NEAR REAL TIME ESTIMATION OF NONLINEAR AERODYNAMICS

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Abstract

This paper describes a novel technique for near real time estimation of non-linear aerodynamic coefficient dependencies on one or more independent variables. It is important from the point of view of updating the aerodynamic database of six DOF simulations subsequent to flight test. The technique is validated using the six DOF simulated data. The novelty about this technique is the combined utilization of linear interpolation and recursive least squares estimation to capture nonlinear functional dependencies that makes it suitable for multidimensional near real time estimation. The technique is preceded by data compatibility check using Extended Kalman filter.

Key words: Estimation, Nonlinear Aerodynamic Coefficients, Recursive Least Squares, Extended Kalman Filtering

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 C_{ma} = C_m derivative with respect to pitch rate $C_{m\delta e}$ = C_m derivative with respect to elevator deflection θ $=$ vector of unknown parameters to be estimated $X = \text{regressor}$ $Y =$ output vector

 $P = correlation matrix$

Introduction

Aerodynamic data of an aircraft is updated as a part of flight-testing if significant differences are found between predicted response and measured data. Numerous techniques are available for flight parameter estimation [1,2]. The dynamic model of the aircraft is estimated using response of the aircraft and control inputs. Most of these techniques are applicable for the estimation of linear mathematical models of flight dynamics. Estimation of linear models captures only the average trend of flight characteristics. Many aircraft do not posses linear characteristics. There exists nonlinear dependence of the aerodynamic coefficients with respect to the variables like Mach number, angle of attack, sideslip, angular rates and control effectors. In the case of modern combat aircraft these nonlinearities can be present even before the stall angle of attack is reached [3-4]. It is important to capture this nonlinear dependence during the update process. The mathematical model is validated by comparing outputs obtained from the mathematical model against flight data.

The wind tunnel model of an aircraft can be quite comprehensive. This is constructed by representing the nonlinear variations in the form of table lookup. It is possible to capture the nonlinear characteristics with sufficient fidelity by choosing break points at proper locations and using linear interpolation in between. In this paper, it is shown that this formulation can be the basis for parameter estimation.

Multiple adaptive regression splines are capable of modeling nonlinear aerodynamic coefficient dependencies [5,6]. Spline modeling calls for the stepwise regression approach for adequate model structure determination. Subsequent to the spline modeling, the Expectation Maximization (EM) algorithm [5] or Filter Error Method (FEM) [6] can be used for the estimation of spline coefficients. The EM technique and FEM are time consuming as they involve nonlinear optimization and are suitable for accurate offline estimation purposes. Wind tunnel prediction through incremental coefficients obtained from flight data analysis is also a well-known offline approach [7]. Postulating suitable derivative models in an analytical form will also aid identification of an aerodynamic database [8]. This is again an accurate offline method. The aim of this paper is to propose a technique for near real time non-linear estimation purposes. The term non-linear estimation in this paper implies estimating the functional dependencies without actually postulating a fixed structure model. The table look-up model that is similar to that of a wind-tunnel database is updated directly. As already mentioned, the wind tunnel table look-up mathematical model captures the underlying nonlinearity in most of the flight regimes using interpolation techniques with suitable break points. This is particularly true for a high performance fighter aircraft. The aerodynamics is generally nonlinear in flight regimes beyond stall. In addition, near stall, the aerodynamics can exhibit hysteresis (more than one value for one independent variable). Therefore, the aerodynamics beyond stall is not easily captured in the form of functional dependence as seen in [9]. This method can be applied where there exists a functional dependence of the aerodynamic forces and moments on the flight variables.

Linear least squares method is an efficient tool for the estimation of flight derivatives. The recursive version of least squares has been shown to be computationally efficient and can be used in real time [10]. In this paper an estimation technique is proposed using RLS, which directly updates the aerodynamic coefficient dependency on one or more independent variables represented in table look-up form [11]. The proposed technique offers the following advantages:

- Near real time estimation of nonlinear aerodynamic coefficients
- A priori Model structure determination is not required
- It does not involve non-linear optimization
- The break points for the estimation can be chosen similar to that of wind tunnel break points, as they are optimal
- Easy to handle multiple dimensions
- Possible to apply for unstable aircraft

The technique is validated using 6 DOF simulated data of a light transport aircraft. Additive process noise of standard deviation 2 m/sec is added to the simulated data.

Bias and scale factor errors are added to the data to simulate realistic scenario. The whole estimation procedure consists of two passes. The first pass is the data compatibility check using EKF. The choice of EKF as against EM/FEM is to avoid nonlinear optimization for near real time usage of the technique. The second pass is the proposed nonlinear estimation using RLS.

Postulated Model

To prove the validity of technique, longitudinal aerodynamics is considered in this paper. The longitudinal non-linear aerodynamic coefficients are computed from the measured variables by inverting the 6 DOF equations as follows:

$$
C_X = (A_X M - Ft_X)/(qS)
$$
 (1)

$$
C_{Z} = (A_{Z} M - Ft_{Z})/(qS)
$$
 (2)

$$
C_m = \left((\dot{q}I_Y - (I_Z - I_X)rp - (r^2 - p^2)I_{ZX}) - \left(Ft_X Dzeng - Ft_Z Dxeng \right) \overline{q} S \overline{c} \right)
$$
(3)

After computing the coefficients, the coefficients are modeled as functions of state and control variables. The longitudinal force and moment coefficients are postulated as non-linear models as follows:

$$
C_{X} = C_{X}(\alpha)
$$
\n⁽⁴⁾

$$
C_Z = C_Z(\alpha) + C_{Zq} q \overline{c}/2 V + C_{Z\delta e} \delta e \tag{5}
$$

$$
C_m = C_m(\alpha) + C_{mq}q \overline{c}/2 V + C_{m\delta e} \delta e
$$
 (6)

where $C_X(\alpha)$, $C_Z(\alpha)$ and $C_m(\alpha)$ represents the functional variation of coefficients with angle of attack. C_{Zq} , $C_{Z\delta e}$, C_{mq} and $C_{m\delta e}$ are linear derivatives.

Linear Interpolation

The aerodynamic nonlinearities are commonly represented by linear table look-up method. Suitable choice of breakpoints is necessary to capture the variation in the coefficient. The usual independent variables are angle of attack, sideslip and control surface deflection. The rate dependence becomes important for rotary balance data at high rotation rates. For a typical aircraft it is common to have joint dependence on more than one variable leading

to a multi-dimensional table look-up. The estimation procedure is motivated by linear interpolation.

Consider the one-dimensional interpolation formulae:

$$
y(x) = y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)} \cdot (x - x_1)
$$

=
$$
\frac{(x_2 - x)}{(x_2 - x_1)} \cdot y_1 \frac{(x - x_1)}{(x_2 - x_1)} \cdot y_2
$$

= $W_l(x) \cdot y_1 + W_r(x) \cdot y_2$ (7)
 $x \in [x_1, x_2]$

This equation gives the function value between two breakpoints (x_1, y_1) and (x_2, y_2) . We have recast the formulae in the form of a left weight $W_l(x)$ multiplying the left hand value of the function y_1 and a right weight $W_r(x)$ multiplying the right hand value y_2 . It is noted that the weights are themselves functions of the independent variable *x*. The value of the function $y(x)$ is a linear combination of the weights and the values at breakpoints. The advantage of expressing in this manner is that the linear interpolation can be generalized to multiple dimensions easily. Consider the two dimensional interpolation formulae (Fig. 1).

$$
y(\theta_1, \theta_2) = W_{l1}(\theta_1) \cdot W_{l2}(\theta_2) \cdot y_{i,j} + W_{l1}(\theta_1) \cdot W_{r2}(\theta_2) \cdot y_{i,j+1} + W_{r1}(\theta_1) \cdot W_{l2}(\theta_2) \cdot y_{i,+1,j} + W_{r1}(\theta_1) \cdot W_{r2}(\theta_2) \cdot y_{i,+1,j+1} \theta_1 \in \left[\theta_{1i}, \theta_{1(i+1)}\right], \theta_2 \in \left[\theta_{2j}, \theta_{2(j+1)}\right]
$$
\n(8)

The weights in the Eq. (8) are computed separately for each dimension in an identical manner to the one-dimensional interpolation in Eq. (7). To compute the value of the function, the four possible combinations of the two onedimensional weights are formed and multiplied with the corresponding breakpoint value of the two-dimensional function. This process can be followed for any higher dimensional table look-up with linear interpolation. In the

Fig.1 Two-dimensional interpolation grid

context of the estimation problem posed in this paper, the values of dependent variable are the unknowns (y_{ii}) . It is noted that the value of the function at an intermediate point y (θ ₁, θ ₂) is a linear function of the weights and the unknowns y_{ii} . This is true in general for the multi-dimensional linear interpolation scheme as well.

Estimation Procedure

As already mentioned, the whole estimation is performed in two passes over the data. The flow diagram is given in Fig.2.

Pass 1: Data Compatibility using EKF

In general, the data used in estimation are affected by noise, biases and scale factors. The data compatibility check ensures that the measurements used in the estimation process are consistent and error free. This pass is also called as flight path reconstruction. The kinematic model for the data compatibility check is derived from the translational equations of motion of an aircraft. In flight path reconstruction, basically state estimation is performed using the kinematic model accounting for the systematic errors. The complete set of non-linear system of equations with the inclusion of biases are written as follows:

$$
\dot{u} = -(q_m - \Delta q) w + (r_m - \Delta r) v - g \sin \theta + a_x^{CG} \qquad u(t_0) = u_0
$$

$$
\dot{v} = -(r_m - \Delta r) u + (p_m - \Delta p) v + g \cos \theta \sin \phi + a_y^{CG} \qquad v(t_0) = v_0
$$

Fig.2 Flow diagram of estimation process

$$
\dot{w} = -(p_m - \Delta p) v + (q_m - \Delta q) u + g \cos \theta \cos \phi + a \frac{CG}{z} w (t_0) = w_0
$$

$$
\dot{\phi} = (p_m - \Delta p) + (q_m - \Delta q) \sin \phi \tan \theta
$$

$$
+ (r_m - \Delta r) \cos \phi \tan \theta
$$

$$
\dot{\theta} = (q_m - \Delta q) \cos \phi - (r_m - \Delta r) \sin \phi
$$

$$
\dot{\theta} = (q_m - \Delta q) \sin \phi \sec \theta + (r_m - \Delta r) \cos \phi \sec \theta \quad \psi (t_0) = \psi_0
$$

$$
\dot{\theta} = u \sin \theta - v \cos \theta \sin \phi - w \cos \theta \cos \phi
$$

$$
h (t_0) = h_0
$$

(9)

The subscript 'm' indicates measured quantity. The 'Δ' prefix represents bias present in the measured quantity.

The measurement equations are given by :

$$
V_m = \sqrt{u^2 + v^2 + w^2}
$$

\n
$$
\alpha_m = K_\alpha \tan^{-1} \left(\frac{w}{u}\right) + \Delta \alpha
$$

\n
$$
\beta_m = K_\beta \sin^{-1} \left(\frac{v}{\sqrt{u^2 + v^2 + w^2}}\right) + \Delta \beta
$$

\n
$$
\phi_m = \phi
$$

\n
$$
\theta_m = \theta
$$

$$
\Psi_m = \Psi
$$

\n
$$
h_m = h \tag{10}
$$

The quantities K_{α} and K_{β} represent scale factor errors in the measurement of angle of attack and sideslip respectively. More details on FPR can be found from [1]. Consider the following nonlinear system of equations:

$$
\dot{x}(t) = f[x(t), u_{i}(t)] + Fw_{n}(t)
$$

$$
y(t) = g[x(t), u_{i}(t)]
$$

$$
z(n) = y(n) + Gv_{n}(n)
$$
 (11)

where f and g are system functions of the state and observation equations and F and G are the corresponding noise distribution matrices. The algorithmic steps of EKF for nonlinear state estimation are given as follows [1-2]:

Prediction

$$
\tilde{x}(n+1) = \tilde{x}(n) + \int_{t_n}^{t_{n+1}} f\left[\hat{x}(t), \overline{u}_i(t)\right] dt
$$

$$
P(n+1) \approx \Phi(n+1) \hat{P}(n) \Phi^{T}(n+1) + \Delta t F F^{T}
$$
 (12)

where

$$
\Phi(n+1) = e^{A(n)\Delta t} \approx I + A(n)\Delta t + A^{2}(n)\Delta t^{2}/2! + ...
$$

$$
A(n) = \frac{\partial f[x(t), u_i(t)]}{\partial x} \bigg|_{x = \hat{x}(n)} \tag{13}
$$

Correction

$$
\tilde{y}(n) = g [\tilde{x}(n), u_{i}(n)]
$$
\n
$$
K(n) = \tilde{P(n)} C^{T} [C \tilde{P C}^{T} + R]^{-1}
$$
\n
$$
\tilde{x}(n) = \tilde{x}(n) + K(n) [z(n) - \tilde{y}(n)]
$$
\n
$$
\hat{P}(n) = [I - K(n) C] \tilde{P(n)} [I - K(n) C]^{T} + K(n) R K^{T}(n)
$$
\n(14)

where

$$
C(n) = \frac{\partial g\left[x\left(t\right), u_{i}\left(t\right)\right]}{\partial x}\Bigg|_{x=\tilde{x}(n)}\tag{15}
$$

where *u i* denotes the average or interpolated values of the inputs between the time points n and $n+1$, \sim the predicted variables, \land the updated variables, K the Kalman gain matrix and R the measurement noise covariance matrix.

Pass 2: Non-linear Estimation using RLS

The estimation problem for model structure given in Eq. (4-6) consists of determining the unknown functions $C_X(\alpha)$, $C_Z(\alpha)$, and $C_m(\alpha)$. The estimation of unknown (possibly nonlinear) functions $C_X(\alpha)$, $C_Z(\alpha)$ and $C_m(\alpha)$ is simplified by treating the function values at selected breakpoints as the unknowns and assuming linear interpolation for values between the breakpoints. For example, if 'n' equidistant breakpoints $(\alpha_1, \alpha_2, ..., \alpha_n)$ in the independent variable α are chosen and the corresponding function values for $C_X(\alpha)$ are represented as x_1, x_2, \ldots, x_n .

$$
\hat{\theta} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}
$$
 (16)

Consider thge kth data point of the time response. Using the linear interpolation formulae, the force equation given in Eq. (4) can be written as:

$$
C_X = \begin{bmatrix} 0 & \dots & W_{lk} & W_{rk} & \dots & 0 & 0 & 0 \end{bmatrix} \hat{\theta} + e_k \tag{17}
$$

where W_{lk} and W_{rk} are the weights obtained from the interpolation scheme using Eq. (7) for the ith and i+1th breakpoints (i.e., for α lying between ith and i+1th breakpoints). For e.g., assume there are 20 break points in angle of attack $(-1, 0, 1, 2,..., 18)$ and kth data point has value of angle of attack $= 10.4234$. Clearly, this value falls between breakpoints 10 and 11. This means that the left index is 12

and the right index is 13. The left weight is computed as $W_{1k} = (11-10.4234)/(11-10) = 0.5766$ as per Eq. (7).

Similarly, the right weight is computed as $W_{rk} = 1 - 0.5766$ $= 0.4234$. These weights are placed in the 12 and 13th columns of the regressor. Remaining entries in the regressor are zeros.

The above equation can be extended in the vector form for all 'n' data points and can be written as:

$$
Y = X \hat{\theta} + e \tag{18}
$$

In general, the breakpoints need not be same for multiple unknown functions and they need not be equidistant. The technique allows the user to choose these depending on the problem at hand. The complete regressor X has to be formed considering all the data points.The unknown $\hat{\theta}$ is estimated using the well known least squares solution.

$$
\hat{\theta} = (X^T X)^{-1} (X^T Y) \tag{19}
$$

The regressor for the estimation of $C_m(\alpha)$ and $C_z(\alpha)$ remain the same as above because the calculation of weights depend only on angle of attack. In addition to the estimation of $C_m(\alpha)$, the estimation of C_{mq} , $C_{m\delta e}$ is required to model C_m . Similarly to model C_Z , estimation of C_{Za} , $C_{Z\delta e}$ is required. This can simply be accomplished by adding pitch rate signal and elevator control surface input signal as additional columns in the regressor.

Using the matrix inversion lemma, the recursive form of Eq. (19) can be obtained which results in Recursive Least Squares (RLS) estimation. The unknown parameter vector θ can be estimated recursively using the following steps:

Initialize the algorithm by setting $P_0 = \delta^{-1} I$, where δ is a small positive constant and

$$
\hat{\theta}_0 = 0 \tag{20}
$$

For each instant of time, $n = 1, 2, \dots, N$ compute :

$$
\pi_n = X_n P_{n-1}
$$

$$
\kappa_n = 1 + \pi_n X_n^T
$$

$$
k_{n} = P_{n-1} X_{n}^{T} / \kappa_{n}
$$

\n
$$
\alpha_{n} = Y_{n} - (\hat{\theta}_{n-1} X_{n}^{T})^{T}
$$

\n
$$
\hat{\theta}_{n} = \hat{\theta}_{n-1} + (k_{n} \alpha_{n})^{T}
$$

\n
$$
P'_{n-1} = k_{n} \pi_{n}
$$

\n
$$
P_{n} = (P_{n-1} - P'_{n-1})
$$
\n(21)

where *P* is the correlation matrix. The advantage of recursive estimation lies in avoiding the matrix inversion and numerical ill conditioning. Also it may be used to perform real time estimation. The starting value for $\hat{\theta}_0$ is 0 as equation error estimation techniques do not require them strictly. The correlation matrix is fixed at 10,000*I. where, I is identity matrix [1].

Results

The data for the estimation is obtained from the six DOF simulation software of a light transport aircraft. As already mentioned, the concept of the paper is proven for longitudinal estimation. Process noise of standard deviation 2m/sec is added as turbulence to the simulated data. Further, scale factor and bias is introduced in angle of attack, bias is added to pitch rate, forward acceleration and normal acceleration. The data affected by errors are processed in the data compatibility check. The estimated states in the data compatibility check are given as follows:

$$
\left[u, v, w, \phi, \theta, \psi, K_{\alpha}, \Delta \alpha, \Delta q, \Delta a_x, \Delta a_z \right]
$$

The estimated states are compared with their true values as well as the signal containing noise and systematic errors in Fig. 3-12.

The effect of process noise is seen in lateral velocity, as its magnitude is low. It can be noticed that the flight path reconstruction using EKF removed all the systematic errors. The estimated bias and scale factor errors are compared with true values in Table-1. Since, the longitudinal estimation is addressed in the results, the magnitude of sideslip is negligible in the longitudinal maneuver, the scale factor K_{β} in Eq. (10) is assumed to be 0 and hence not estimated.

After the removal of systematic errors from the responses, the longitudinal force and moment coefficients are computed using Eq. (1, 2, 3). Subsequent to the computation of coefficients, the nonlinear estimation is per-

Fig.3 Forward velocity from FPR compared with true value

Fig.4 Lateral velocity from FPR compared with true value

Fig.5 Vertical velocity from FPR compared with true value

Fig.6 Roll angle from FPR compared with true value

Fig.7 Pitch angle from FPR compared with true value

Fig.8 Yaw angle from FPR compared with true value

Fig.9 Corrected angle of attack compared with true and measured values

Fig.11 Corrected normal acceleration compared with true and measured values

Fig.12 Corrected forward acceleration compared with true and measured values

formed as a function of angle of attack using Eq. (4, 5, 6). The estimated coefficients as a function of angle of attack are compared with wind tunnel coefficients and plotted in Fig.13. It can be noted that all coefficients exhibit nonlinearity for angle of attack greater than 13 degrees.

Once the estimation is over the estimated model needs to be validated. To accomplish this, the wind-tunnel database is replaced with the estimated database. Again the longitudinal simulation is performed for the same flight condition as before and the responses are compared. The comparison is given in Figs.14-16. The comparison is satisfactory.

Multi-dimensional Estimation

Having proved the technique for estimation of coefficients as a function of single variable, we also proceed to prove the capability of the technique for multidimensional estimation by considering the following model postulate for longitudinal flight data :

$$
\dot{\alpha} = f_1(\alpha, q, \delta e)
$$

$$
\dot{q} = f_2(\alpha, q, \delta e)
$$
 (22)

Fig.13 Estimated coefficients as a function of angle of attack

It is noted that Eq. (22) contains functions which depend simultaneously on three independent variables.For the estimation problem, the measurements of α , *q* and δ*e* were taken from short period flight data excited with large amplitude input. 'n' break points of α , '*m*' break points of *q* and '1' break points of δ*e* were chosen to estimate *f*¹ and f_2 . At any point of time one sample of α , *q* and δe are available. The left and right indices for each of these variables for interpolation are determined. Hence there will be six indices at a point of time corresponding to the three variables. Hence, using the six indices six weights are computed using Eq. (7) . Eight (2^3) combinations of weights can be formed and the index in the regressor for those combinations can be found as follows:

$$
index = i + (j-1)*n + (k-1)*n*m
$$
 (23)

where n is the number of break points in α , *m* is the number of break points in *q*, *i* denotes the left/right index of α for interpolation, *j* denotes the left/right index of *q* for interpolation, *k* denotes the left/right index of δ*e* for interpolation.

The functions f_1 and f_2 were estimated as 3 dimensional look up tables. It is noted that the data points required for the estimation increases in direct proportion with number of dimensions (data points covering l^*m^*n combinations in this case) . In this paper, one segment of the flight data is used for the estimation, merely to show the extendibility of the technique to multi-dimensions. In practical applications, several segments have to be concatenated to cover almost all combinations of break points. The plots for nonlinear functional variations are not shown, as it is difficult to cover all combinations of break

Fig.17 Comparison of linear and nonlinear estimated responses with measured data

points with one flight segment data. However, the measured responses were compared with simulated responses in Fig.17 according to the postulated model given in Eq. (22). In addition to this, a linear estimation is carried out by postulating a second order short period state space model. This additional exercise brought out the importance of estimating the underlying nonlinearity in the time response, which the linear estimation could not capture as shown in Fig.17.

Conclusion

A novel technique is proposed to perform near real time estimation of nonlinear aerodynamics. Although, the present aircraft example required only one-dimensional estimation, the technique is tested for multidimensional estimation and the results are also reported. The multidimensional formulation is found to be much simpler for implementation as against multidimensional splines. The linear interpolation in conjunction with RLS does not involve any nonlinear optimization to capture the nonlinearity that makes it suitable for near real time estimation.

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