$[K_T] \left| d\delta \right| = \left| dP \right|. \tag{13}$$

These incremental equations are solved using the Newton-Raphson iteration method [15] with the help of Gauss elimination technique [15]. Knowing the incremental displacements $\{d\delta\}$, the total displacements at any load level are obtained by adding the incremental displacements to displacements at the earlier load level. From the known displacements at any load level, the strains of the shell $\{\chi\}$ are evaluated from equations (3) and the expressions for displacements at any point in the element in terms of the nodal displacements [12] and then the stress resultants are obtained from

$$\{F\} = [D] \{\chi\}, \tag{14}$$

where

$$\begin{split} & \{\chi\} = \{\varepsilon_{x0}, \varepsilon_{y0}, \gamma_{xy0}, K_x, K_y, K_{xy}, \varepsilon_{x0}^*, \varepsilon_{y0}^*, \gamma_{xy0}^*, \\ & K_x^*, K_y^*, K_{xy}^*, \phi_x, \phi_y, \gamma_{xz}^*, \gamma_{yz}^*, \phi_x^*, \phi_y^*\}^T, \\ & \{F\} = \{N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, N_x^*, N_y^*, N_{xy}^*, \\ & M_x^*, M_y^*, M_{xy}^*, Q_x, Q_y, S_x, S_y, Q_x^*, Q_y^*\}^T. \end{split}$$

Results and Discussion

In the present investigation, the non-linear behaviour and the collapse pressures are presented for symmetrically and anti-symmetrically laminated cylindrical shell panels $(R_x/a = 3, a/b = 1)$ subjected to uniform normal pressure p_0 . The straight edges of the panels are assumed as simply supported whereas the curved edges are assumed as free. Mid-surface linear displacements are restrained along the supported edges. The following lamina material properties are used throughout the investigation. $E_1/E_2 = 25$, $G_{12}/E_2 = 0.5$, $G_{13}/E_2 = 0.5$, $G_{23}/E_2 = 0.2$ and $v_{12} = 0.25$. The entire cylindrical panel is discretised with 64 elements (8 x 8). The accuracy of the present finite element analysis and the necessity of a higher-order theory are shown in Ref.[12]. The fibre orientation angle θ in each lamina is measured with respect to the generator.

The normalized central deflection, $W_n = W/t$, and the normalized uniform normal pressure, $p_n = p_0 (a/t)^4/E_2$, are plotted for symmetrically and anti-symmetrically laminated cylindrical shell panels subjected to uniform normal pressure for a/t (arc-length/thickness) ratios equal to 20 and 50 and the plots are shown in Figs.2 to 5. The normalized collapse pressures p_{nc} are also given in these figures.

The collapse pressure of $[90^{\circ}/0^{\circ}/90^{\circ}]$ cylindrical shell panel with a/t = 20 is about 4.1 times the collapse pressure of $[0^{\circ}/90^{\circ}/0^{\circ}]$ cylindrical shell panel with the same a/t ratio whereas the collapse pressure of $[90^{\circ}/0^{\circ}/90^{\circ}]$ cylindrical shell panel with a/t = 50 is about 10 times the collapse pressure $[0^{\circ}/90^{\circ}/0^{\circ}]$ cylindrical shell panel with the same a/t ratio. The reason for this is as follows. In the above panels, the first (top) lamina is subjected to maximum compression. In $[90^{\circ}/0^{\circ}/90^{\circ}]$ cylindrical shell panel, the fibres are along the direction of compression in the first lamina whereas the fibres are across the direction of compression in the first lamina of $[0^{\circ}/90^{\circ}/0^{\circ}]$. Since the Young's modulus of a lamina along the direction of fibres is more than that across the direction of fibres, $[90^{\circ}/0^{\circ}/90^{\circ}]$ has more collapse pressure than $[0^{\circ}/90^{\circ}/0^{\circ}]$. The collapse pressure of $[\theta/-\theta/\theta]$ cylindrical shell panel increases with the increase in the value of θ from 15° to 75°. This is due to increase in the stiffness of the cylindrical shell panel along the directrix with the increase in the value of θ .

The collapse pressure of $[90^{\circ}/0^{\circ}/90^{\circ}/0^{\circ}]$ cylindrical shell panel with a/t = 20 is about 1.96 times the collapse pressure of $[0^{\circ}/90^{\circ}/0^{\circ}]$ cylindrical shell panel with the same a/t ratio whereas the collapse pressure of $[90^{\circ}/0^{\circ}/90^{\circ}/0^{\circ}]$ cylindrical shell panel with a/t = 50 is about 1.165 times the collapse pressure of $[0^{\circ}/90^{\circ}/0^{\circ}/90^{\circ}]$ cylindrical shell panel with the same a/t = ratio. The collapse pressure of $[\theta/-\theta/\theta-\theta]$ cylindrical shell panel also increases with the increase in the value of θ from 15^o to 75^o. The explanation given for the variation of collapse pressure for symmetrically laminated cylindrical shell panels is valid for anti-symmetrically laminated cylindrical shell panels also.



Fig. 2 Buckling response of laminated composite cylindrical shell panels with a/t=20



Fig. 3 Buckling response of laminated composite cylindrical shell panels with a/t=50

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Fig. 4 Buckling response of laminated composite cylindrical shell panels with a/t=20

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Fig. 5 Buckling response of laminated composite cylindrical shell panels with a/t=50

The collapse pressures of $[\theta/-\theta/\theta-\theta]$ cylindrical shell panels are higher than the collapse pressures of corresponding $[\theta/-\theta/\theta]$ cylindrical shell panels, i.e. for the same a/t ratio. It is observed that the deformed shapes of the cylindrical shell panels with a/t ratios 20 and 50 at the limit point remain cylindrical.

Conclusions

The buckling of laminated composite cylindrical shell panels is studied using a geometric non-linear finite element analysis based on a higher-order shear deformation theory. The non-linear behaviour and the collapse pressures are presented for symmetrically and anti-symmetrically laminated cylindrical shell panels with straight edges 'simply supported' and curved edges 'free' subjected to uniform normal pressure. The following conclusions may be made from the results.

In the case of three layered symmetric cross-ply cylindrical panels, the collapse pressures are more when the fibre orientation in the top and bottom laminae is along the directrix compared to the collapse pressures when the fibre orientation in the top and bottom laminae is along the generator. In the case of four layered anti-symmetric cross-ply cylindrical panels, the collapse pressures are more when the fibre orientation in the first (top) and third laminae is along the directrix compared to the collapse pressures when the fibre orientation in these layers is along the generator. The collapse pressure of an angle-ply cylindrical panel (both symmetric and anti-symmetric) increases with the increase in fibre orientation angle (measured with respect to generator). Three layered symmetric angle-ply cylindrical panels have less collapse pressures compared to four layered anti-symmetric angleply cylindrical panels for the same arc-length to thickness ratio. It is observed that the deformed shapes of both the thick and thin composite cylindrical panels at the limit point remain cylindrical.

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