ASYMMETRIC DYNAMIC BUCKLING OF ISOTROPIC/ ANISOTROPIC SPHERICAL CAPS

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Abstract

This paper deals with nonlinear asymmetric dynamic buckling of clamped isotropic/anisotropic spherical shells under suddenly applied pressure loads. The formulation is based on first-order shear deformation theory and Lagrange's equation of motion. The nonlinearity due to finite deformation of the shell considering von Karman's assumptions is included in the formulation. The buckling loads are obtained through dynamic response history using Newmark's numerical integration scheme coupled with a Newton-Raphson iteration technique. An axisymmetric curved shell element is used to investigate the dynamic characteristics of the spherical caps. The pressure value beyond which the maximum average displacement response shows significant growth rate in the time history of the shell structure is considered as critical dynamic load. Detailed numerical results are presented to highlight the influences of shell geometric parameter, orthotropicity, ply-angle, number of layers and asymmetric mode on the critical load of spherical caps.

Keywords : Dynamic buckling, Asymmetric, Isotropic, Angle-ply, Cross-Ply, Spherical Caps, Nonlinear response

Introduction

Thin spherical shells have been interesting topics in structural engineering not only for their widespread applications but also the complicated stability problems. These shells often subjected to snap-through buckling. Buckling analysis of such shells under dynamic loads has received considerable attention in the literature. It is known, in general, that shells subjected to dynamically applied loads usually buckle at load levels that are lower than the corresponding quasi-static buckling load.

The available work on axisymmetric dynamic buckling behavior of spherical shells is mainly concerned with isotropic case subjected to the step pressure load of infinite duration. Budiansky and Roth [1] have analyzed the problem employing the Galerkin method whereas Simitses [2] adopted Ritz-Galerkin procedure. Haung [3], Stephens and Fulton [4], and Ball and Burt [5] have investigated using the finite difference scheme while Stricklin and Martinez [6] utilized more efficient finite element procedure. The effect of geometric imperfection on the dynamic buckling load, by employing buckling criterion based on the displacement response, is examined by Kao and Perrone [7], and Kao [8] based on finite difference method whereas Saigal et.al [9] and Yang and Liaw [10] analyzed using finite element technique. Lock et.al [11] have carried out experimental study on the buckling of spherical caps.

Quite often, the asymmetric modes of these shells may be excited due to the introduction of slight deviation in perfect axisymmetric loading, geometric imperfection and/or initial displacement/velocity to the shells, leading to asymmetric type of buckling behavior. The buckling of isotropic spherical shell under such modes has received very limited attention in the literature. Ball and Burt [5], Stricklin and Martinez [6] and Stricklin et.al [12] have assumed imperfection in the step load to excite the asymmetric modes and presented results for a shell geometry with few asymmetric modes. Klosner and Longhitano [13], while obtaining the response of dynamically loaded spherical shells, have considered an asymmetric initial velocity to the shell but the numerical results are not presented for the dynamic buckling study. Akkas [14] has

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examined the asymmetric dynamic buckling behavior of spherical caps by perturbing few asymmetric modes through the initial displacement and presented very few results based on the response of asymmetric part of the displacement. It may be inferred that the effect of asymmetric modes of spherical shell with step load of infinite duration on the dynamic buckling characteristics could not be well established with a few available work in the literature in comparison with those of axisymmetric dynamic buckling case.

In recent years, advanced composite materials have found rapid growth in engineering structural applications. However, the complexity of the analysis due to the inherent directional properties of the composite materials has limited the study to axisymmetric dynamic buckling behavior of single-layered orthotropic spherical case using classical lamination theory [15-18], except the work of Ganapathi and Varadan [19]. Alwar and Sekhar Reddy [15], and Dumir et.al [17] have examined the problem using the method of orthogonal collocation whereas Chao and Lin [18] have obtained the critical loads based on finite difference scheme including the influence of geometric imperfection. Ganapathi and Varadan [19] have solved the problem employing shear deformation theory coupled with finite element technique. However, to author's knowledge, work on the asymmetric dynamic buckling behavior of laminated composite spherical shells under externally applied pressure seems to be scarce in the literature.

In the present work, a three-noded shear flexible axisymmetric curved shell element based on semi-analytical approach and the field-consistency principle [20, 21] is extended to analyze the asymmetric dynamic buckling of isotropic/anisotropic spherical caps under externally applied pressure load. Geometric nonlinearity is assumed in the present study using von Karman's strain-displacement relations. The nonlinear governing equations derived are solved employing Newmark numerical integration method in conjunction with the modified Newton-Raphson iteration scheme. For axisymmetric case, the dynamic buckling pressure is defined as the pressure corresponding to a sudden jump in the maximum average displacement in the time history of the shell structure [1, 22]. However, the load associated with the threshold value of pressure beyond which the asymmetric component of displacement response of shell shows significant growth rate with time is taken as the critical load for asymmetric buckling case [14,23]. A detailed investigation is carried out to bring out the influences of geometric parameters, orthotropicity, lamination scheme, ply-angle and different asymmetric modes of excitation on the dynamic buckling characteristics of clamped spherical caps.

Formulation

An axisymmetric laminated composite shell of revolution is considered with the coordinates s, θ and z along the meridional, circumferential and radial/thickness directions, respectively. The displacements u, v, w at a points (s, θ, z) from the median surface are expressed as functions of middle-surface displacements u_o, v_o and w and independent rotations β_s and β_θ of the meridional and hoop sections, respectively, as

$$u(s,\theta,z,t) = u_o(s,\theta,t) + z\,\beta_s(s,\theta,t)$$
$$v(s,\theta,z,t) = v_o(s,\theta,t) + z\,\beta_\theta(s,\theta,t)$$
$$w(s,\theta,z,t) = w(s,\theta,t) \qquad (1)$$

where t is the time.

Using the semi-analytical approach, u_o, v_o, w, β_s and β_θ are represented by a Fourier series in the circumferential angle θ. For the n^{th} harmonic, these can be written as

$$u_o(s,\theta,t) = u_o^o(s,t) + \sum_{i=1}^{4}\left[u_o^{c\,i}(s,t)\cos(in\theta) + u_o^{s\,i}(s,t)\sin(in\theta)\right]$$

$$v_o(s,\theta,t) = v_o^o(s,t) + \sum_{i=1}^{4}\left[v_o^{c\,i}(s,t)\cos(in\theta) + v_o^{s\,i}(s,t)\sin(in\theta)\right]$$

$$w(s,\theta,t) = w^o(s,t) + \sum_{i=1}^{2}\left[w^{c\,i}(s,t)\cos(in\theta) + w^{s\,i}(s,t)\sin(in\theta)\right]$$

$$\beta_s(s,\theta,t) = \beta_s^o(s,t) + \sum_{i=1}^{2}\left[\beta_s^{c\,i}(s,t)\cos(in\theta) + \beta_s^{s\,i}(s,t)\sin(in\theta)\right]$$

$$\beta_\theta(s,\theta,t) = \beta_\theta^o(s,t) + \sum_{i=1}^{2}\left[\beta_\theta^{c\,i}(s,t)\cos(in\theta) + \beta_\theta^{s\,i}(s,t)\sin(in\theta)\right] \qquad (2)$$

The above displacement variations in the circumferential direction are chosen according to the physics of the large amplitude asymmetric vibrations of shells of revolution i.e. participation of axisymmetric mode and higher asymmetric modes [24-26]. Additional terms in the in-plane displacements, compared to radial displacement, are added to keep the nonlinear membrane strains consistent.

Using von Karman's assumption for moderately large deformation, Green's strains can be written in terms of mid-plane deformations as,

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_p^L \\ 0 \end{Bmatrix} + \begin{Bmatrix} z\varepsilon_b \\ \varepsilon_s \end{Bmatrix} + \begin{Bmatrix} \varepsilon_p^{NL} \\ 0 \end{Bmatrix} \tag{3}$$

where, the membrane strains $\{\varepsilon_p^L\}$, bending strains $\{\varepsilon_b\}$, shear strains $\{\varepsilon_s\}$ and nonlinear in-plane strains $\{\varepsilon_p^{NL}\}$ in the Eq. (3) are written as [17]

$$\{\varepsilon_p^L\} = \begin{Bmatrix} \dfrac{\partial u_o}{\partial s} + \dfrac{w}{R} \\ \dfrac{u_o \sin\phi}{r} + \dfrac{\partial v_o}{r\partial\theta} + \dfrac{w\cos\phi}{r} \\ \dfrac{\partial u_o}{r\partial\theta} - \dfrac{v_o \sin\phi}{r} + \dfrac{\partial v_o}{\partial s} \end{Bmatrix};$$

$$\{\varepsilon_b\} = \begin{Bmatrix} \dfrac{\partial \beta_s}{\partial s} + \dfrac{\partial u_o}{R\partial s} \\ \dfrac{\beta_s \sin\phi}{r} + \dfrac{\partial\beta_\theta}{r\partial\theta} + \dfrac{u_o \sin\phi}{R\,r} \\ \dfrac{1}{R}\dfrac{\partial u_o}{r\partial\theta} + \dfrac{\partial v_o}{\partial s}\dfrac{\cos\phi}{r} + \dfrac{\partial\beta_s}{r\partial\theta} + \dfrac{\partial\beta_\theta}{\partial s} - \dfrac{\beta_\theta \sin\phi}{r} \end{Bmatrix};$$

$$\{\varepsilon_s\} = \begin{Bmatrix} \beta_s + \dfrac{\partial w}{\partial s} \\ \beta_\theta + \dfrac{\partial w}{r\partial\theta} - \dfrac{v_o\cos\phi}{r} \end{Bmatrix}; \quad \{\varepsilon_p^{NL}\} = \begin{Bmatrix} \dfrac{1}{2}\left(\dfrac{\partial w}{\partial s}\right)^2 \\ \dfrac{1}{2}\left(\dfrac{\partial w}{r\partial\theta}\right)^2 \\ \dfrac{\partial w}{\partial s}\dfrac{\partial w}{r\partial\theta} \end{Bmatrix} \tag{4}$$

where r, R and ϕ are the radius of the parallel circle, radius of the meridional circle and angle made by the tangent at any point in the shell with the axis of revolution.

If {N} represents the stress resultants $(N_{ss}, N_{\theta\theta}, N_{s\theta})$ and $\{M\}$ the moment resultants $(M_{ss}, M_{\theta\theta}, M_{s\theta})$, one can relate these to membrane strains $\{\varepsilon_p\}\ (=\{\varepsilon_p^L\} + \{\varepsilon_p^{NL}\})$ and bending strains $\{\varepsilon_b\}$ through the constitutive relations as

$$\{N\} = [A]\{\varepsilon_p\} + [B]\{\varepsilon_b\} \text{ and } \{M\} = [B]\{\varepsilon_p\} + [D]\{\varepsilon_b\} \tag{5}$$

where [A], [D] and [B] are extensional, bending and bending-extensional coupling stiffness coefficients matrices of the composite laminate. Similarly, the transverse shear force {Q} representing the quantities $(Q_{sz}, Q_{\theta z})$ are related to the transverse shear strains $\{\varepsilon_s\}$ through the constitutive relation as

$$\{Q\} = [E]\{\varepsilon_s\} \tag{6}$$

where [E] is the transverse shear stiffness coefficients matrix of the laminate.

For a composite laminate of thickness h, consisting of N layers with stacking angles ϕ_i $(i = 1, ..., N)$ and layer thicknesses h_i $(i = 1, ..., N)$, the necessary expressions to compute the stiffness coefficients, available in the literature [28] are used here.

The potential energy functional $U(\delta)$ is given by,

$$U(\delta) = \frac{1}{2}\int_A \Big[\{\varepsilon_p\}^T[A]\{\varepsilon_p\} + \{\varepsilon_p\}^T[B]\{\varepsilon_b\} + \{\varepsilon_b\}^T[B]\{\varepsilon_p\} + \{\varepsilon_b\}^T[D]\{\varepsilon_b\} + \{\varepsilon_s\}^T[E]\{\varepsilon_s\}\Big]\,dA - \int_A q\,w\,dA \tag{7}$$

where δ is the vector of degrees of freedom associated to the displacement field in a finite element discretisation and q is the applied external pressure load.

The kinetic energy of the shell is given by

$$T(\delta) = \frac{1}{2}\int_A \left[p\left(\dot{u}_o^2 + \dot{v}_o^2 + \dot{w}^2\right) + I\left(\dot{\beta}_s^2 + \dot{\beta}_\theta^2\right)\right] dA \tag{8}$$

where $p = \int_{-h/2}^{h/2} \rho \, dz$... and ... $I = \int_{-h/2}^{h/2} \rho z^2 \, dz$ and ρ is the mass density. The dot over the variable denotes derivative with respect to time.

Following the procedure given in the work of Rajasekaran et.al [29], the potential energy functional U given in Eq.(7) is rewritten as

$$U(\delta) = \{\delta\}^T \left[(1/2[K] + [(1/6)[N_1(\delta)] + (1/12)[N_2(\delta)] \right] \{\delta\}$$
$$= \{\delta\}^T \{F\} \tag{9}$$

where [K] is the linear stiffness matrix, $[N_1]$ and $[N_2]$ are non-linear stiffness matrices linearly and quadratically dependent on the field variables, respectively and {F} is the load vector. Substituting Eqs. (8) and (9) in Lagrange's equation of motion, the governing equation for the shell are obtained as :

$$[M]\{\ddot{\delta}\} + \left[[K] + \frac{1}{2}[N_1(\delta)] + \frac{1}{3}[N_2(\delta)] \right] \{\delta\} = \{F\} \tag{10}$$

where [M] is the mass matrix.

The Eq. (10) is solved using the implicit method [30]. In this method, equilibrium conditions are considered at the same time step for which solution is sought. If the solution is known at time t and one wishes to obtain the displacements, etc. at time $t+\Delta t$, then the equilibrium equations considered at time $t+\Delta t$ are given as

$$[M]\{\ddot{\delta}\}_{t+\Delta t} + [[N(\delta)]\{\delta\}]_{t+\Delta t} = \{F\}_{t+\Delta t} \tag{11}$$

where $\{\ddot{\delta}\}_{t+\Delta t}$ and $\{\delta\}_{t+\Delta t}$ are the vectors of the nodal accelerations and displacements at time $t+\Delta t$ respectively. $[[N(\delta)]\{\delta\}]_{t+\Delta t}$ is the internal force vector at time $t+\Delta t$ and is given as

$$[[N(\delta)]\{\delta\}]_{t+\Delta t} = ([[K] + (1/2)[N_1(\delta)$$
$$+ (1/3)[N_2(\delta)]]\{\delta\})_{t+\Delta t}, \tag{12}$$

In developing equations for the implicit integration, the internal forces $[N(\delta)]\{\delta\}$ at the time $t+\Delta t$ are written in terms of the internal forces at time t, by using the tangent stiffness approach, as

$$[[N(\delta)]\{\delta\}]_{t+\Delta t} = [[N(\delta)]\{\delta\}]_t + [K_T(\delta)]_t\{\Delta\delta\}, \tag{13}$$

where $[K_T(\delta)]_t = [[K] + [N1] + [N2]]$ is the tangent stiffness matrix and $\{\Delta\delta\} = \{\delta\}_{t+\Delta t} - \{\delta\}_t$. Substituting Eq. (13) into Eq.(11), one obtains the governing equation at $t+\Delta t$ as

$$[M]\{\ddot{\delta}\}_{t\Delta t} + [K_T(\delta)]_t\{\Delta\delta\} = \{F\}_{t+\Delta t} - [N(\delta)]\{\delta\}]_t, \tag{14}$$

To improve the solution accuracy and to avoid numerical instabilities, it is necessary to employ iteration within each time, thus maintaining the equilibrium.

The non-linear equations obtained by the above procedure are solved by the Newmark's numerical integration method. Equilibrium is achieved for each time step through modified Newton-Raphson iteration until the convergence criteria [31] are satisfied within the specific tolerance limit of less than one percent.

Dynamic Buckling Criterion

Criteria for the static buckling of axisymmetric shallow spherical shell are well defined whereas it is not so for the dynamic case. It requires the evaluation of the transient response of the shell for different load amplitudes. However, the dynamic buckling criterion suggested by Budiansky and Roth [1] is generally accepted because the results obtained by various investigators by different numerical techniques using the criterion are in reasonable agreement with each other. This criterion is based on the plots of the peak nondimensional average displacement in the time history of the structure with respect to the amplitude of the pressure load. The average displacement Δ is defined as

$$\Delta = \frac{\int_0^a r\,w\,dr}{\int_0^a r\,Z\,dr}$$

The numerator is the volume generated by the shell deformation and the denominator corresponds to the original volume under the spherical cap. Z is the height of a point on the middle-surface of the shell. There is a load range where a sharp jump in peak average displacement occurs for a small change in load magnitude. The inflection point of the load-deflection curve is considered as the dynamic buckling load.

For asymmetric dynamic buckling analysis of spherical shells, there is no well-understood and generally accepted criterion available so far. Furthermore, the available numerical results are very few to obtain a reasonable conclusion on the criterion, compared to those of axisymmetric case. The criterion adopted by Akkas [14] and Fulton and Barton [23] is somewhat similar to that of axisymmetric case. It is based on the plots of the peak nondimensional average asymmetric component of the displacement in the time history of the structure against the amplitude of the pressure load. However, there is no occurrence of a sudden jump in peak average displacement associated with asymmetric part of the deformation over a load range. Hence, the load corresponding to the inflection point on the load-deflection curve beyond which the asymmetric part of the displacement response reveals significant growth rate is considered as dynamic buckling load.

Element Description

The laminated axi-symmetric shell element employed here is a C^0 continuous shear flexible curved element with three nodes. It needs thirty-three degrees of freedoms per node for the filed variables (u_o, v_o, w, β_s and β_θ) described in Eq.(2).

If the interpolation functions for three-noded element are used directly to interpolate the five field variables u_o, v_o, w, β_s and β_θ in deriving the transverse shear and membrane strains, the element will lock and show oscillations in the shear and membrane stresses. Field consistency requires that the membrane and transverse shear strains must be interpolated in a consistent manner. Thus, β_s term in the expression for $\{\varepsilon_s\}$ given in Eq. (4) has to be consistent with field function $\left\{\frac{\partial w}{\partial s}\right\}$ as shown in the works of Balakrishna and Sarma [20], Ganapathi et.al [21], and Prathap and Ramesh Babu [32]. Similarly the w and (u_o, v_o) terms in the expression of $\{\varepsilon_p^L\}$ (first and third strain components) have to be consistent with the field functions $\frac{\partial u_o}{\partial s}$ and $\frac{\partial v_o}{\partial s}$, respectively. This is achieved by using the field redistributed substitute shape functions to interpolate those specific terms that must be consistent as described by Prathap and Ramesh Babu [32]. The element derived in this fashion behaves very well for both thick and thin situations, and permits the greater flexibility in the choice of integration order for the energy terms. It has good convergence and has no spurious rigid modes. For the sake of brevity, the development and the performance of the element are omitted, as they are available in the literature [20, 21, 32].

Results and Discussion

The study here deals with asymmetric dynamic buckling behavior of clamped isotropic/anisotropic spherical caps. Since the finite element used is based on the field consistency approach, an exact integration is employed to evaluate all the strain energy terms. The shear correction factor which is required in a first-order theory to account for the variation of transverse shear stresses, is taken as 5/6. For the present analysis, based on progressive mesh refinement, 15 elements idealization is found to be adequate in modeling the spherical caps. The initial conditions for the nonlinear asymmetric dynamic response analysis are considered as non-zero values for displacements and zero values for the velocities. The initial displacement vectors are assumed to be proportional to the normalized linear flexural asymmetric mode vectors and then scaled up by multiplying the mode vectors with a very small value for the perturbation of asymmetric mode of the spherical shell. From the dynamic response curves, the load amplitudes and the corresponding maximum average displacements are obtained for applying the buckling criterion. The constants α and β (controlling parameters for stability and accuracy of the solution) in the Newmark's integration are taken as 0.5 and 0.25, which correspond to the unconditionally stable scheme in linear analysis. Since there is no estimate of the time step for the non-linear dynamic analysis available in the literature, the critical time step of a conditionally stable finite difference schemes [33, 34] is introduced as a guide and a convergence study was conducted to select a time step which yields a stable and accurate solution.

The material properties assumed in the present analysis are

$$E_L/E_T = 25.0,\ G_{LT}/E_T = 0.5,\ G_{TT}/E_T = 0.2,$$

$$\nu_{LT} = 0.25,\ E_T = 1\ GPa,\ \rho = 1600\ Kg/m^3$$

where E, G and ν are Young's modulus, shear modulus and Poisson's ratio. Subscripts L and T are the longitudinal and transverse directions respectively with respect to the fibres. All the layers are of equal thickness. The ply-angles are measured with respect to the meridional axis. Results of non-dimensional dynamic pressure P_{cr}, are presented for isotropic, cross-and angle-ply shells for different values of the geometrical parameter λ. P_{cr} and λ are given by

$$P_{cr} = \frac{1}{8}\left[3\,(1-\nu_{LT}\,\nu_{TL})\right]^{1/2}\left(\frac{h}{H}\right)^2 \frac{q\,a^4}{E_L\,h^4}$$

$$\lambda = 2\left[3\,(1-\nu_{LT}\,\nu_{TL})\right]^{1/4}\left(\frac{H}{h}\right)^{1/2}$$

Here, H, a are the central shell rise and base radius, respectively. For the chosen shell parameter and lamination scheme, the dynamic buckling study is conducted for step loading of infinite duration. The length of response calculation time

$$\tau = \left[\left(\frac{D_{11}}{\rho\,h\,a^4}\right)^{1/2} t\right]$$

in the present study is varied between 1 and 2 with the criterion that in the neighborhood of the buckling, τ is large enough to allow deflection-time curves to fully develop. The time step selected, based on the convergence study, is $\delta\tau = 0.002$. The values selected for τ and $\delta\tau$ are of same order as considered in the work of Ball and Burt [5], Kao and Perrone [7] and Chao and Lin [18].

Before proceeding for the detailed study of the nonlinear asymmetric dynamic buckling characteristics, the formulation developed herein is validated against the axisymmetric buckling of isotropic spherical shells subjected to uniform external pressure of infinite duration. The nonlinear axisymmetric dynamic response history with time for the geometric shell parameter λ = 7 is shown in Fig.1 for different externally applied pressure. Further, using such plots, the variation of maximum average displacement with applied load obtained for λ = 7, is also highlighted in Fig.1 for predicting the critical load. The critical dynamic pressure calculated for various geometrical parameter values are presented in Fig.2 along with those of available analytical/numerical results [3, 4, 15, 19] and they are, in general, found to be in good agreement.

Next, for the isotropic spherical caps, considering different values for the geometrical parameter, λ, the dynamic buckling loads are evaluated based on asymmetric nonlinear dynamic response of shells subjected to externally applied pressure. This study is carried out perturbing the different asymmetric modes (circumferential wave number, n). Fig.3 exhibits the dynamic response pattern of average asymmetric displacement component of shell (λ = 7) with time and in turn, the variation of maximum average displacement with applied loads. The influence of various asymmetric modes on the critical dynamic load is also investigated and depicted in Fig.4. It is revealed from Fig.4 that the results available in the literature [5, 14] are fairly in good agreement with the present solutions and the buckling criterion used in the present study is based on the growth rate of asymmetric mode response with time. However, for λ = 7.5 case, the present model predicts high value for the critical load than those of Akkas [14], and is more than those of axisymmetric case. Furthermore, it can

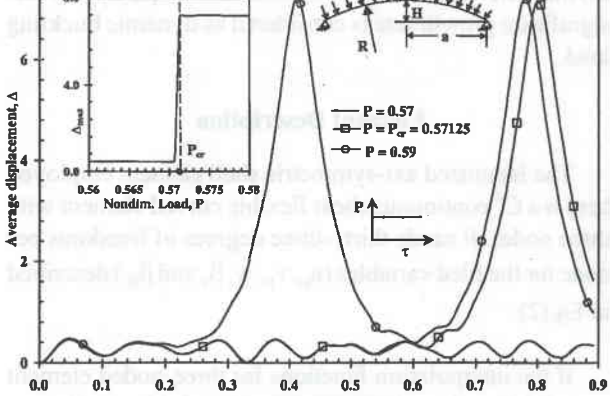

Fig. 1 Average displacement versus nondimensional time for isotropic shperical cap (λ=7, Axisymmetric case)

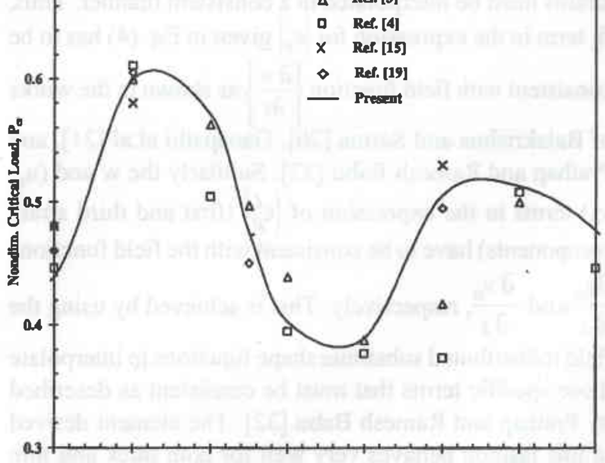

Fig. 2 Comparision of axisymmetric nondimensional critical load for isotropic spherical cap

be seen from Fig.4 that this particular shell appears to be buckled in axisymmetric mode of vibration rather than in asymmetric deformation as pointed out in the work of Stricklin and Martinez [6] while examining the asymmetric mode of buckling. The predicted axisymmetric critical load is fairly in good agreement with the available experimental result [11]. It is further noticed from Fig.4 that the values of critical load corresponding to λ = 6 and 12 obtained by Ball and Burt [5] are in very close agreement with those of present results pertaining to asymmetric mode n=3. However, the mode numbers reported in Ref.[5] for the shell parameter λ = 6 and 12 are n=2, and 5 or 6, respectively. It can be observed from Fig.4 that the axisymmetric type of dynamic buckling occurs for very shallow shells, and also for shell parameters in the transition region between moderately shallow and deep cases. The asymmetric buckling deformation, in general, dominates the moderately shallow shells under lower asymmetric modes and deep shell regions under higher modes. It can be further inferred that, among the asymmetric cases, the difference in the predicted critical loads is high for the shell geometrical parameter in the shallow region and it reduces with the increase in the value of shell parameter. It is however, largely depends on the perturbation of asymmetric vibration mode.

Next, for the laminated eight-layered angle-ply $(\phi^{o}/-\phi^{o})_4$ spherical caps considering different values for the geometrical parameter λ, the dynamic buckling loads are evaluated based on asymmetric nonlinear dynamic response of shells subjected to externally applied pressure.

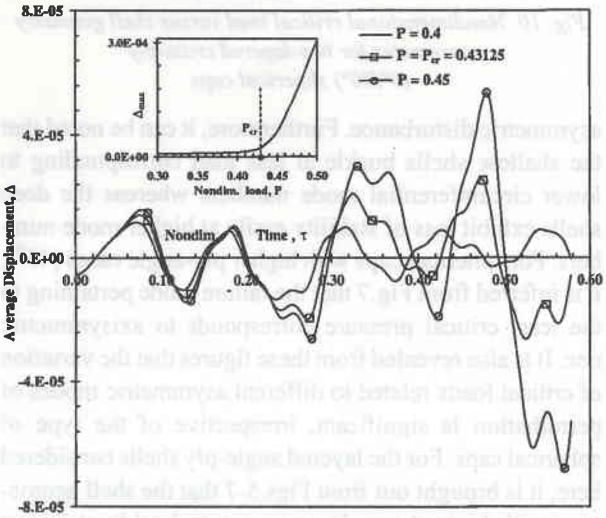

Fig. 3 Average displacement versus nondimensional time for isotropic shperical cap (λ=7, Asymmetric case with n=3)

The influence of various asymmetric modes (circumferential wave number, *n*) on the critical dynamic load is investigated and highlighted in Figs.5-7 for various angle-ply laminates. It is observed from Fig.5 that, for low angle-ply case (15°) considered here, the lowest critical load of very shallow shell corresponds to axisymmetric mode. But, the asymmetric mode of buckling behavior dominates the stability of deep spherical caps. However, the differences in the critical loads predicted among different asymmetric mode cases decrease with increase in the geometric parameter value. For 30° angle-ply case, it is viewed from Fig.6 that, irrespective of geometric shell parameter values, the asymmetric mode produces the lowest critical load. It is also noticed from Fig.6 that the value of critical pressure significantly depends on the type of

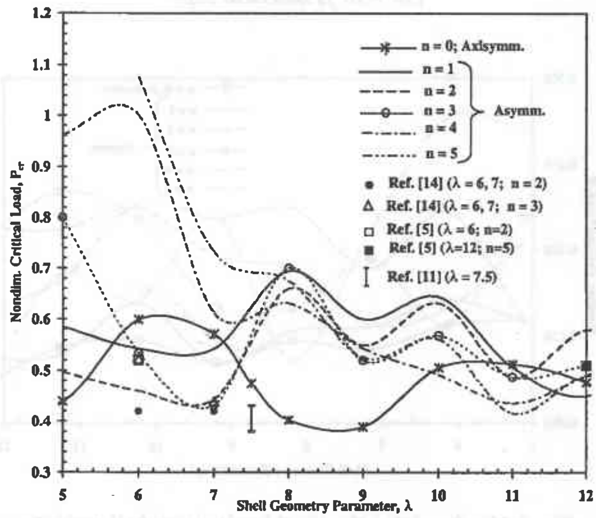

Fig. 4 Nondimensional critical load (for Axisymmetric and Asymetric cases) versus shell geometry for isotropic shperical cap

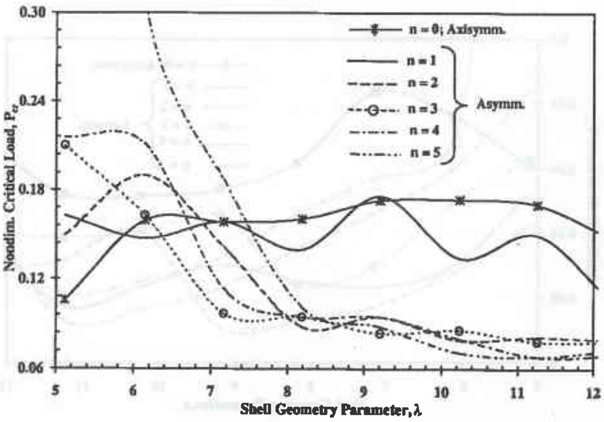

Fig. 5 Nondimensional critical load versus shell geometry parameter for eight -layared angle-ply $(15^{o}/-15^{o})_4$ shperical caps

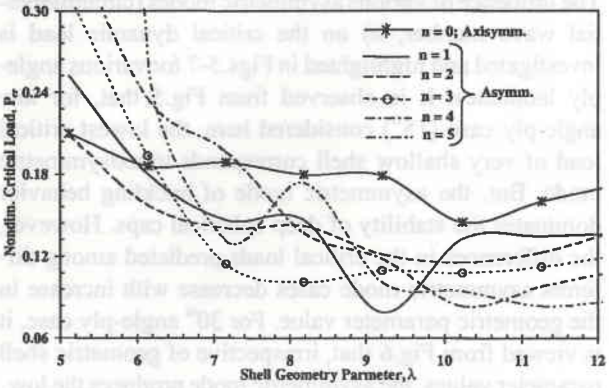

Fig. 6 Nondimensional critical load versus shell geometry parameter for eight-layered angle-ply (30°/-30°)$_4$ spherical caps

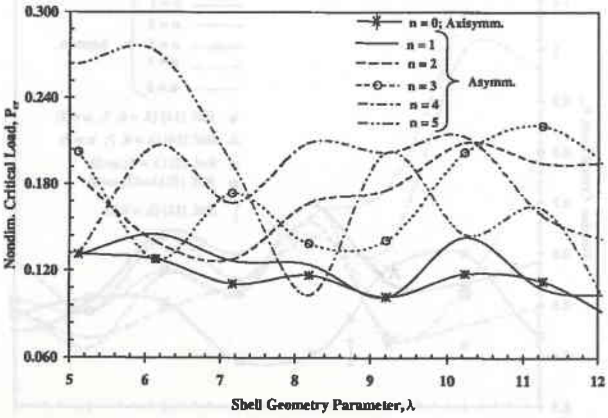

Fig. 7 Nondimensional critical load versus shell geometry parameter for eight-layered angle-ply (45°/-45°)$_4$ shperical caps

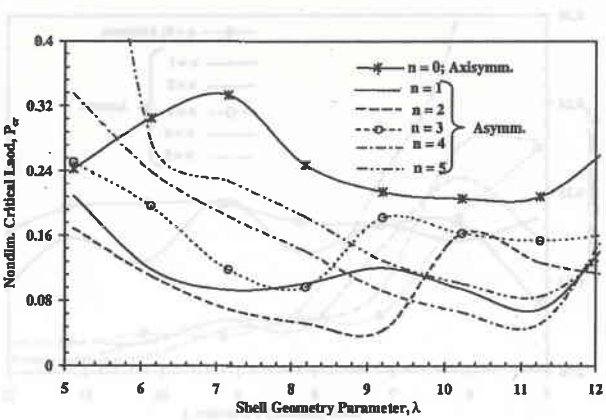

Fig. 8 Nondimensional critical load versus shell geometry parameter for eight-layered cross-ply (0°/90°)$_4$ shperical caps

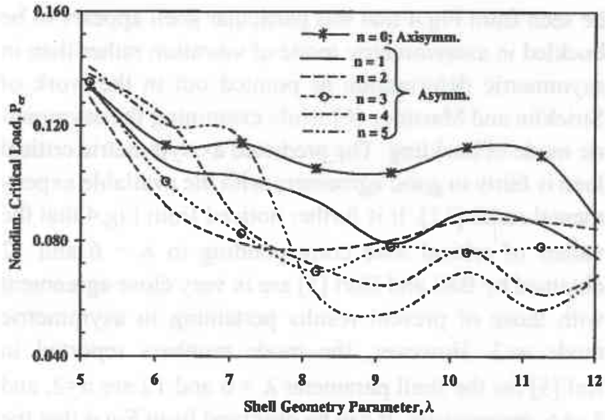

Fig. 9 Nondimensional critical load versus shell geometry parameter for two-layered angle-ply (30°/-30°) shperical caps

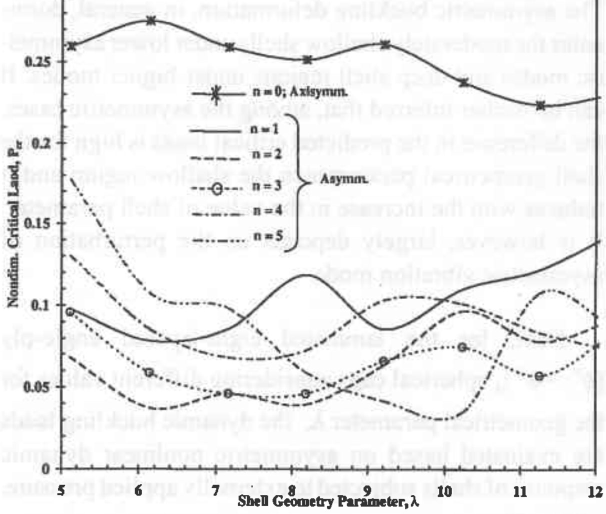

Fig. 10 Nondimensional critical load versus shell geometry parameter for two-layered cross-ply (0°/90°) shperical caps

asymmetric disturbance. Furthermore, it can be noted that the shallow shells buckle at less load corresponding to lower circumferential mode numbers whereas the deep shells exhibit loss of stability easily at higher mode numbers. For spherical caps with higher ply-angle cases (45^0), it is inferred from Fig.7 that the failure mode pertaining to the least critical pressure corresponds to axisymmetric one. It is also revealed from these figures that the variation of critical loads related to different asymmetric modes of perturbation is significant, irrespective of the type of spherical caps. For the layered angle-ply shells considered here, it is brought out from Figs.5-7 that the shell geometries with high ply-angle are prone to buckle at lowest critical loads. It is worthwhile to mention here that the shallow shells with certain ply-angles may not fail under asymmetric modes of large circumferential wave number, as the shell response characteristics do not reveal any significant growth with time. Similar study is also carried out for cross-ply case. The dynamic buckling values obtained for eight-layered cross-ply shells are presented in Fig.8. It can be seen from these investigations that the variation of buckling load of cross-ply shells is qualitatively similar to those of low angle-ply laminates.

The effect of bending-stretching coupling due to lamination scheme is also examined and brought out in Figs.9 and 10 for angle and cross-ply spherical caps. The predicted asymmetric dynamic critical loads for two-layered case are, in general, less and the buckling behavior is qualitatively similar to those of eight-layered shells. In general, it can be opined that the lowest critical dynamic buckling load of a given spherical cap significantly depends on its geometrical parameter value, ply-angle and type of mode of excitation.

Conclusions

Asymmetric dynamic buckling of clamped isotropic/anisotropic spherical caps subjected to externally applied pressure has been investigated through nonlinear transient dynamic response analysis. A three-noded axisymmetric curved shell element based on field consistency principle has been employed for this purpose. Numerical results obtained here for an isotropic case are found to be fairly in good agreement with the previous findings. From the detailed study, the following observations can be made :

1. The critical load decreases with increase in the value of orthotropicity, E_L/E_T.
2. For low angle-ply case, the difference in the buckling values predicted among various asymmetric modes decreases rapidly with increase in the value of the shell parameter and the lowest critical pressure corresponds to either axisymmetric or asymmetric modes depending on the shell parameter value.
3. With increase in ply-angle, the influence of asymmetric mode on the dynamic buckling load is significant for all the shell parameter but the axisymmetric buckling mode yields the lowest critical values.
4. Asymmetric buckling at large circumferential wave numbers is not observed for shallow shells with certain ply-angles.
5. The shells with higher lamination angles are prone to buckle at lower critical loads.
6. The bending-stretching coupling due to lay-up, in general, reduces the buckling loads and the asymmetric buckling mode dominates the failure of various shell geometric parameters considered here.
7. Shallow shells exhibits the onset of asymmetric buckling at lower circumferential mode numbers whereas the occurrence of instability of deep shells associates with the higher mode number.
8. For a given spherical cap, the critical dynamic buckling load depends significantly on the initial conditions, geometrical parameter and ply-angle.

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