

DYNAMIC STABILITY ANALYSIS OF SQUARE ISOTROPIC/LAMINATED COMPOSITE PLATES WITH CIRCULAR CUTOUT SUBJECTED TO NON-UNIFORM FOLLOWER EDGE LOAD WITH DAMPING

L. Ravi Kumar*, P.K. Datta* and D.L. Prabhakara**

Abstract

The present investigation deals with the study of vibration and dynamic instability behaviour of square isotropic/laminated composite plates with circular hole subjected to partially distributed follower edge forces using finite element method. The first order shear deformation theory is used to model the plate, considering the effects of shear deformation and rotary inertia. The modal transformation technique is applied to the resulting equilibrium equation for subsequent analysis. Structural damping is introduced into the system in terms of equivalent viscous damping to study the significance of damping on stability characteristics. The effects of cutout size, load width, boundary condition, ply orientation, direction control of the load and damping parameters are considered for the stability behaviour of the plates. The results show that under follower loading, the system is susceptible to instability due to flutter alone or due to both flutter and divergence, depending on system parameters.

Key words: cutout, dynamic stability, follower loading, structural damping, finite element method

Notation

a,b = dimensions in the x and y directions respectively
c = load width parameter
d = diameter of the circular cutout
h = thickness
r = radius of the circular cutout
T = kinetic energy
 U_1 = strain energy associated with bending with transverse shear
 U_2 = work done by the initial in-plane stresses and the non-linear strain
{q} = nodal displacement vector
[K] = elastic stiffness matrix
[K_G] = stress stiffness matrix
[K_{NC}] = nonconservative matrix
[M] = mass matrix
[C] = damping matrix
 ν = Poisson's ratio
[ϕ] = modal matrix
P = edge load
[F] = flexural rigidity matrix

[B] = strain displacement matrix
 γ = non-dimensional load
 λ = non-dimensional frequency
 η = damping loss factor
 φ = load direction control parameter
 ω = angular frequency of transverse vibration
 δW_{NC} = variational work done by nonconservative force

Introduction

Lightweight isotropic and laminated composite structural members have been extensively used in many engineering fields such as in aerospace, mechanical, civil, automotive and automobile applications. The stability behaviour of such structural members is of increasing importance. The applied loads are considered as nonconservative forces if the work done by the system is path dependent. Some of the important practical illustrations of nonconservative forces are the aerodynamic drag forces acting on the body of rockets, missiles and other flight vehicles, the wing of an aircraft carrying jet engines subjected to concentrated follower forces (engine thrust),

*Department of Aerospace Engineering, Indian Institute of Technology, Kharagpur-721 302, India
Email : ravibmsce@yahoo.com; pkdatta@aero.iitkgp.ernet.in

** Department of Civil Engineering, Malnad College of Engineering, Hassan-573 201, India
Email : cacep2001@yahoo.com

Manuscript received on 06 May 2004; Paper reviewed, revised and accepted on 26 Aug 2004

a cantilever pipe conveying fluid, the forces acting on the rotor of a gas turbine, the forces acting on the links and elements in automatic control system applications and the frictional forces acting on the break shoes of automobiles etc. The nonconservative loading is generated by considering a load which follows the rotation of the point of application in the prescribed manner. The ratio of rotational angle of the load to that of the point of application is called the tangency parameter of the loading. By changing this tangency parameter, a wide range of nonconservative loading cases can be obtained and it permits the study of the elastic systems under nonconservative loading.

Two types of instability may occur in structural systems under the application of nonconservative loading, either divergence (static instability)/flutter (dynamic instability) or both. The presence of nonconservative loads makes the governing equation of the system mathematically non-self-adjoint and corresponding eigenvalue problem is ruled by a non-symmetric matrix and can exhibit complex eigenvalues.

The stability of nonconservative systems has been extensively studied by Bolotin [1-2] and Ziegler [3]. Many investigations have been reported on stability of columns, beams and other systems subjected to follower forces. Some of the important contributions are as follows: Herrmann and co-authors [4-5] studied the stability of elastic systems subjected to nonconservative forces and the effect of damping on the system. Sankaran and Venkateswara Rao [6] studied stability of tapered cantilever columns subjected to follower forces. Leipholz [7] proposed a variational principle for a clamped-free rod subjected to a tangential follower force. Sugiyama and co-investigators [8-11] studied both experimentally and theoretically stability behaviour of columns and beams subjected to nonconservative follower forces. Argyris et al. [12] studied static and dynamic stability of non-linear elastic systems under nonconservative forces. Graspardini et al. [13] studied the stability and instability regions of nonconservative continuous systems under partially follower forces. Investigations on the stability characteristics of plates under follower loading are relatively few. Farshad [14] studied the stability of cantilever plates subjected to biaxial subtangential loading. Adali [15] studied the stability of a plate under nonconservative forces. Culkowski and Reismann [16] presented an analytical method for stability analysis of a plate under two follower forces. Leipholz [17] studied the stability of rectangular simply supported plates subjected to non-increasing tangential

forces. Leipholz and Pfendt [18] used extended Galerkin's theory to analyze a plate with distributed follower forces on the surface of the plate. Higuchi and Dowell [19] investigated the effects of structural damping on flutter behaviour of plates. Ashwini Kumar and Srivastava [20] studied the stability of rectangular elastic plates under a follower force. Kishore [21] studied on the stability of an orthotropic plate under a follower force. Kim and his co-authors present extensive studies on the dynamic stability of a plate under follower force at the edge and at intermediate locations [22-23]. Datta and Deolasi [24] studied the dynamic instability characteristics of an isotropic plate subjected to partially distributed follower edge loading with damping. Herrmann [25], Komarakul and Arora [26], Bismarck [27], Langthjem and Sugiyama [28] and Bazant [29] have presented extensive reviews on different problems involving nonconservative forces.

In the literature it is found that the researchers mainly focused their attention on structural elements subjected to uniformly distributed follower forces, which cause an uniform stress field. In many practical applications, however, structural members are also subjected to non-uniform/partial load having different nonconservative parameters. For example skin (panel) of the wing structure of an aircraft carries non-uniform partial in-plane load, making the system susceptible to buckling. Further, in-plane forces change their directions following the deformation of the structure.

The studies on stability behaviour of plates with hole subjected to follower force are not found in the literature. It is felt that this is an important area of research involving stability behaviour of structural elements. In the present study an attempt as been made to study the effect of follower force on isotropic/laminated composite plates with a hole considering different parameters like structural damping, load direction control, partial load and the results are presented.

Theory and Formulations

The extended Hamilton principle is applied to a plate/panel to account for the work done by nonconservative forces. The extended Hamilton principle [15] can be expressed as

$$\delta \int_{t_1}^{t_2} (T - U) dt + \int_{t_1}^{t_2} \delta W_{NC} dt = 0 \quad (1)$$

The kinetic energy T of the plate in the above equation can be expressed as

$$T = \frac{1}{2} \rho \iiint (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) dx dy dz \quad (2)$$

The strain energy expressions are expressed as follows:

For a laminate under flexural and shear deformation, the strain energy with bending and shear can be expressed by integrating the sum of the strain energies in the element of the individual lamina over the entire area as

$$U_1 = \frac{1}{2} \iint \left(\sum_{k=1}^p \int_{z_{k-1}}^{z_k} \{\epsilon_1\}_k^T [\bar{Q}]_k \{\epsilon_1\}_k dz \right) dx dy, \quad (3)$$

where p is the number of layers, Z_{k-1} and Z_k are the distances from the middle plane to the bottom and top of k^{th} lamina. For the k^{th} lamina with general ply orientation, the terms $\{\epsilon_1\}_i$ and $[\bar{Q}]_i$ have been represented [30]. The

work done by the initial in-plane stresses and the nonlinear strain can be expressed as

$$U_2 = \frac{1}{2} \iiint \{\sigma^0\} \{\epsilon_n\} dx dy dz \quad (4)$$

Using the classical lamination theory and standard finite element procedures [31, 32], expressions for the energies can be written in finite form as

$$U_1 = \frac{1}{2} \{q\}^T [K] \{q\} \quad (5)$$

$$U_2 = \frac{1}{2} \{q\}^T [K_G] \{q\} \quad (6)$$

$$T = \frac{1}{2} \{\dot{q}\}^T [M] \{\dot{q}\} \quad (7)$$

where $\{q\} = [u_i \ v_i \ w_i \ \theta_{xi} \ \theta_{yi}]^T$, $i = 1, 2, \dots, n$ (n is the number of nodes) is the overall displacement vector, $[K]$, and $[K_G]$ are elastic and geometric stiffness matrices and $[M]$ is consistent mass matrix [30] respectively.

Further, $\delta W_{NC} =$ Variation of the work done by the nonconservative forces, which consists of two parts: follower forces and damping forces, and can be expressed as

$$\delta W_{NC} = \delta W_F + \delta W_D \quad (8)$$

where, W_D and W_F are work done by the damping and follower forces respectively and are expressed as follows

$$\delta W_F = \sum_{i=1}^1 \delta w_i (-P_i \theta_{y,i})$$

where 1 is the number of nodes,

$$\begin{aligned} &= \{\delta q\}^T [N]^T [P] [N_d] \{q\} \\ &= \{\delta q\}^T [K_{NC}] \{q\} \end{aligned} \quad (9)$$

$$\delta W_D = \{\delta q\}^T [C] \{\dot{q}\} \quad (10)$$

where P_i , w_i and $\theta_{y,i}$ are the force, deflection and rotation about y axis at the node n , 1 is the number of nodes and $[C]$ is the damping matrix.

$[N]$, $[N_d]$ and $[P]$ are written as follows:

$$[N]^T = [\{N_1\}, \{N_2\}, \dots, \{N_1\}] \text{ in which } \{N_i\} = [0, 0, 1, 0, 0]^T$$

$$[N_d]^T = [\{N_d\}_1, \{N_d\}_2, \dots, \{N_d\}_1];$$

in which $\{N_d\}_i = [0, 0, 0, 0, 1]^T$

$$[P] = -diag [P_1, P_2, \dots, P_1]$$

the follower force matrix $[K_{NC}]$ in equation (9) is given by

$$[K_{NC}] = [N]^T [P] [N_d]$$

where $[K_{NC}]$ is the nonconservative loading matrix which takes into account the nonconservative component of the follower load.

Substituting energy expressions in the equation (1), the following equilibrium equation for the plate is obtained.

$$[M] \{\ddot{q}\} + [C] \{\dot{q}\} + [K] \{q\} - P [K_G] + [K_{NC}] \{q\} = 0 \quad (11)$$

where P is the magnitude of the applied load.

For sinusoidal motion of the plate, structural damping can be expressed in terms of an equivalent viscous damping matrix as follows

$$[C] = \frac{\eta}{\omega} [K] \quad (12)$$

where η is the loss factor for the plate material and (J) is frequency of flexural vibration of the plate. It is obvious from the formulation of $[K_{NC}]$ that the matrix $[K_{NC}]$ is non-symmetric. Hence the equilibrium equation (11) leads to a non-self-adjoint eigenvalue problem for non-zero P .

Modal Transformation

The orders of the finite element matrices in equation (11) are very large and the solution of this equation in its original form may be prohibitive, particularly for determination of the flutter load. Hence a modal transformation is applied to equation (11) to reduce its size and to retain only the most dominant modes of vibration. The modal transformation is performed by means of the first few normal modes of vibration.

The equilibrium equation for free vibration of an undamped unloaded plate can be written as

$$-\omega^2 [M] \{q_0\} + [K] \{q_0\} = 0 \quad (13)$$

where ω is the angular natural frequency of vibration and $\{q_0\}$ corresponds to the mode shape of free vibration. Equation (13) is solved for the first few modes of vibration by means of a subspace iteration method. Let $[\phi]$ be a modal matrix, which contains the first few normal modes of the free vibration problem (13.), and let

$$\{q\} = [\phi] \{\xi\} \quad (14)$$

where $\{\xi\}$ are normal coordinates. Substituting equation (14) in equation (11) and premultiplying by $[\phi]^T$, the following modal equilibrium equation is obtained.

$$\{\ddot{\xi}\} + [\hat{C}] \{\dot{\xi}\} + [\Lambda - P [\hat{K}_G]] \{\xi\} = 0 \quad (15)$$

where

$$[\hat{C}] = [\phi]^T [C] [\phi] \text{ and } [\hat{K}_G] = [\phi]^T [K_G + K_{NC}] [\phi];$$

$[\Lambda]$ is a diagonal matrix containing the eigenvalues of equation (13), that is squares of the natural frequencies of free vibration of the unloaded plate. From equation (12),

$$[\hat{C}] = \frac{\eta}{\omega} [\Lambda] \quad (16)$$

Now considering the motion of the plate in the form

$$\{\xi\} = \{\xi_0\} e^{i\omega t}, \text{ Equation (16) changes to}$$

$$-\omega^2 \{\xi_0\} + i\eta [\Lambda] \{\xi_0\} + ([\Lambda] - P [\hat{K}_G]) \{\xi_0\} = 0 \text{ or}$$

$$-\omega^2 \{\xi_0\} + [(1 + i\eta) [\Lambda] - P [\hat{K}_G]] \{\xi_0\} = 0 \quad (17)$$

Equation (17) is an eigenvalue problem with eigenvalues ω^2 , which are the squares of the natural frequencies of free vibration under follower load P . Equation (17) can be solved by using standard eigenvalue routine for a complex general matrix. The imaginary part of ω corresponds to the exponential increment or decrement of the amplitude of vibration. The values of ω^2 depends on the magnitude of the applied load P . Depending on the nature of ω^2 , the system may loose stability either by divergence or by flutter. If ω^2 reduces to zero with the increase in the value of P , then the loss of stability is termed as divergence. Instead, if any two frequencies coalesce each other and become complex conjugate, then the loss of stability is by flutter. The system is unstable when any of the values of ω in equation (17) has a negative imaginary part. Further, if during the transition from stability to instability of the real part of ω is zero, then instability occurs due to divergence. Other wise instability occurs due to flutter.

Description of the Problem

The general problem considered here consists of a rectangular cantilever plate ($a \times b$) having cutout diameter (d) and thickness (h) subjected to partial follower edge load which is acting opposite to the clamped edge side as shown in Fig.1. The load direction control parameter (φ) is used. The inclination of the follower load with reference to x -direction is then defined by the product $\varphi \theta_y$, where $\varphi = 1$ defines a tangential follower load and $\varphi = 0$ defines a non-follower load. Fig.2(a) shows the specific case of a square ($a = b$) CFFF plate (plan view) subjected to partial follower edge load and Fig.2(b) represents the finite element mesh discretization of the plate with hole. An eight noded isoparametric quadratic element is employed in the present analysis by using Mindlin's plate theory considering five degrees of freedom u , v , w , θ_x , and θ_y per node.

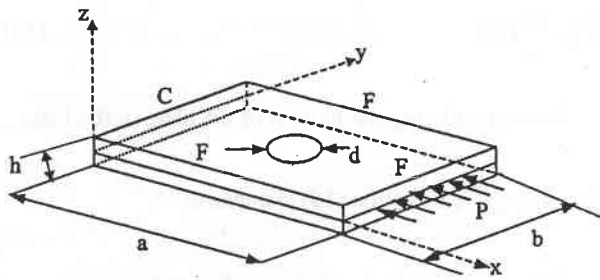


Figure 1(a)

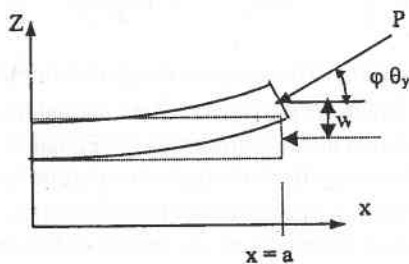


Figure 1(b)

Fig. 1 Plate under follower force
(a) Geometry of the plate with circular hole
(b) Plate under tangential follower force

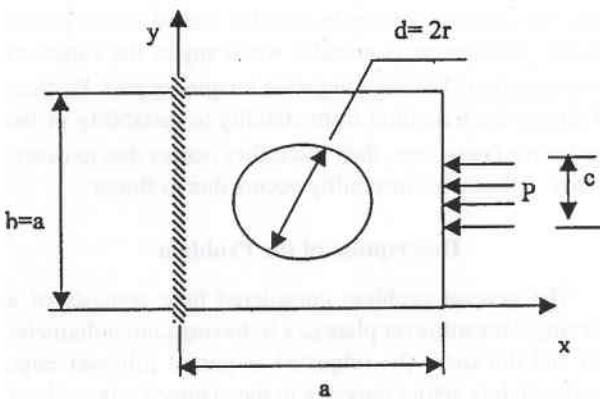


Fig. 2a Cantilever plate with circular cutout subjected to partial edge load extending from the centre (plan view)

The shear correction factor of $\frac{5}{6}$ [33] has been employed to account for the nonlinear distribution of shear strain through the thickness. In the present problem two types of material properties (M_1, M_2) are considered with C-F-F-F and C-F-S-S boundary conditions as follows:

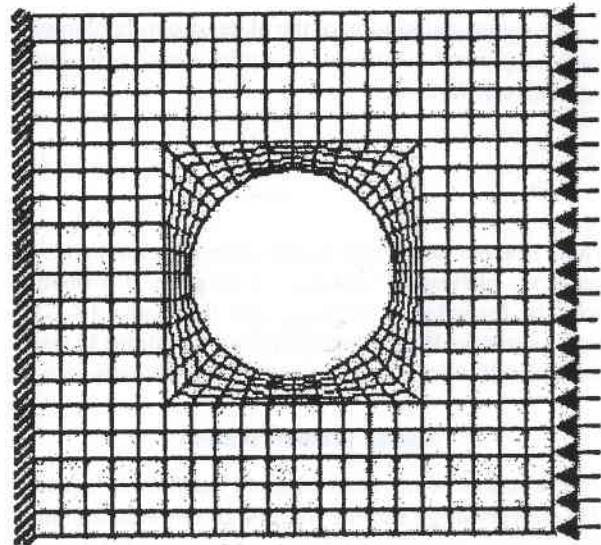


Fig. 2b Finite element mesh pattern for C-F-F-F curved panels with in-plane load (plan view)

The elastic constants of isotropic material (M_1) used in the present investigation are

$$\frac{E_{11}}{E_{22}} = 1.0, G_{12} = G_{13} = G_{21} = 0.3846E_{11}, \nu = 0.3$$

and that of laminated composite material

$$(M_2) \text{ are } \frac{E_{11}}{E_{22}} = 40, G_{12} = G_{13} = 0.6 E_{22},$$

$G_{23} = 0.5E_{22}, \nu_{12} = 0.25$. The orientation (θ) of fibers is measured with reference to x-direction (counter-clockwise positive). In the formulation of stiffness matrix for composite plate element, all the possible coupling effects have been considered.

Boundary Conditions:

- a) On edges parallel to y axis
 - i) Simply Supported, S : $w = \theta_x = 0$,
 - ii) Clamped, C: $u = v = w = \theta_y = \theta_x = 0$
 - iii) Free, F: no restraints

- b) On edges parallel to x axis
 - i) Simply Supported, S: $w = \theta_y = 0$,
 - ii) Clamped, C: $u = v = w = \theta_y = \theta_x = 0$
 - iii) Free, F: no restraints

In the present problem damping parameter (η) load direction control parameter (ϕ), load width parameter c ,

cutout ratio (r/b), (where, r is the radius of the circular cutout) are considered.

A computer program has been developed to perform all the necessary computations. The element elastic stiffness and mass matrices are obtained by using standard procedure of assembly. The subspace iteration method and complex eigenvalue solver is adopted throughout to solve the eigenvalue problems. In the present analysis, the non-dimensional frequencies and buckling loads are represented by the following definitions.

For Material (M1)

$$\lambda = \omega b^2 \sqrt{\left(\frac{\rho h}{D}\right)} \quad \text{and} \quad \gamma = N_x b^2 / D$$

where $D = E_{11} h^3 / 12 (1 - \nu^2)$

For Material (M2)

$$\lambda = \omega b^2 \sqrt{\left(\frac{\rho}{(E_{22} h^2)}\right)} \quad \text{and} \quad \gamma = N_x b^2 / E_{22} h^3$$

where $\lambda =$ non-dimensional complex frequency, $\gamma =$ non-dimensional load

Results and Discussions

Table-1 shows the convergence and comparison of fundamental frequencies of vibration for a cantilever plate with 0/90/0, 45/-45/45 orientations. The results are compared with those of Qatu [34]. Fig.3 shows the comparison of nondimensional buckling load for an isotropic simply supported square plate with cutout subjected to uniaxial compression load with Lee et al. [35]. Further to check the validity of the present model subjected to follower loading the results for CFSS boundary conditions are compared with Adali [15] and Deolasi [24] in Table-2 for an isotropic plate. The comparison shows that the flutter loads and flutter frequencies subjected to uniformly distributed follower loads are in good agreement with the published results. In the present paper, the results are mainly concerned with the first flutter mode and aspect ratio of $a/b=1$, breadth to thickness ratio of $b/h=100$ for all the cases unless otherwise mentioned.

Effect of Follower Force on Isotropic Plates with Cut-outs

Figures 4(a-b) and 5(a-b) show non-dimensional real and imaginary part of the frequency versus nondimen-

Table-1 : Convergence of non-dimensional frequencies without in-plane load of cross-ply laminated plate ($a/b = 1, b/t=100, E_{11}=138\text{Gpa}, E_{22}=8.96\text{Gpa}, \nu_{12}=0.3, \lambda=\omega b^2 \sqrt{(\rho/E_{11} t^2)}$)

Non-dimensional Frequencies (λ)		
Element Size	0/90/0	45/-45/45
6 x 6	0.9994	0.4603
8 x 8	0.9993	0.4596
10 x 10	0.9993	0.4591
Qatu [34]	0.9998	0.4607

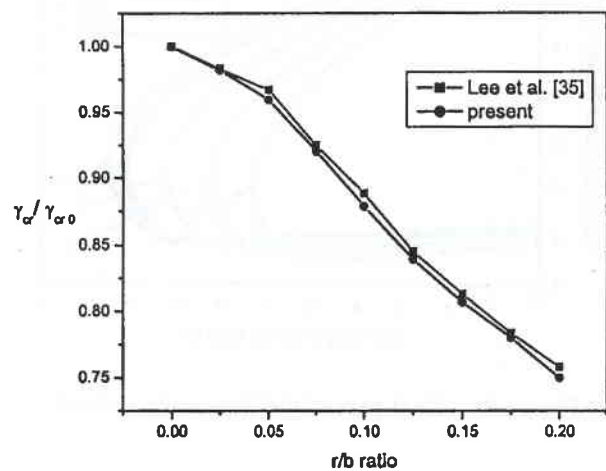


Fig. 3 Non-dimensional buckling load with cutout size ratio for different r/b

sional load (γ) for C-F-F-F and C-F-S-S plates respectively having direction control parameter (ϕ) equal to 1 (purely tangential) subjected to follower edge force with $c/b = 1$ and material property M_1 . In this analysis the first flutter mode is observed between the second and third eigenvalues for CFFF and the first and second frequencies for CFSS cases respectively. From the figures (real part vs load), it is observed that the critical flutter loads are gradually decreasing with increasing cutout size. From the figures (imaginary part of the frequency vs load), it is observed that the eigen frequencies are having zero imaginary roots until flutter occurs, and changing of their magnitude is noticed after the flutter mode. Figs.6 and 7 show the variation of flutter frequencies with load width parameter for CFFF and CFSS plate subjected to partial edge load [Fig.2b] with different cutout ratio for the first flutter mode. From the figures it is observed that the flutter frequencies are gradually decreasing with increasing load width parameter (c/b).

Table-2 : Comparison of non-dimensional flutter loads γ_{cr} and non-dimensional flutter frequencies λ_{cr} ($c/b = 1.0$ and $\nu = 0.3$)

a/b	Flutter Load γ_{cr}			Flutter Frequency λ_{cr}		
	Present	Adali [15]	Deolasi [24]	Present	Adali	Deolasi
1.0	51.651	51.650	52.06	16.696	16.67	16.33
0.5	26.923	27.110	27.20	49.567	49.58	49.30

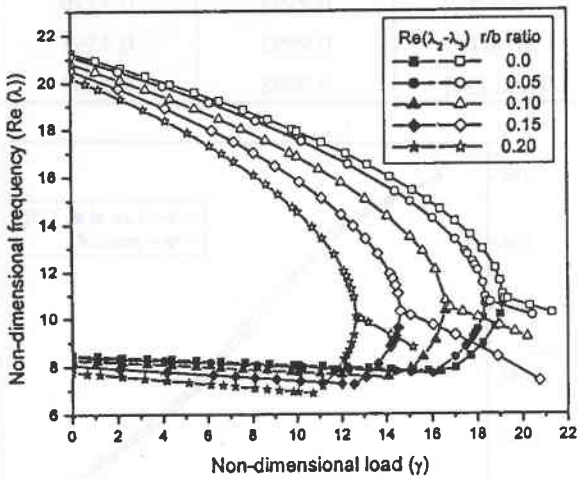


Figure 4(a) Real part of the frequency vs load

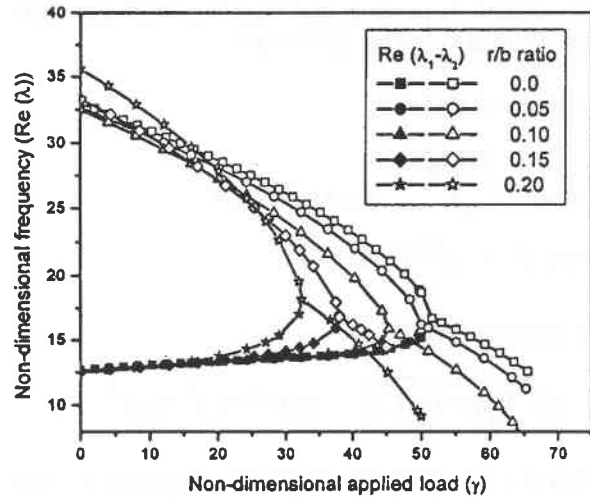
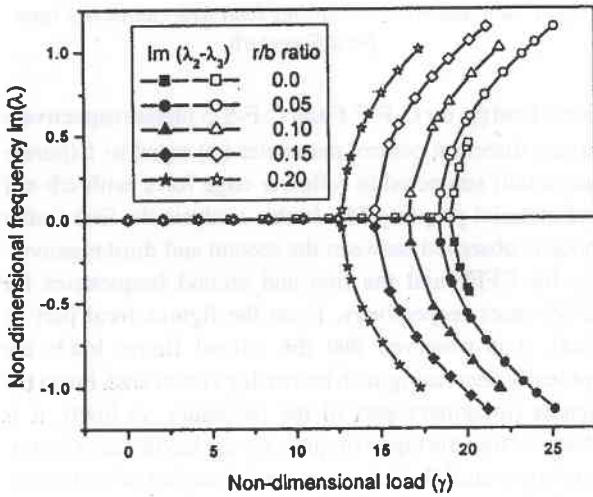
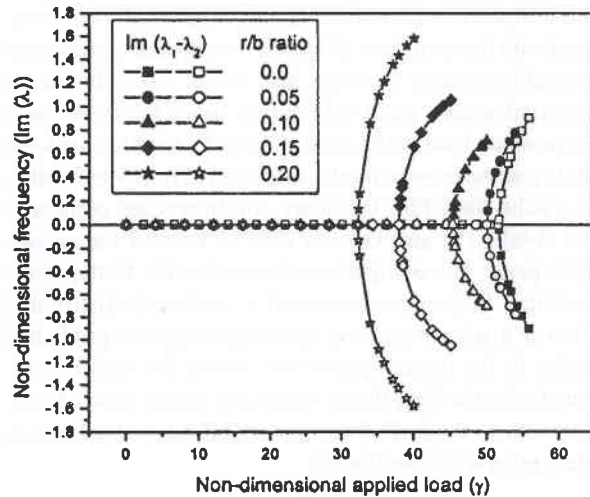


Figure 5(a) Real part of the frequency vs load



4(b) Imaginary part of the frequency vs load

Fig. 4 Non-dimensional frequency vs non-dimensional load for CFFF plate for different cutout ratios subjected follower force with $c/b = 1$



5(b) Imaginary part of the frequency vs load

Fig. 5 As figure 4, but for CFSS plate for different cutout ratios

Table-3 : Non-dimensional flutter loads γ_{cr} with different direction control parameters for C-F-S-S plate subjected to partial follower edge load extending from the centre

Non-dimensional flutter load γ_{cr}						
c/b ratio	Load direction control parameter (ϕ)					
	0.9	0.8	0.7	0.6	0.5	0.4
r/b ratio = 0.0						
0.2	24.8976	22.4592	20.5296	19.1100	18.1272	17.6904
0.4	26.5356	24.2424	22.4952	21.1848	20.3112	20.0928
0.6	29.9208	27.6276	25.7712	24.3516	23.5872	23.4780
0.8	35.5992	32.8692	30.576	28.9380	27.8460	27.5184
1.0	46.6284	42.0420	38.3292	35.5992	33.6336	32.5416
r/b ratio = 0.1						
0.2	21.1848	19.4376	18.018	16.9260	16.2708	16.0524
0.4	22.7136	21.0756	19.7652	18.7824	18.2364	18.1272
0.6	25.6620	23.9148	22.6044	21.6216	21.0756	21.0756
0.8	30.5760	28.5012	26.754	25.5528	24.7884	24.6792
1.0	40.1856	36.3636	33.4152	31.3404	29.9208	29.1564
r/b ratio = 0.2						
0.2	16.9260	15.9432	15.0696	14.5236	14.0868	14.1960
0.4	18.2364	17.3628	16.7076	16.1616	15.8340	16.0524
0.6	20.7480	19.8744	19.2192	18.6732	18.4548	18.7824
0.8	24.6792	23.5872	22.7136	22.1676	21.8400	22.0584
1.0	30.5760	29.0472	27.8460	26.8632	26.2080	25.9896

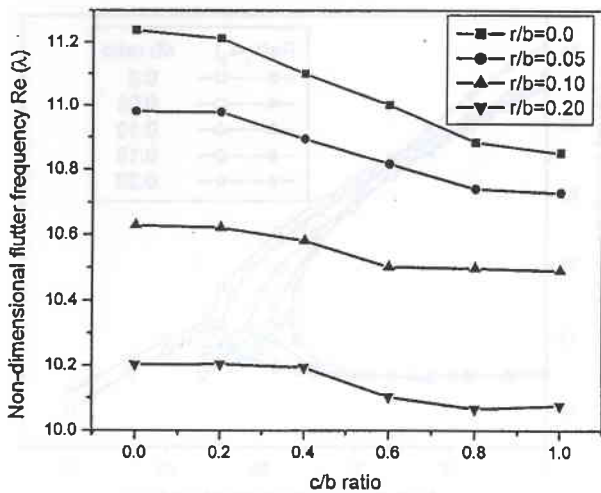


Fig. 6 Non dimensional flutter frequency (Real part) vs load width ratio for CFFF plate (material M1) with different r/b ratio

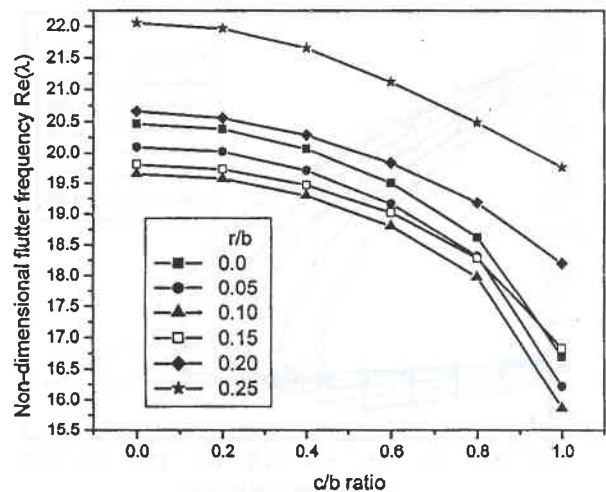


Fig. 7 As figure 6, but for CFSS plate

Effect of Load Direction Control and Damping Parameter

Figure 1(b) shows the plate subjected to applied follower force with directional control parameter ϕ . The magnitude of the direction control parameter is varied from 0 to 1. If $\phi = 0$ the effect of the normal component of the follower force is zero and the problem is similar to conservative load problem and only divergence instabilities exist. For $\phi = 1$, the force remains tangential to the deformed state of the plate. The system exhibits flutter or divergence types of instability based on the direction control parameter, ϕ . In the analysis, free vibration, divergence and flutter problems are solved simply by replacing the matrix $[K_{NC}]$ in equation (11) by $[K_{NC}]\phi$.

In the presence of damping, eigen frequencies of the plates are complex quantities having positive imaginary part initially (before flutter). When $\eta=0$, the two real frequencies merge together into a pair of conjugate complex frequency ($\lambda = \text{Re}(\lambda) \pm \text{Im}(\lambda)$) at γ_{cr} to form a flutter mode. As γ is increased beyond γ_{cr} , there is a rapid increase in the imaginary parts of the natural frequencies. Since one of the frequencies has a negative imaginary part, the plates undergo strong flutter. If $\eta \neq 0$, then the natural frequencies are always complex for all levels of load γ , and the two curves for the real part of the frequencies do not merge, but approach each other and initially both eigencurves have positive imaginary parts. However, as γ is increased, the imaginary part of the first frequency gradually changes

from positive to negative (crosses zero frequency line) at γ_{cr} . This phenomenon can be clearly observed later in the investigation of the effect of damping in composite plates. On this basis, the critical flutter loads have been found out for isotropic square C-F-S-S plates with cutout and are given in Table-4.

Figures 8 and 9 show the variation of real part of the eigen frequency vs applied load for CFFF and CFSS plates having different rib ratio with load direction control parameter (ϕ) of 0.8 and 0.6 respectively subjected to end follower force with $c/b = 1$. Table-3 shows the numerical result of CFSS plate subjected to partial follower edge load extending from the centre (Fig. 2(a)) with different load direction control parameters having rib ratio equals to 0.0, 0.1 and 0.2. Comparing the numerical results, it is observed that the critical flutter load is gradually increasing with increasing load direction parameter.

Table-4 shows the numerical results for CFSS plate with/without circular cutout subjected to uniformly distributed follower force ($c/b = 1$) with different load direction control parameter (ϕ) varying from 0.5 to 1.0 for the different damping factors (η) 0.0, 0.02 and 0.1 respectively. From the table it is noticed that the effect of damping is very significant on the stability characteristics of the plate with cutout and it shows the destabilizing effect of damping. Further it is also noticed that the critical flutter loads are gradually decreasing with decreasing direction control parameter.

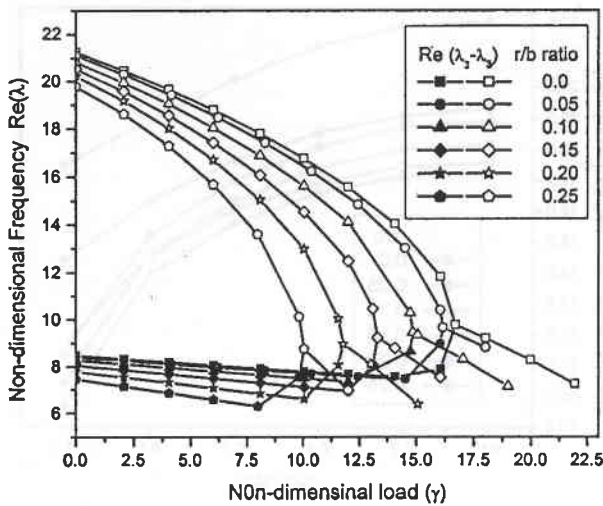


Fig. 8 Non-dimensional frequency (Real part) vs non-dimensional follower load with $\phi = 0.8$ for CFFF plate for different r/b ratios

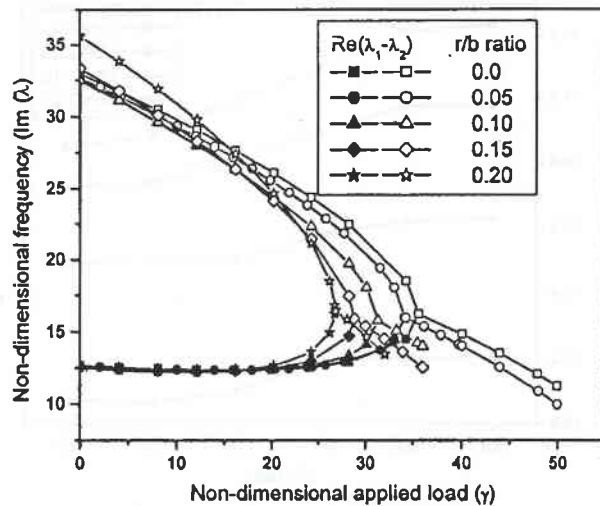


Fig. 9 As figure 8, but for CFSS plate and $\phi = 0.6$

Effect of Follower Force on Laminated Composite Plates with Cut-outs

Table-5 shows numerical results of critical flutter load and frequencies for laminated CFFF composite plate with different cutout sizes having 0/90, 0/90/0, 0/90/90/0 and 0/90/0/90 ply orientations respectively subjected to uniformly distributed follower edge load having $c/b=1$. Figs.10 and 11 show the variation of flutter frequencies with different cutout sizes for CFSS cross-ply and CFFF $\theta/-\theta/\theta$ angle ply laminated plates. Fig.12 shows non-dimensional flutter load with load direction control parameter for 0/90/0-laminated plate. From the figure it is observed that the flutter load is gradually increasing with direction control parameter and also observed that the magnitude of the critical flutter load is changing significantly with the cutout size ratio. Table-6 shows numerical results for 0/90, 0/90/0 and 0/90/90/0 laminated plate with cutout subjected to follower edge load having $c/b=1$, and damping parameters 0.0, 0.02 and 0.10. It is generally

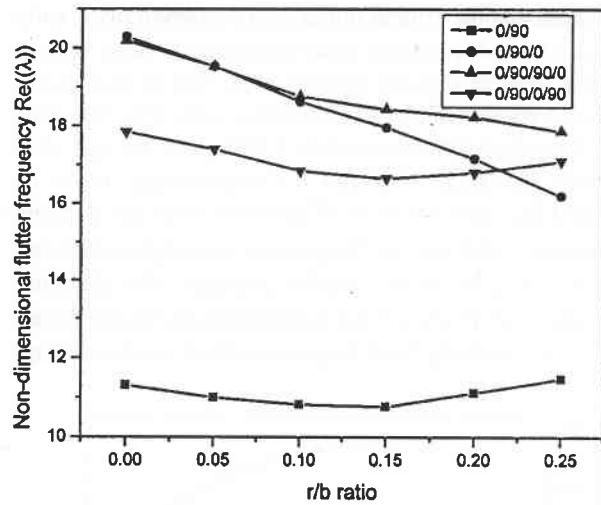


Fig. 10 Non-dimensional flutter frequency (Real part) vs cut-out ratio for CFSS cross-ply laminated plate

Table-4 : Effect of load direction control and damping parameter on non-dimensional flutter loads γ_{cr} C-F-S-S plate with cutout subjected follower edge load with $c/b = 1.0$

r/b ratio	Damping factor (η)	Load direction control parameter (ϕ)					
		1.0	0.9	0.8	0.7	0.6	0.5
0.0	0.0	51.6516	46.6284	42.042	38.3292	35.5992	33.6336
	0.02	47.3928	42.0420	37.128	33.6336	31.2312	29.7024
	0.10	49.1400	43.5432	38.1108	34.398	31.8864	30.4668
0.10	0.0	45.4272	40.1856	36.3636	33.4152	31.3404	29.9208
	0.02	42.8064	36.6912	32.5416	29.7024	27.8460	26.754
	0.10	44.9904	37.8924	33.4152	30.4668	28.6104	27.5184
0.20	0.0	32.4324	30.576	29.0472	27.846	26.8632	26.208
	0.02	28.1736	26.3172	24.8976	23.9148	23.2596	23.0412
	0.10	28.8288	26.8632	25.4436	24.3516	23.6964	22.5872

Table-5 : Non-dimensional flutter loads (γ_{cr}) and non-dimensional frequency (λ_{cr}) a laminated cross-ply plate with different cutout sizes subjected to uniformly distributed follower load for C-F-F-F case

r/b ratio	0/90		0/90/0		0/90/90/0		0/90/0/90	
	γ_{cr}	λ_{cr}	γ_{cr}	λ_{cr}	γ_{cr}	λ_{cr}	γ_{cr}	λ_{cr}
0.0	11.0	8.1341	63.40	19.4585	57.80	18.5767	28.40	12.9914
0.05	10.6	7.9934	57.40	19.4744	51.40	20.7038	26.80	12.8424
0.10	9.40	7.8361	44.40	19.4764	40.20	21.0422	23.80	12.4354
0.15	8.20	7.6590	33.60	19.2974	31.40	21.1170	20.40	12.1867
0.20	7.00	7.5059	24.80	18.8523	25.20	20.8986	17.20	12.0019

noticed that the critical flutter load for 0/90/0 ply orientation having the highest value compared to other orientations considered in the analysis. Figs.13(a-b) and 14(a-b) show variation of real and imaginary part of the frequency with applied follower load for CFSS plate having cutout ratios r/b equal to 0 and 0.1 respectively. From the Figs.13(a) and 14(a), it is observed that for damping parameter $\eta=0$, the two frequencies are merging with each other and going to be complex conjugate after the flutter is observed. For $\eta \neq 0$ the natural frequencies are always complex. Initially both frequencies have positive imagi-

nary parts. However as load increased, the imaginary part of the frequency gradually changes from positive to negative and crosses the zero frequency line at γ_{cr} . From the Figs.13(b) and 14(b) it is observed that the flutter in presence of damping occurs at much lower load as compared to without damping. The negative imaginary part of the first natural frequency implies that there is negative effective damping in the first mode for load γ greater than γ_{cr} . Thus damping has a destabilising effect on flutter behaviour of plates under follower load.

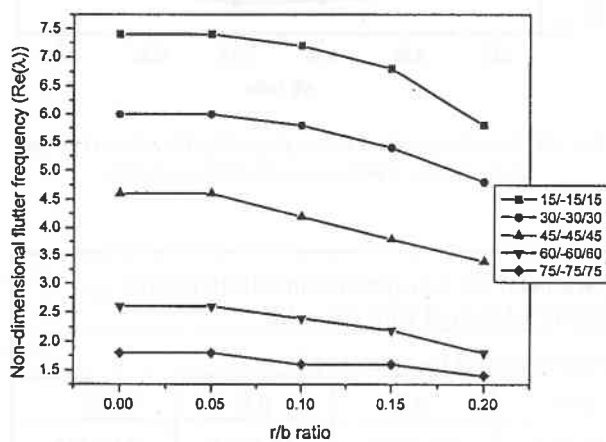


Fig. 11 Non-dimensional flutter frequency (Real part) vs cut-out ratio for CFFF angle-ply laminated plate having $\theta/-\theta/\theta$ ply-orientation

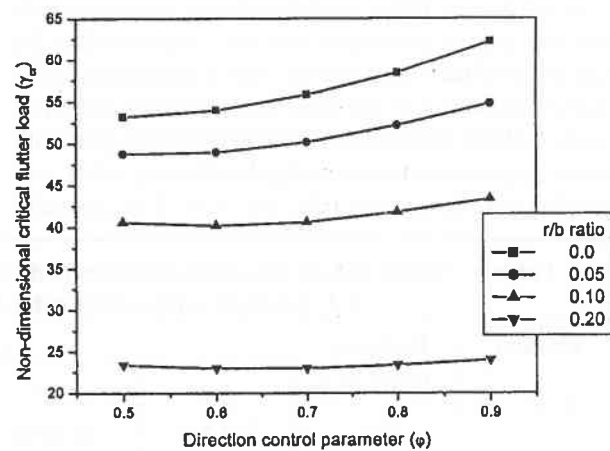


Fig. 12 Non-dimensional flutter load vs direction control parameter for CFSS 0/90/0 plate with cutout

Table-6 : Effect of damping on non-dimensional flutter loads γ_{cr} for a cross-ply plate subjected to uniformly distributed follower edge load for C-F-S-S case						
Ply-orientation	Damping factor (η)	Non-dimensional critical flutter load				
		Cut-out size ratio (r/b)				
		0.0	0.05	0.10	0.15	0.20
0/90	0.0	14.6	13.8	11.4	9.6	8.0
	0.02	12.8	12.0	10.2	8.4	7.0
	0.10	13.2	12.2	10.4	8.6	7.2
0/90/0	0.0	67.0	58.6	45.6	34.6	24.8
	0.02	42.8	38.2	30.2	22.6	16.0
	0.10	43.2	38.6	30.4	22.8	16.1
0/90/90/0	0.0	61.4	54.2	43.0	33.8	25.8
	0.02	43.2	38.8	31.2	24.2	18.2
	0.10	43.6	39.2	31.6	24.6	18.3

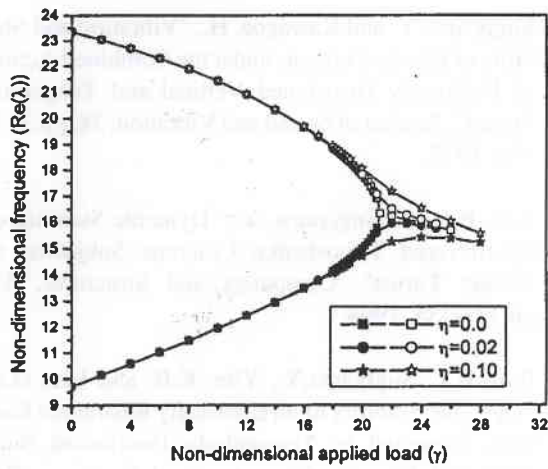


Figure 13(a) Real part vs Non-dimensional load

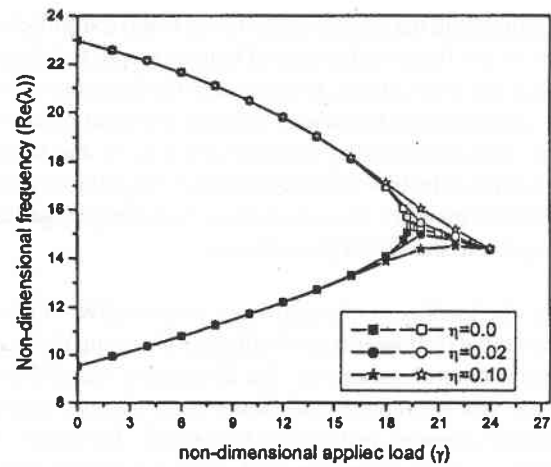


Figure 14(a) Real part vs Non-dimensional load

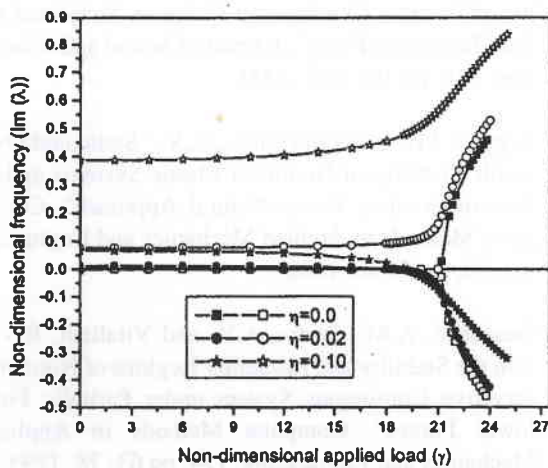


Figure 13(b) Imaginary part vs. Non-dimensional load

Fig. 13 Non-dimensional frequency vs non-dimensional follower load for CFSS 45/-45/45 angle-ply plate having $r/b = 0$ with damping parameters.

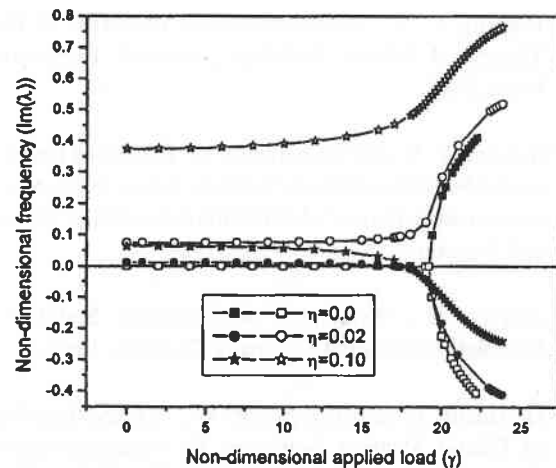


Figure 14(b) Imaginary part vs. Non-dimensional load

Fig. 14 As figure 13, but for $r/b = 0.1$

Conclusions

The results from dynamic instability behaviour of laminated composite plates subjected to partially distributed follower loading can be summarized as follows:

The follower loading on the free edges may undergo flutter type of instability beyond a certain value of the follower load due to coalescence of two frequencies into a complex conjugate pair. Flutter is observed to be more common than divergence under follower type of loading.

The cutout is very much influences on the dynamic stability of both isotropic and laminated plates. The magnitude of the flutter load is gradually decreasing with the increasing cutout size.

The ply-orientation, load bandwidth and type of load conditions have a significant influence on the flutter and divergence characteristics of both isotropic and laminated plate having different boundary conditions.

Damping in the system is perceived to have significant effect on the flutter behaviour of both isotropic and laminated plates with cutouts. In most cases the damping effect gives destabilizing behaviour, making the plate prone to flutter. The destabilizing effect of damping is due to the fact that the effective damping becomes negative beyond certain load in one of the two modes, which merge together giving rise to the flutter phenomenon.

As the loading is changed from conservative to non-conservative (follower type) loading by changing the load direction control parameter, the divergence instabilities become less prominent and flutter instabilities initiate, indicating dynamic instability phenomena. The effect of direction control parameter with damping significantly affects the critical load depending on the type of material and type of ply orientation.

References

1. Bolotin, V. V., "Nonconservative Problems of the Theory of Elastic Stability", Oxford, Pergamon Press, 1963.
2. Bolotin, V. V. and Zhinzher, N.I., "Effects of Damping on Stability of Elastic Systems Subjected to Non-conservative Forces", International Journal of Solids and Structures, 5, pp.965-989, 1969.
3. Ziegler, H., "Principles of Structural Stability", Blaisdell Publication Company, Toronto, 1968.
4. Herrmann, G. and Bungay, R. W., "On the Stability of Elastic Systems Subjected to Nonconservative Forces", Journal of Applied Mechanics, 31, pp. 435-440, 1964.
5. Herrmann, G and Jong, I-C., "On the Destabilizing Effect of Damping in Nonconservative Elastic Systems". Journal of Applied Mechanics 32, pp. 592-597, 1965.
6. Sankaran, G.V. and Venkateswara Rao, G., "Stability of Tapered Cantilever Columns Subjected to Follower Forces", Computers and Structures 6, pp.217-220, 1976.
7. Leipholz, H.H.E., "On a Variational Principle for Clamped-Free Rod Subjected to Tangential Follower Forces", Mechanics of Research Communications, 5, pp.335-339, 1978.
8. Sugiyama, Y. and Kawagoe, H., "Vibration and Stability of Elastic Columns under the Combined Action of Uniformly Distributed Vertical and Tangential Forces", Journal of Sound and Vibration, 38, pp.314-355, 1975.
9. Ryu, B.J. and Sugiyama, Y., "Dynamic Stability of Cantilevered Timoshenko Columns Subjected to Rocket Thrust", Computers and Structures, 51, pp.331-335, 1994.
10. Ryu, B.J., Sugiyama, Y., Yim, K.B. and Lee, G.S. "Dynamic Stability of an Elastically Restrained Column Subjected to Triangularly Distributed Subtangential Forces", Computers and Structures 76, pp.611-619, 2000.
11. Sugiyama, Y., Katayama, K., Kariyama, K. and Ryu, B.J., "Experimental Verification of Dynamic Stability of Vertical Cantilevered Columns Subjected to Sub-Tangential Force", Journal of Sound and Vibration, 236, pp.193-207, 2000.
12. Argyris, J.H. and Symeonidis, K.S., "Static and Dynamic Stability of Nonlinear Elastic Systems under Nonconservative Forces-Natural Approach", Computer Methods in Applied Mechanics and Engineering, 32, pp.59-83, 1982.
13. Gasparini, A.M., Saetta, A.V. and Vitaliani, R.V., "On the Stability and Instability Regions of Nonconservative Continuous System under Partially Follower Forces", Computer Methods in Applied Mechanics and Engineering, 124, pp.63- 78, 1995.
14. Farshad, M., "Stability of Cantilever Plates Subjected to Biaxial Subtangential Loading", Journal of Sound and Vibration 58, pp.555-561, 1978.
15. Adali, S., "Stability of Rectangular Plates under Non-conservative and Conservative Forces", International Journal of Solids and Structures, 18, pp.1043-1052, 1982.
16. Culkowski, P.M. and Reismann, H., "Plate Buckling due to Follower Edge Force", Transactions of ASME, Journal of Applied Mechanics, 44, pp. 768-769, 1977.
17. Leipholz, H.H.E., "Stability of a Rectangular Simply Supported Plate Subjected to Nonincreasing Tangen-

- tial Follower Forces", *Transactions of AS ME, Journal of Applied Mechanics*, 45, pp.223-224, 1978.
18. Leipholz, H.H.E. and Pfenndt, F., "Application of Extended Equations of Galerkin to Stability problems of Rectangular Plates with Free Edges Subjected to Uniformly Distributed Follower Forces", *Computer Methods in Applied Mechanics and Engineering*, 37, pp.341-365, 1983.
 19. Higuchi, K. and Dowell, E.H., "Effect of Structural Damping on Flutter of Plates with Follower Force", *AIAA Journal*, 30, pp.820-825, 1992.
 20. Ashwini Kumar and Srivastava, A.K., "Stability of Thin Rectangular Elastic Plates under Follower Force", *Mechanics Research Communications*, 13, pp.165-168, 1986.
 21. Kishore, Y.K., "Stability of an Orthotropic Plate under Follower Force", *Mechanics Research Communications*, 15, pp.249-252, 1988.
 22. Kim, J.H. and Kim, H.S., "A Study on the Dynamic Stability of Plate under Follower Force", *Computers and Structures* 74, pp.351-363, 2000.
 23. Kim, J.H. and Park, J.H., "On the Dynamic Stability of Rectangular Plates Subjected to Intermediate Follower Forces", *Journal of Sound and Vibration, Letters to the Editor* 209, pp.882-888, 1998.
 24. Datta, P.K. and Deolasi, P.J., "Dynamic Instability Characteristics of Plates Subjected to Partially Distributed Follower Edge Loading with Damping". *Proc. Inst. Mech Engrs (U.K)* 210, pp.445-452, 1996.
 25. Herrmann, G., "Stability of Equilibrium of Elastic Systems Subjected to Nonconservative Forces", *Applied Mechanics Review*, 20, pp.103-108, 1967.
 26. Komarakul-Na-Nakoran, A. and Arora, J.S., "Stability Criteria: A Review", *Computers and Structures* 37, pp.35-49, 1990.
 27. Bismarck-Nasr, M.N., "Finite Element Analysis of Aeroelasticity of Plates and Shells", *ASME, Applied Mechanics Review*, 45, pp.461-482, 1992.
 28. Langthjem, M.A. and Sugiyama, Y., "Dynamic Stability of Columns Subjected to Follower Loads: A Survey", *Journal of Sound and Vibration*, 238, pp.809-851, 2000.
 29. Bazant, Z.P., "Structural Stability", *International Journal of Solids and Structures* 37, 55-67, 2000.
 30. Ravi Kumar, L., Datta, P.K. and Prabhakara, D.L., "Dynamic Instability Characteristics of Laminated Composite Plates Subjected to Partial Follower Edge Load with Damping", *International Journal of Mechanical Sciences*, 45, pp.1429-1448, 2003.
 31. Jones, R., "Mechanics of Composite Material", McGraw-Hill, New York, 1998.
 32. Cook, R.D., "Concepts and Applications of Finite Element Analysis", John Wiley, 1989.
 33. Moorthy, J., Reddy, J.N. and Plaut, R.H., "Parametric Instability of Laminated Composite Plates with Transverse Shear Deformation", *International Journal of Solids and Structures*, 26, pp.801-811, 1990.
 34. Qatu, M.S. and Leissa, A.W., "Natural Frequencies for Cantilevered Doubly Curved Laminated Composite Shallow Shells", *Composite Structures* 17, pp.227-255, 1991.
 35. Lee, Y.J., Lin, H.J., and Lin, C.C., "A Study of Buckling Behaviour of an Orthotropic Square Plate with Central Circular Hole", *Composite Structures*, 13, pp.173-188, 1989.