

STRESS SEPARATION IN DIGITAL PHOTOELASTICITY PART A - PHOTOELASTIC DATA UNWRAPPING AND SMOOTHING

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Abstract

The advent of digital photoelasticity saw the development of various techniques to get photoelastic data of isoclinics and isochromatics at every pixel in the domain. Unlike in other interferometric techniques, in photoelasticity one gets phase data of two quantities and their mutual dependence and interaction affect their evaluation. For stress separation studies, one requires both isochromatics and isoclinics accurately free of any kinks in the domain. With this in view, a robust approach is evolved to get these parameters. Initially wrapped isoclinics are obtained using a plane polariscope based phase shifting technique. These are unwrapped by a quality guided approach. However, the result has several kinks due to isochromatic-isoclinic interaction. An outlier smoothing algorithm is proposed for getting a smooth variation of the isoclinics. Six-step phase shifting method using a circular polariscope is used for determining the isochromatic data. The smoothed isoclinic values obtained are used in isochromatic calculation to get isochromatic phasemap free of ambiguities. This phasemap is also unwrapped by a quality guided approach and smoothed by the outlier algorithm. These methodologies are validated for two benchmark problems of a disk under diametral compression and a ring subjected to internal pressure. The models are subjected to moderate loads showing a high level of isochromatic-isoclinic interaction.

Notations

I_i	= intensity of light transmitted for arbitrary positions of optical elements in a polariscope	η	= orientation of the II quarter wave plate axis w.r.t. x-axis
I_a	= light intensity accounting for the amplitude of light vector and the proportionality for circular and plane polariscopes arrangements respectively	$\Delta^x i, j$	= partial derivatives of the wrapped phase differences along x- direction
I_b	= background light intensity for circular and plane polariscopes arrangements respectively	$\Delta^y i, j$	= partial derivatives of the wrapped phase differences along y- direction
λ	= actual wavelength of light used	$k \times k$	= window size usually 3×3
α	= orientation of polarizer axis w.r.t. x-axis	$\overline{\Delta^x} m, n$	= average of partial derivative values along x- direction centred around $k \times k$ window
β	= orientation of analyzer axis w.r.t. x-axis	$\overline{\Delta^y} m, n$	= average of partial derivative values along y- direction centred around $k \times k$ window
δ	= fractional retardation in radians introduced by the model		
δ_c	= calculated value of fractional retardation in radians		
θ	= orientation of principal stress direction w.r.t. x-axis		
θ_c	= calculated value of isoclinic parameter w.r.t. x-axis		
ξ	= orientation of the I quarter wave plate axis w.r.t. x-axis		

Introduction

Photoelasticity is an optical technique for experimental stress analysis. It is widely used for 2-D and 3-D analysis of components for getting the information of principal stress difference and principal stress direction at every point in the domain. In the last two decades, computer based image processing has revolutionized the data acquisition techniques in photoelasticity. With the advent of digital computers, recording of images as intensity data became easier and a separate branch of photoelasticity namely *digital photoelasticity* came into existence [1]. In

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digital photoelasticity, intensity information of the captured image is used for evaluating the isoclinic and isochromatic parameters. Thus, in principle one gets values of principal stress orientation (θ) and principal stress difference in the form of fringe order (N) for the whole-field in a true sense. The various techniques of digital photoelasticity can be broadly classified as polarization stepping, phase shifting, load stepping and Fourier transform approach [1]. Among the various digital photoelastic techniques, phase shifting technique (PST) is the most widely used for isochromatic data evaluation due to its simplicity and accuracy. Of all the PST methods, six-step method stands out as it effectively accounts for the quarter-wave plate mismatch error [1, 2] and also the role of background intensity [3]. Recently, Ramji et. al [4] have shown that plane polariscope based algorithms give better isoclinic values than the methods that use a quarter wave plate.

The isoclinics and isochromatics are obtained only in a wrapped form directly. The wrapped values are represented as an image called phasemap, which are different from the conventional fringe patterns of photoelasticity. Both isoclinic and isochromatic phasemaps have interdependence. The phasemaps have to be unwrapped in different ways for getting the continuous phase values. In the case of isoclinic phasemaps, unwrapping refers to the process of obtaining the direction of either σ_1 or σ_2 consistently over the domain [4]. Phase unwrapping of isochromatic phasemap refers to the suitable addition of integral value to the fractional retardation values for making it as a continuous fringe order data [1]. One of the main issues in isochromatic phasemap is how to interpret the sign of the fractional retardation calculated while unwrapping. This is referred to as the presence of ambiguous zones in the isochromatic phasemap. In isoclinic phasemap, the values are not defined on isochromatic skeletons leading to estimation of unreliable values in zones close to these skeletons. The problem is more accentuated if there are a large number of isochromatics.

One of the simplest approaches for unwrapping of isochromatic phasemap is by a raster scanning approach [1, 6]. Removal of ambiguous zones in isochromatic phasemap prior to unwrapping was recognized [6] and several methodologies have been reported on how to correct these ambiguous zones [6-9]. Ramesh and Tamrakar [10] and Ekman and Nurse [11] have used load stepping to get isochromatic phasemaps free of ambiguities but it requires eighteen images. Extension of raster scanning to unwrap phasemaps of multiply connected

objects became possible with the development of new approaches for boundary encoding [12]. There have been only fewer attempts to unwrap isoclinics as a need to do this was not recognized. Simple raster scanning for isoclinic unwrapping has been reported [11, 13-15]. Quiroga and Gonzalez-Cano [16] developed regularized phase tracking algorithm for isoclinic unwrapping. But it is computationally intensive. Use of quality guided approach for phase unwrapping that has been developed in other optical techniques [17] has gained prominence in photoelasticity in the recent years [18, 19].

Hitherto, isochromatics and isoclinics were mostly used for independent applications and depending on the need, several methods were developed independently. For an application like stress separation, one needs both isochromatics and isoclinics free of noise and even in the presence of a considerable number of isochromatics, the isoclinic data has to vary smoothly over the domain. Thus, a comprehensive approach to evaluate both isochromatics and isoclinics with an optimal effort is needed. It has been pointed out that one may have to choose different optical arrangements for accurately evaluating the isoclinics and isochromatics. Recognizing this fact, there have been efforts to propose hybrid techniques where quarter-wave plates have been removed optically [20, 21] to determine isoclinics. As the quarter-wave plates are affected by manufacturing error [22] which is non-uniformly distributed over the plate [23], optical elimination becomes troublesome. In addition there could be error introduced due to optical misalignment. In view of this, authors have resorted to the technique, which has no quarter wave plate for isoclinic parameter estimation. In this paper, plane polariscope based algorithm of Mangal and Ramesh [24] is used for isoclinic data evaluation and six-step phase shifting algorithm [2] based on a circular polariscope is used for isochromatic data. Use of independent optical arrangements though result in capturing slightly more images (totally ten), the methods adopted are time-tested, are comparatively less sensitive to optical misalignment and an existing polariscope could be used. The problem of removal of ambiguity in isochromatic phasemaps is approached differently in this paper.

Mathematically, isoclinic values are undefined at isochromatic skeletons [1]. This is termed as isochromatic-isoclinic interaction in the case of an isoclinic phasemap. The isochromatic-isoclinic interaction increases with increased load and this is a major source of error in evaluating the isoclinic parameter [15]. Multiple wavelength approach [25-27] or multiple load approach [24] has been

used in reducing the noise occurring in isoclinic phasemap due to these interactions. Since, a large quantum of data is available, one could reduce the noise due to isochromatic-isoclinic interaction by developing appropriate methods for data smoothing. Ramji and Ramesh [15] proposed a polynomial based smoothing of isoclinic data. The polynomial fitting is a global smoothing technique which in principle can modify the trend of experimental isoclinic data in some zones.

In this paper, a quality guided algorithm of phase derivative variance along with domain delimiting has been developed for both isoclinic and isochromatic phasemap unwrapping. For domain delimiting, an advanced boundary encoding algorithm has been adopted [19]. A robust outlier algorithm is implemented for isoclinic and isochromatic data interpolation and smoothing. The methodology is validated for both simply and multiply connected objects, which are loaded such that the isochromatic-isoclinic interaction is significant. For completeness, the optical arrangements used to get primary isoclinic and isochromatic data as well as the quality map by phase derivative variance are presented.

Optical Arrangements to Get Primary Isoclinic and Isochromatic Data

Figure 1a shows a generic plane polariscope and Fig. 1b shows a generic circular polariscope. The whole field

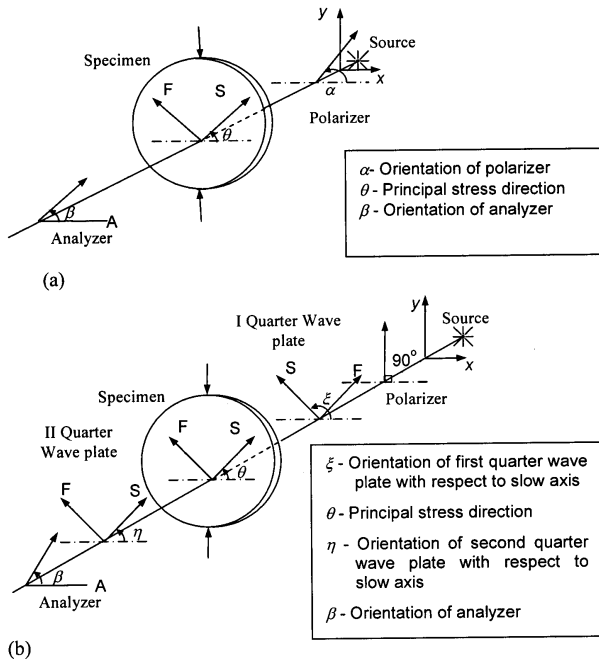


Fig.1 Generic polariscope arrangement
(a) Plane polariscope (b) Circular Polariscope

digital photoelastic parameters are obtained using a ten-step method in this paper as shown in Table-1. The first four steps in Table-1 correspond to the optical arrangement of the plane polariscope based algorithm of Mangal and Ramesh [24] and they are used for the evaluation of isoclinic values. The next six arrangements are that of the six-step PST algorithm [2] based on a circular polariscope arrangement and they are used for the evaluation of isochromatic values. The isoclinic value is obtained by

$$\theta_c = \frac{1}{4} \tan^{-1} \left(\frac{I_3 - I_2}{I_4 - I_1} \right) \quad (1)$$

In Eq. (1), the subscript *c* indicates that the principal value of the inverse trigonometric function is used and the intensity values correspond to the plane polariscope arrangements. Reference [1] recommends that θ_c is to be evaluated by atan2 () function. The isoclinic value obtained by Eq. (1) is unwrapped and then smoothed as discussed in the later part of this paper. These smoothed isoclinic values (θ) are used for isochromatic phasemap evaluation (Eq. (2)). The isochromatic value is obtained by

$$\delta_c = \tan^{-1} \left(\frac{(I_9 - I_7) \sin 2\theta + (I_8 - I_{10}) \cos 2\theta}{(I_5 - I_6)} \right) \quad (2)$$

where the intensity values correspond to the circular polariscope based six-step phase shifting techniques. Reference [1] recommends that δ_c is to be evaluated by atan2 () function. This approach of using the unwrapped and smoothed isoclinic values in isochromatic evaluation ensures the removal of the ambiguous zones occurring in the isochromatic phasemap. This isochromatic phasemap has to be unwrapped for getting the total fringe order. The phase shifted images are experimentally recorded with a monochromatic light source (Sodium vapour, $\lambda = 589.3$ nm) and the intensity of light transmitted is recorded by a monochrome CCD camera (DC-700 SONY) having a resolution of 768 x 576 pixels.

Quality Based Isoclinic Phasemap Unwrapping

A new tile based quality guided unwrapping of isoclinic phasemap is proposed in this paper. The tile chosen can be of any arbitrary shape. This has been possible with the adaptation of a new approach for boundary encoding [12]. It is very useful in handling problems to delimit zones containing an isotropic point / line or ninety degree

Table-1 : Polariscope arrangements and intensity equations for isoclinic and isochromatic evaluation - Ten-step method

α	ξ	η	β	Intensity Equation
0	-	-	$\pi/2$	$I_1 = I_b + I_a \sin^2 \frac{\delta}{2} \sin^2 2\theta$
$\pi/4$	-	-	$\pi/2$	$I_2 = I_b + \frac{I_a}{2} \left[1 - \sin^2 \frac{\delta}{2} \sin 4\theta \right]$
0	-	-	$\pi/4$	$I_3 = I_b + \frac{I_a}{2} \left[1 + \sin^2 \frac{\delta}{2} \sin 4\theta \right]$
$\pi/4$	-	-	$3\pi/4$	$I_4 = I_b + I_a \sin^2 \frac{\delta}{2} \cos^2 2\theta$
$\pi/2$	$3\pi/4$	$\pi/4$	$\pi/2$	$I_5 = I_b + \frac{I_a}{2} (1 + \cos \delta)$
$\pi/2$	$3\pi/4$	$\pi/4$	0	$I_6 = I_b + \frac{I_a}{2} (1 - \cos \delta)$
$\pi/2$	$3\pi/4$	0	0	$I_7 = I_b + \frac{I_a}{2} (1 - \sin 2\theta \sin \delta)$
$\pi/2$	$3\pi/4$	$\pi/4$	$\pi/4$	$I_8 = I_b + \frac{I_a}{2} (1 + \cos 2\theta \sin \delta)$
$\pi/2$	$\pi/4$	0	0	$I_9 = I_b + \frac{I_a}{2} (1 + \sin 2\theta \sin \delta)$
$\pi/2$	$\pi/4$	$3\pi/4$	$\pi/4$	$I_{10} = I_b + \frac{I_a}{2} (1 - \cos 2\theta \sin \delta)$

jump in isoclinics that happen in problems such as a ring under diametral compression or a ring subjected to internal pressure. Quality maps are arrays of values that indicate the quality or goodness of the phase data. They are usually scaled to yield values that range from 0 to 1. Lower values are considered to be of inferior quality and the higher values closer to one are measured to be of superior quality. This quality information from the phase data is used to guide the integration path while doing unwrapping. It is essentially a region growing approach that identifies the highest quality pixels, unwraps them first, and avoids the low quality pixels until the end of the unwrapping procedure. Lastly, the low quality pixels are unwrapped. Further details can be obtained from Ref. [17]. For starting the unwrapping process, a seed point of high quality has to be given within a selected tile. The unwrapping process consistently evaluates the isoclinic direction corresponding to the principal stress in the selected tile. The choice of the tile is left to the user to identify so that one can get the direction corresponding to σ_1 or σ_2 as the need demands.

Quality guided unwrapping has the following steps: (i) first step involves identification of boundary / selected tile over the domain (ii) Generation of quality map (iii) Seed point selection within the boundary / selected tile for effective unwrapping.

Quality Map by Phase Derivative Variance

This quality map gives the measure of the statistical variance of the phase derivatives. Since it involves derivatives, this quality map is more suitable for phase data with steep gradients. It is defined as [19]

$$\frac{\sqrt{\sum (\Delta^x i, j - \bar{\Delta}^x m, n)^2} + \sqrt{\sum (\Delta^y i, j - \bar{\Delta}^y m, n)^2}}{k^2} \quad (3)$$

where for each sum, the indices (i, j) range over a $k \times k$ neighborhood of each center pixel (m, n) and $\Delta^x i, j$ and $\Delta^y i, j$ are the partial derivatives of the phase (i.e., the wrapped phase differences). The terms $\bar{\Delta}^x m, n$ and

$\bar{\Delta}^y m, n$ are the averages of these partial derivatives in the $k \times k$ windows. Eq.(3) is a root-mean-square measure of the variances of the partial derivatives in the x and y directions. The phase derivative variance map differs indicates the "badness" rather than the goodness of the phase data. The quality value is negated consequently to represent goodness.

Isoclinic Phasemap Unwrapping Algorithm

The procedure of phase unwrapping is explained by applying it to the benchmark problem of a disk under diametral compression (dia = 60mm, thickness = 6 mm, Fsigma = 11 N/mm/fringe, load = 574 N). Experimentally generated isoclinic phasemap obtained using the first four images of Table-1 is shown in Fig.2a. Fig.2e shows the analytically obtained isoclinic phasemap [1]. When comparing Fig.2a with Fig.2e, near the loading points (left and right) there is an abrupt jump in the grey values which needs to be corrected. In these zones the phasemap represents principal stress direction of the other principal stress and they are referred to as inconsistent zones. Fig.2b shows the quality map obtained using phase derivative variance over the entire domain. In the quality map, black colour represents zone of poor quality and white is of good quality. As isoclinic values suddenly shift by $\pm\pi/2$ when one moves from a zone of one principal stress to the other principal stress, one could clearly see in the quality map, the boundaries of those zones are captured as pixels of poor quality. Further, isoclinic is not defined in isochromatic skeletons and these zones are also identified as zones of poor quality, which appears as isochromatic skeletons in the quality map.

Domain masking [19] is used to perform the algorithm within the selected tile. A seed point of high quality is selected in a zone decided by the user for isoclinic unwrapping. For the problem of disk, the central zone away from the loading point is selected. Four neighbouring pixels adjacent to the selected pixel are unwrapped based upon the conditions mentioned in Table-2. The integration path is dictated by the quality. To identify the $\pi/2$ jump in isoclinic values, a tolerance in theta value (*theta_tol*) is used for capturing this. A value of $\pi/3$ for theta tolerance is found to be suitable for a variety of problems. A value of $\pi/2$ is added or subtracted depending upon the sign of the gradient of the adjacent pixels. Using the quality guided algorithm, the entire domain is unwrapped in a single step.

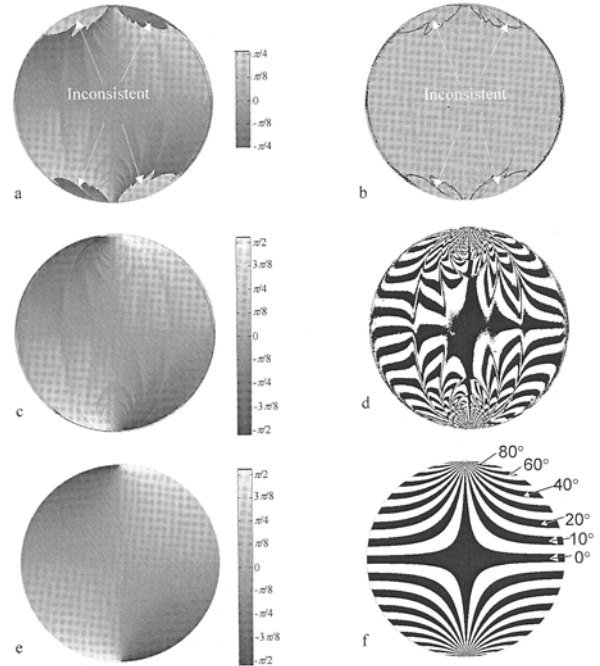


Fig.2

Steps involved in unwrapping isoclinic phasemap obtained for the problem of a disk under diametral compression obtained using Mangal and Ramesh phase shifting algorithm (a) wrapped isoclinic (with arbitrary tile for inconsistency removal) (b) phase derivative variance within respective tiles (c) unwrapped isoclinic phasemap (d) isoclinic plots in steps of 10° for experimentally obtained unwrapped isoclinic values (e) unwrapped isoclinic phasemap obtained using analytical solution [1] (f) isoclinic plots in steps of 10° obtained analytically.

Figure 2c shows the grey level representation of the unwrapped isoclinic phasemap. One could see a semblance of isochromatic fringes in the phasemap. This is actually noise due to isochromatic-isoclinic interaction. Fig. 2d and Fig.2f shows a binary representation of isoclinic plot in steps of 10° obtained experimentally and analytically. The binary representation (Fig.2d) has brought out more forcefully the bad impact of noise due to isochromatic - isoclinic interaction on the isoclinic phasemap. Thus smoothing of raw unwrapped isoclinic data is needed for further applications like stress separation studies [4].

Smoothing of Isoclinic Data

The smoothing algorithm is developed for handling both simply and multiply connected models. For smoothing, the information of isoclinic data row by row within the model domain is required. Usually the phase data is

Table-2 : Table showing checking conditions for different pixel position in isoclinic unwrapping		
Sl. No.	Pixel Position	Checking Condition
1	Left Pixel (LP)	$abs(\theta_{i,j} - \theta_{i-1,j}) > theta_tol, then$ $\theta_{i-1,j} = \begin{cases} \theta_{i-1,j} - \frac{\pi}{2} & \theta_{i-1,j} - \theta_{i,j} > 0 \\ \theta_{i-1,j} + \frac{\pi}{2} & \theta_{i-1,j} - \theta_{i,j} < 0 \end{cases}$
2	Right Pixel (RP)	$abs(\theta_{i,j} - \theta_{i+1,j}) > theta_tol, then$ $\theta_{i+1,j} = \begin{cases} \theta_{i+1,j} - \frac{\pi}{2} & \theta_{i+1,j} - \theta_{i,j} > 0 \\ \theta_{i+1,j} + \frac{\pi}{2} & \theta_{i+1,j} - \theta_{i,j} < 0 \end{cases}$
3	Bottom Pixel (BP)	$abs(\theta_{i,j} - \theta_{i,j+1}) > theta_tol, then$ $\theta_{i,j+1} = \begin{cases} \theta_{i,j+1} + \frac{\pi}{2} & \theta_{i,j+1} - \theta_{i,j} < 0 \\ \theta_{i,j+1} - \frac{\pi}{2} & \theta_{i,j+1} - \theta_{i,j} > 0 \end{cases}$
4	Top Pixel (TP)	$abs(\theta_{i,j} - \theta_{i,j-1}) > theta_tol, then$ $\theta_{i,j-1} = \begin{cases} \theta_{i,j-1} + \frac{\pi}{2} & \theta_{i,j-1} - \theta_{i,j} < 0 \\ \theta_{i,j-1} - \frac{\pi}{2} & \theta_{i,j-1} - \theta_{i,j} > 0 \end{cases}$

stored as an array and from this the retrieval of sequence of pixel coordinate values row by row within the model boundary is required for implementing the smoothing algorithm. The information thus retrieved will also be of use to implement the shear difference scheme digitally.

The *xbn* file format is suited for vertical scanning algorithms and *ybn* is suited for horizontal scanning algorithms [12]. As smoothing of the data is to be done row by row, one needs the co-ordinates of the boundary pixels as well as the number of pixels in between them. These are extracted from the boundary information file with extension **.ybn*. If needed, vertical smoothing of the data is done column by column where **.xbn* file is used for extracting the boundary information.

Robust Outlier Smoothing Algorithm

The outlier smoothing algorithm belongs to a class of local regression techniques. The smoothing procedure is termed local because each smoothed value is determined by the neighbouring data points defined within a span. The span defines a window of neighbouring points to be included in the smoothing calculation for each data point. The larger the span, the smoothed curve will follow the trend better. The data points lying outside the trend are omitted and a local curve fitting is done by least squares analysis [28]. The omission of data points, which are present outside the trend is done by a weighting process. The weights are calculated based upon the statistical parameter of median absolute deviation (MAD) and scaled between 0-1. Data points with weights nearer to one are those which are lying outside the data trend locally and

with weights nearer to zero are those that exactly follow the data trend. Thus the algorithm avoids the near unity weighted data points and appropriately chooses the data points closer to zero for local least squares curve fitting. One can use either linear or quadratic polynomial for least squares curve fitting. Likewise, the whole procedure is completed for the rest of the line. The algorithm is written in Matlab software and the smoothing function is available in the curve fitting module of the Matlab. More details regarding this algorithm can be found in Ref. [28].

Smoothing has the following steps: (i) Identification of boundary / selected tile over the domain (ii) Generate *.ybn or *.xnb files depending upon scanning direction for the identified boundary / selected tile (iii) Give span width and order of polynomial for least squares curve fitting.

Isoclinic Data Smoothing

The unwrapped isoclinic data obtained is fed into the smoothing algorithm. Smoothing is done along both the horizontal and vertical directions. In all five hundred and thirty five rows and four hundred & seventy three columns are present inside the model domain. Initially the experimental data is smoothed row-wise and then column-wise. An important factor here is the selection of the span width and the order of the polynomial for least squares curve fitting. It varies from problem to problem but usually, the longer the span width it is better. In case of horizontal smoothing, a span of one hundred and forty five pixels and a quadratic polynomial for curve fitting are found to be sufficient over the entire domain. Similarly for vertical smoothing, a span of one hundred and thirty pixels and a quadratic polynomial for curve fitting are found to be sufficient over the entire domain. For each row, smoothed data is obtained using Matlab function. Fig.3a is the iso-

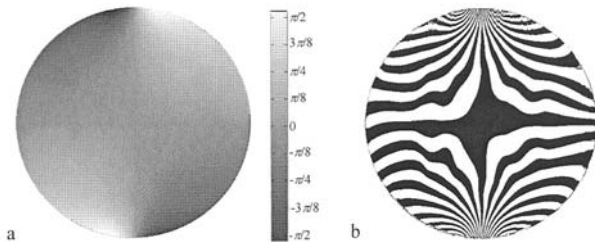


Fig.3

Smoothed unwrapped isoclinic phasemap obtained using robust outlier smoothing algorithm for the problem of a disk under diametral compression (a) smoothed isoclinic phasemap (b) isoclinic plots in steps of 10° obtained after smoothing.

clinic phasemap obtained after smoothing. One can clearly notice that a semblance of isochromatic fringes seen in Fig. 2c is not seen here. After applying the smoothing algorithm as mentioned above, there is total improvement in the binary isoclinic plot of Fig. 3b when compared to Fig. 2d.

Isochromatic Phasemap Unwrapping and Smoothing

Experimentally obtained isochromatic phasemap using the last six images of Table- 1 for the problem of a disk under diametral compression is shown in Fig.4a, which contains ambiguous zones near the loading points. This is because the isochromatic phasemap is obtained by using θ calculated by the six-step PST algorithm which corresponds to the last six images in Table-1. Normally, the isochromatic phasemap needs to be corrected for ambiguity before doing unwrapping. Since, the formation of ambiguous zone is linked to inconsistent determination of isoclinics over the domain, in this paper, the unwrapped and smoothed isoclinic values obtained from the first four images of Table-1 are used in Eq. (2) for generating the isochromatic phasemap. This has resulted in a phasemap free of ambiguous zones (Fig. 4b). Unwrapped isochromatic phase values obtained by quality guided approach [29] is shown in Fig. 4c.

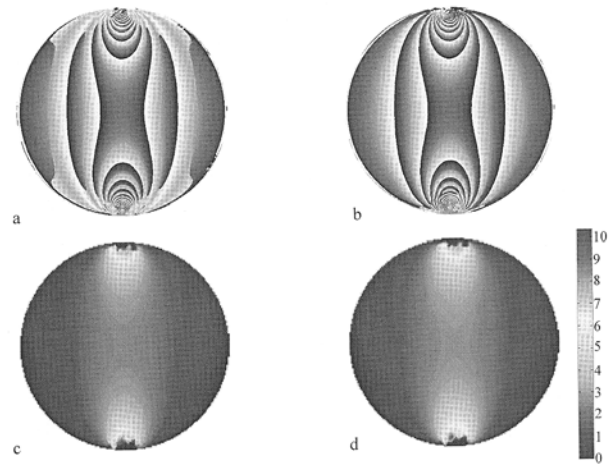


Fig.4

Steps involved in unwrapping isochromatic phasemap obtained for the problem of a disk under diametral compression obtained using six-step phase shifting algorithm (a) phasemap obtained using θ calculated by the six step algorithm (b) phasemap obtained using unwrapped and smoothed isoclinic value obtained by Mangal and Ramesh algorithm in Eq.(2) (c) Matlab plot of unwrapped isochromatic phasemap obtained using phase derivative variance quality guided path follower unwrapping algorithm (d) Matlab plot of smoothed isochromatic phasemap obtained using robust outlier algorithm.

Isochromatic data smoothing is similar to that of isoclinic data smoothing. The only change is in the selection of the extent of the span width specified and the smoothing polynomial. Here a span of fifteen pixels is found to be sufficient. Smoothing is done along both the horizontal and vertical directions using the same span width. A quadratic polynomial is selected for least squares curve fitting in both the directions. Fig. 4d shows the Matlab plot of smoothed isochromatic values.

Application of the Methodology to Multiply Connected Objects

The problem of a thick ring subjected to internal pressure (Outer dia = 80 mm, Inner dia = 40 mm, thickness = 6 mm, $F_{\text{sigma}} = 11.54 \text{ N/mm/fringe}$, load = 3.93 MPa), which is a multiply connected object is chosen. Though this problem has symmetry, the problem as a whole is analysed to develop a suitable algorithmic structure. Experimentally obtained isoclinic phasemap is shown in Fig. 5a. Fig. 5e shows the analytically obtained isoclinic phasemap [30]. When comparing Fig. 5a with Fig. 5e, there is an abrupt jump in the grey values on both left and right sides of the ring. These are nothing but the inconsistent zone.

Need for Developing a Domain Delimiting Algorithm

Domain delimiting is a technique, which sub divides the problem of interest as an assembly of smaller problem zones. The sub divisions need not be of rectangular zones but can take arbitrary shapes too. When the inconsistent zones are of arbitrary shape or spread across lines of symmetry, there is a need for domain delimiting within the problem domain for performing the unwrapping process. The boundary extraction method mentioned in Ref. [12] is used here for delimiting the zones of any arbitrary shape. For the unwrapping process to perform within the chosen zone, the concept of domain masking is used. Usually a binary image file containing the masked area is created by any general image processing softwares such as Adobe Photoshop. The data generated is given as boundary mask input file while unwrapping. The unwrapping algorithm reads this binary image file and the masked regions are flagged [19]. In this way, unwanted zones are identified and left out while unwrapping. This concept is utilized in domain delimiting where the regions outside the required zones are masked out using arbitrary tiles and the algorithm performs within the zone. For delimiting, one requires the tile's boundary coordinates information which is obtained using the boundary extraction technique

developed in-house. A stand alone software has been developed using VC++ for isoclinic unwrapping with domain delimiting. Domain delimiting in conjunction with domain masking can be used to solve complicated problems having arbitrary ambiguous zones. Domain delimiting is also useful for unwrapping simply connected models wherein the ambiguous zones become complex.

Isoclinic Phasemap Unwrapping and Data Smoothing

The ring is delimited into two rectangular regions for unwrapping (Fig. 5a). Quality map is obtained for these two regions separately and are assembled as one figure in Fig. 5b. One could clearly see in the quality map, the boundaries of zones depicting the direction of different principal stresses are captured as zones of poor quality.

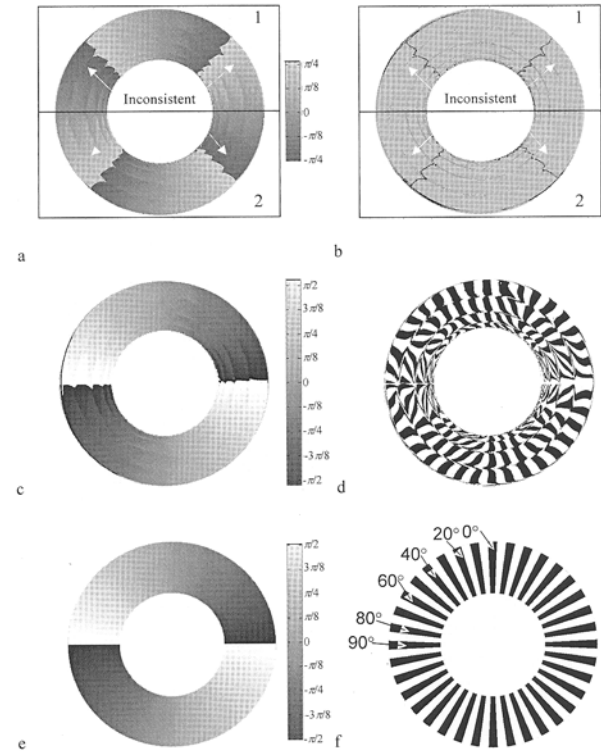


Fig.5

Steps involved in unwrapping isoclinic phasemap obtained for the problem of a ring subjected to internal pressure obtained using Mangal and Ramesh phase shifting algorithm (a) wrapped isoclinic (with tiles for inconsistency removal) (b) phase derivative variance within respective tiles (c) unwrapped isoclinic phasemap (d) isoclinic plots in steps of 10° for experimentally obtained unwrapped isoclinic values (e) unwrapped isoclinic phasemap obtained using analytical solution [30] (f) isoclinic plots in steps of 10° obtained analytically.

Further, one can also notice isochromatic skeletons indicating that isoclinic is of poor quality in these zones. Unwrapping is done region-wise individually. The unwrapped isoclinic phasemap is shown in Fig. 5c which is similar to the analytically obtained phasemap [30] (Fig. 5e). However, the grey level representation is deceptive in not reflecting the errors due to isochromatic-isoclinic interaction. Fig. 5d and Fig.5f show the binary isoclinic plot in steps of 10° obtained experimentally and analytically. In Fig. 5d one could see abrupt jumps in isoclinic bands clearly indicating the need for smoothing.

For data smoothing, the multiply connected region of the ring need to be divided into an assembly of simply connected regions. Four simply connected regions as shown in Fig. 6a are identified. For each of these regions, the outlier algorithm is applied row-wise using a span length of eighty five pixels and a linear polynomial for the least squares curve fitting. Fig. 6b shows the grey level isoclinic plot obtained after smoothing, which is free of noise seen in Fig. 5c. The binary isoclinic plot (Fig. 6c) shows a remarkable change compared to Fig. 5d indicating that a binary representation gives a better visual picture of the quality of data. Only at the horizontal axis of symmetry, the isoclinic lines are slightly perturbed.

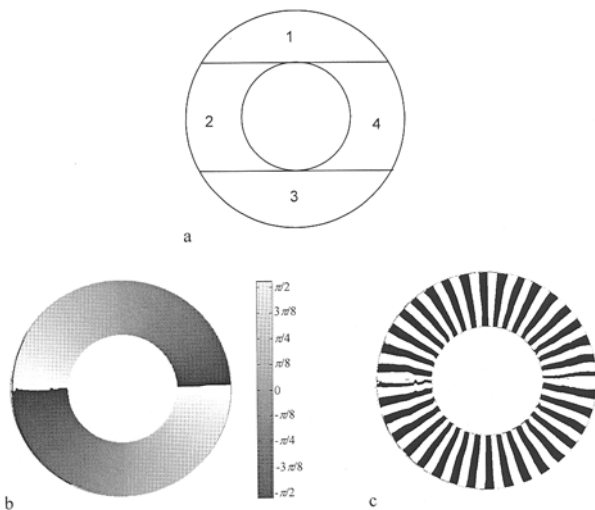


Fig.6

Smoothed unwrapped isoclinic phasemap obtained using robust outlier smoothing algorithm for the problem of a ring subjected to internal pressure (a) figure showing domain split for data smoothing (b) smoothed isoclinic phasemap (c) isoclinic plots in steps of 10° obtained after smoothing.

Isochromatic Phasemap Unwrapping and Data Smoothing

Direct use of last six images in Table-1 for the problem of a thick ring subjected to internal pressure gives the phasemap with ambiguous zones as shown in Fig. 7a. Using the before mentioned procedure, a phasemap free of ambiguities (Fig. 7b) is obtained. Fig. 7c shows the Matlab plot of unwrapped raw isochromatic phasemap. For isochromatic data smoothing, the ring is divided into an assembly of four simply connected regions as before. A span of sixty five pixels and a linear polynomial for least squares is used for the outlier algorithm. Fig. 7d shows the Matlab plot of smoothed isochromatic values.

Conclusion

A new methodology for isoclinic phasemap unwrapping using quality guided path follower algorithm has been developed. The unwrapping methodology is validated for the problem of a disk under diametral compression, which is a simply connected body. A versatile smoothing methodology using outlier algorithm has been

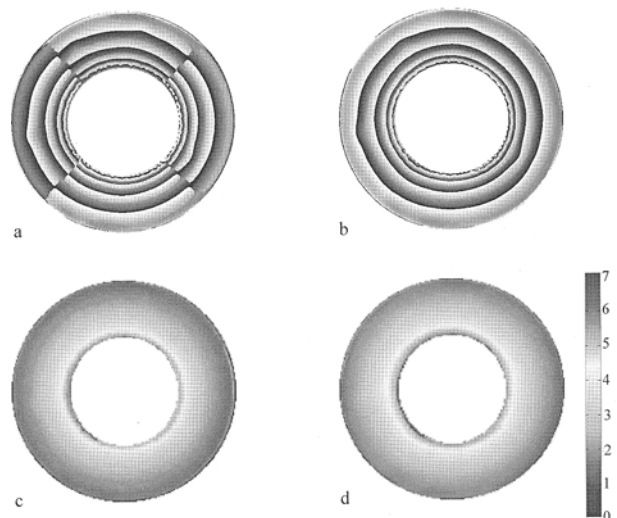


Fig.7

Steps involved in unwrapping isochromatic phasemap obtained for the problem of a ring subjected to internal pressure obtained using six-step phase shifting algorithm (a) phasemap obtained using θ calculated by the six-step algorithm (b) phasemap obtained using unwrapped and smoothed isoclinic value obtained by Mangal and Ramesh algorithm in Eq.(2) (c) Matlab plot of unwrapped isochromatic phasemap obtained using phase derivative variance quality guided path follower unwrapping algorithm (d) Matlab plot of smoothed isochromatic phasemap obtained using robust outlier algorithm.

developed for isoclinics initially and later adopted for isochromatic smoothing. The smoothing methodology drastically reduces the noise in the isoclinic parameter due to isochromatic-isoclinic interaction yielding smooth values comparable with theory. The smoothed isoclinic values are used in isochromatic calculation and one obtains an isochromatic phasemap free of ambiguity. The application of the smoothing procedure for a multiply connected body is demonstrated using the problem of a ring under internal pressure. The concept of domain delimiting is used to sub-divide the problem domain into simply connected regions. The smoothing algorithm developed is of generic nature and can be applied to any other optical methods where data smoothing is required.

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