DYNAMIC RESPONSE OF THICK LAMINATED SHELLS USING HIGHER-ORDER THEORY

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Abstract

In this paper, the dynamic characteristics of thick cross-ply laminated composite cylindrical shells are studied using a higher-order displacement model. The formulation accounts for the nonlinear variation of the in-plane and transverse displacements through the thickness, and abrupt discontinuity in slope of the in- plane displacements at any interface. The effect of inplane and rotary inertia terms is included. The analysis is carried out using finite element approach. The governing equations derived using Lagrange's equations of motion are solved employing Newmark's direct integration technique for transient response analysis. The influences of various terms in the higher-order displacement field on the free vibrations, and transient dynamic response characteristics of cylindrical composite shells subjected to thermal and mechanical loads are analysed.

Keywords : Laminated shell, Cross-ply, Free vibration, Transient response, Higher-order, Finite element.

Introduction

The increased use of composite materials in high temperature environment, high strength and stiffness applications have made the mechanical/thermal analysis of composite structures necessary. Laminated fiber reinforced composites are characterized by low transverse shear modulus compared to the in-plane Young's moduli and therefore the classical theory of non-deformable normals based on neglecting transverse shear strains is not acceptable for laminated composite structures.

Various structural theories proposed for the analysis of composite laminates have been reviewed and assessed in the literature [1-2]. It is brought out that the first-order theory is inadequate for the accurate estimation of higher-order frequencies, mode shapes and distribution of stresses, and the thickness has pronounced effects on the behavior of composite structures. This has necessitated the introduction of higher- order function in the displacement model, and layer-wise theory for the study of plates/shells [3-9]. Recently, a new kinematics for higher-order theory for composite plates incorporating through the thickness approximation for in-plane and transverse displacements have been introduced in Ali et. al. [10] and it yields accurate results for static analysis of thick laminates. To the authors' knowledge, there is no work available in the literature on the application of the higher-order formulation, in general, in predicting the dynamic response characteristics of thick laminated shells subjected to thermal/mechanical load.

Here, the dynamic analysis is carried out using finite element approach employing the kinematics proposed in Ref. [10], to demonstrate the influence of higher-order terms introduced in the kinematics on the free vibration characteristics, and transient response behavior of simply supported laminated cross-ply circular cylindrical shell subjected to thermal/mechanical loads.

Formulation

A laminated composite general shell of revolution is considered with the co-ordinates x along the meridional direction, y along the circumferential direction and z along the thickness direction having origin at the mid-plane of the shell. The in-plane displacements u^k and v^k and the

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transverse displacement w^k for the kth layer, are assumed as

$$u^k(x,y,z) = u_o(x,y) + z\,\theta_x(x,y) + z^2\,\beta_x(x,y) + z^3\,\phi_x(x,y) + S^k\,\psi_x(x,y)$$

$$v^k(x,y,z) = v_o(x,y) + z\,\theta_y(x,y) + z^2\,\beta_y(x,y) + z^3\,\phi_y(x,y) + S^k\,\psi_y(x,y)$$

$$w^k(x,y,z) = w_o(x,y) + z\,w_1(x,y) + z^2\,\Gamma(x,y) \qquad (1)$$

Here, u_o, v_o, w_o are the displacements of a generic point on the reference surface; θ_x, θ_y are the rotations of normal to the reference surface about the y and x axes, respectively; w_1, β_x, β_y, Γ, ϕ_x, ϕ_y are the higher order terms in the Taylor's series expansions, defined at the reference surface. The terms w_1 and Γ in the expression for w^k are for accounting the through the thickness stretching/contraction for thick laminates. ψ_x and ψ_y are generalized variables associated with the zig-zag function, S^k. The zig-zag function, S^k, as given in Ref. [11], is defined by

$$S^k = 2(-1)^k\, z_k/h_k \qquad (2)$$

where z_k is the local transverse coordinate with its origin at the centre of the kth layer and h_k is the corresponding layer thickness. Thus, the zig-zag function is piecewise linear with values of -1 and 1 alternately at the different interfaces. The 'zig-zag' function, as defined above, takes care of the inclusion of the slope discontinuity of u and v at the interfaces of the laminate as observed in exact three-dimensional elasticity solutions of thick laminated composite structures. The strains in terms of mid-plane deformation, rotations of normal, and higher order terms associated with displacements for kth layer are as,

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_{bm} \\ \varepsilon_s \end{Bmatrix} - \begin{Bmatrix} \bar{\varepsilon}_t \\ 0 \end{Bmatrix} \qquad (3)$$

The vector $\{\varepsilon_{bm}\}$ includes the bending and membrane terms of the strain components and vector $\{\varepsilon_s\}$ contains the transverse shear strain terms. These strain vectors can be defined as

$$\{\varepsilon_{bm}\} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} (u^k_{,x} + w/R_1)/(1+z/R_1) \\ (v^k_{,y} + (u^k/r)Cos\phi + (w^k/r)Sin\phi)/(1+z/R_2) \\ w^k_{,z} \\ (u^k_{,y} - (v^k/r)Cos\phi/(1+z/R_2) + v^k_{,x}/(1+z/R_1) \end{Bmatrix} \qquad (4a)$$

$$\{\varepsilon_s\} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} u^k_{,z} + (w^k_{,x} - u/R_1)/(+z/R_1) \\ v^k_{,z} + (w^k_{,y} - (v/r)\,Sin\,\phi)/(1+z/R_2) \end{Bmatrix} \qquad (4b)$$

where R_1, R_2 are the principal radii of curvature in meridional and hoop directions, respectively; r is the radius of the parallel circle; and ϕ is the angle between normal to reference surface and axis of revolution. The subscript comma denotes the partial derivative with respect to the spatial coordinate succeeding it. The thermal strain vector $\{\bar{\varepsilon}_t\}$ is represented as

$$\{\bar{\varepsilon}_t\} = \begin{Bmatrix} \bar{\varepsilon}_{xx} \\ \bar{\varepsilon}_{yy} \\ \bar{\varepsilon}_{zz} \\ \bar{\varepsilon}_{xy} \end{Bmatrix} = T \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \\ \alpha_{xy} \end{Bmatrix} \qquad (4c)$$

where T is the rise in temperature and is generally represented as function of x, y, and z. α_x, α_y, α_z and α_{xy} are thermal expansion coefficients in the shell coordinates and can be related to to the thermal coefficients (α_1, α_2 and α_3) in the material principal directions.

The constitutive relations for an arbitary layer k, in the laminated shell (x,y,z) coordinate system can be expressed as

$$\{\sigma\} = \{\sigma_{xx}\ \sigma_{yy}\ \sigma_{zz}\ \tau_{xy}\ \tau_{xz}\ \tau_{yz}\}^T$$

$$= [\bar{Q}_k]\,\{\varepsilon_{xx} - \bar{\varepsilon}_{xx}\ \varepsilon_{yy} - \bar{\varepsilon}_{yy}\ \varepsilon_{zz} - \bar{\varepsilon}_{zz}\ \gamma_{xy} - \bar{\gamma}_{xy}\ \gamma_{xz}\ \gamma_{yz}\}^T \qquad (5)$$

where the terms of $[\bar{Q}_k]$ matrix of kth ply are referred to the laminated shell axes and can be obtained from the $[Q_k]$ corresponding to the fibre directions with the appropriate transformation, as outlined in the literature [12]. $\{\sigma\}$, $\{\varepsilon\}$ and $\{\bar{\varepsilon}_t\}$ are stress, strain, and thermal strain vectors due to rise in temperature, respectively. The superscript T refers the transpose of a matrix/vector.

The strain energy of the laminate can be expressed in terms of u_o, v_o, w_o, θ_x, θ_y, w_1, β_x, β_y, Γ, ϕ_x, ϕ_y, ψ_x and ψ_y and their derivatives. Work done by the externally applied pressure load is also considered in the total potential energy expression. All the inertia terms, due to the parts resulting from first-order model, the higher order displacement function, and the coupling between the different order displacement are included in evaluating the kinetic energy. Then the governing equations are obtained using Lagrange's equation of motion. These governing equations are solved using the finite element approach based on C^0 continuous element developed based on the above theory.

An eight-noded serendipity quadrilateral shear flexible shell element with thirteen degrees of freedom as per the kinematics given in Eq. 1 (HSDT13 : u_o, v_o, w_o, θ_x, θ_y, w_1, β_x, β_y, Γ, ϕ_x, ϕ_y, ψ_x and ψ_y) is employed. Four more alternate discrete models are proposed, to study the influence of higher order terms in the displacement functions, whose displacement fields are deduced from the original element by deleting the appropriate degrees of freedom (HSDT11a : w_1 and $\Gamma = 0$; HSDT11b : ψ_x and $\psi_y = 0$; HSDT7 : z^2 terms, ψ_x, ψ_y, w_1 and $\Gamma = 0$; FSDT5: dropping all the higher order terms).

Using Eqs. (3) - (5) and following the standard procedure [13], the finite element equations are derived as

$$[M]\{\ddot{\delta}\} + [K]\{\delta\} = \{F_T\} + \{F_M\} \tag{6}$$

where [M] and [K] are global mass and stiffness matrices, respectively, and $\{F_T\}$, $\{F_M\}$ are the global thermal and mechanical load vectors. $\{\delta\} = \{\delta_1, \delta_2, \ldots\ldots, \delta_l, \ldots\ldots, \delta_n\}^T$ is the vector of the degree of freedoms/generalized coordinates. A dot over the variables represents the derivative with respect to time. The solutions of Eq.(6) can be obtained using either standard eigenvalue algorithm or employing Newmark's direct integration method, depending on free vibration or forced response analysis.

Results and Discussion

From the results obtained for the moderately thick and thin laminated composite shells, it has been observed that the agreement between the available work in the literature and the present analysis is excellent for predicting the frequency values. For the sake of brevity, such results are not given. However, for thick circular cylindrical shell case, the frequency values obtained here match well with the three-dimensional analysis [14] as shown in Table-1. Based on progressive: mesh refinement, 16 x 4 grid size (circumferential and meridional directions) is found to be adequate to model a 1/8 th of the thick laminated composite shells considered here for the free and forced vibration analysis. The accuracy of the present model is assessed considering different models. The material properties used, unless otherwise mentioned, are

$$E_1/E_2 = 40,\ G_{12}/E_2 = G_{13}/E_2 = 0.6,\ G_{23}/E_2 = 0.5,$$
$$\nu_{12} = \nu_{23} = \nu_{13} = 0.25,\ \alpha_2/\alpha_1 = 1125,\ E_2 = E_3 = 1\ GPa,$$
$$\alpha_1 = 10^{-5}/°C,\ \rho = 1500\ Kg/m^3$$

where 1, 2 and 3 denotes the material principal directions. All the layers are of equal thickness and the ply- angle is measured with respect to the x axis (meridional axis).

The simply supported boundary conditions considered for this problem are:

$$v_0 = w_0 = \theta_y = w_1 = \Gamma = \beta_y = \phi_y = \psi_y = 0 \quad at\ x = 0, L$$

Next, the influence of various higher-order terms on the natural frequencies of thick laminated circular cylindrical shells is studied and presented in Table-2. It is observed from Table-2 that, irrespective of short or long cylinder, model HSDT7 over predicts the frequency values whereas FSDT5 under predicts the results for the short and thick case in comparison with those of complete model HSDT13. Also, the difference in the values, in general, increases with the increase in the circumferential wave number. Furthermore, it is inferred that, for a short cylinder, HSDT11a having zig-zag variation through the thickness for in-plane displacements predicts frequency values very close to complete model HSDT13 whereas HSDT11b with thickness variation in transverse displacement is more close to the HSDT13 for long cylinder case.

The transient response analysis is carried out considering eight-layered unsymmetric thick cross-ply shell [L/R = 0.5, R/h = 5; $(0^o/90^o)_4$] subjected to thermal [$T=T_o$ $(2\ z/h)\sin(\pi x/L)\cos(3\ y/R)$; T_o=1] and internal pressure

Table-1 : Comparison of natural frequency parameter $\Omega\ (= \omega R \sqrt{\rho/E_2})$ of a three-layered symmetric cross-ply circular cylindrical shell (E_1 / E_2 = 25; L/R = 5; Longitudinal mode number, m = 1)

Circum. Wave Number n	R/h					
	5		10		20	
	Present	Ref.[14]	Present	Ref.[14]	Present	Ref.[14]
1	0.339297	0.339	0.331522	0.332	0.329408	0.329
2	0.306985	0.306	0.224928	0.225	0.197009	0.197
3	0.594289	0.591	0.330063	0.329	0.194639	0.194

Table-2 : Lowest frequency parameter $\Omega\ (= \omega R \sqrt{\rho/E_2})$ of an eight-layered unsymmetric cross-ply circular cylindrical shell (Longitudinal mode number, m = 1)

Circum. Wave Num-ber n	Theory	L/R					
		5			0.5		
		R/h			R/h		
		2.5	5	10	2.5	5	10
1	HSDT13	0.352448	0.336093	0.331732	4.503126	4.325594	3.971877
	HSDT11a	0.352408	0.336097	0.331739	4.505317	4.323748	3.970198
	HSDT11b	0.352506	0.336098	0.331733	4.505275	4.332821	3.977382
	HSDT7	0.352681	0.336118	0.331740	4.583386	4.383107	3.997469
	FSDT5	0.352982	0.336156	0.331744	4.487248	4.355006	3.996548
2	HSDT13	0.758660	0.566201	0.371140	4.387114	4.098246	3.568073
	HSDT11a	0.780684	0.576207	0.373572	4.385639	4.095270	3.566235
	HSDT11b	0.760016	0.567005	0.371324	4.391072	4.108857	3.575911
	HSDT7	0.793617	0.580614	0.374374	4.500067	4.179835	3.604629
	FSDT5	0.794385	0.581616	0.374551	4.357779	4.139150	3.603539
3	HSDT13	1.527944	1.244130	0.835968	4.531517	4.165393	3.502635
	HSDT11a	1.550800	1.259912	0.841047	4.529267	4.162229	3.501198
	HSDT11b	1.530750	1.247065	0.837028	4.536939	4.177905	3.511649
	HSDT7	1.587034	1.277661	0.845840	4.660843	4.257282	3.544421
	FSDT5	1.575407	1.279348	0.846796	4.501162	4.212103	3.543059
4	HSDT13	2.307072	1.986646	1.451409	4.829248	4.414821	3.645450
	HSDT11a	2.324346	2.003967	1.458411	4.827397	4.412307	3.644882
	HSDT11b	2.310461	1.992089	1.454149	4.835591	4.428982	3.655415
	HSDT7	2.385855	2.041247	1.471331	4.973065	4.516423	3.691595
	FSDT5	2.342163	2.038262	1.473214	4.796745	4.468223	3.690103

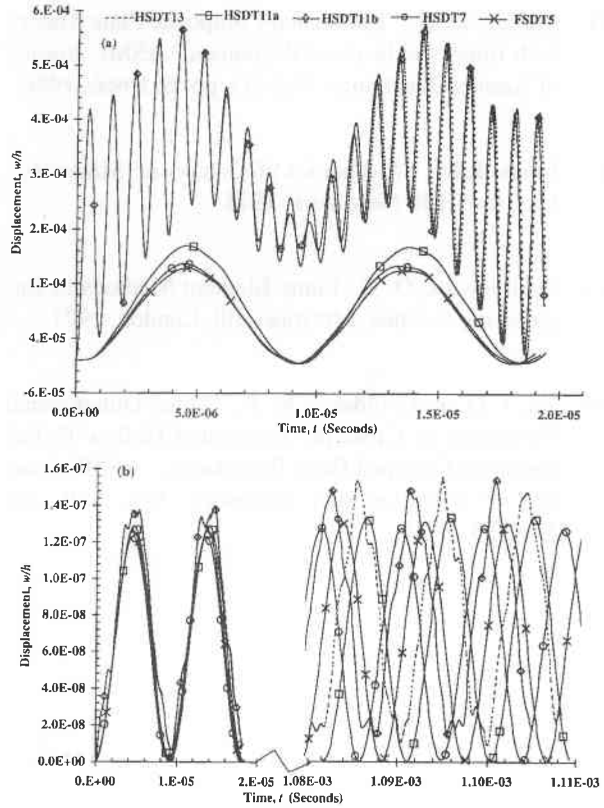

Fig. 1 Transverse response of an eight-layered unsymmetric cross-ply (0°/90°)$_4$ circular cylindrical shell, (L/R=0.5, R/h=5) under: (a) Thermal load; (b) Internal pressure

load [$q=q_o \sin(\pi x/L)\cos(3\,y/R)$; q_o=50]. The variation of transverse displacement evaluated using different models is described in Fig.1. For the thermal loading, it is noticed from Fig.la that the responses calculated using FSDT5, HSDT7 and even HSDT11a are very low compared to that of HSDT11b/HSDT13. The amplitudes predicted by HSDT11b/HSDT13 are high and the response shows high frequency oscillations due to the participation of thickness stretch modes. However, it appears that retaining the thickness stretch terms (w_1 and Γ) in the transverse displacement is more important than the inclusion of zig-zag terms (ψ) in in-plane displacement description.

For the internal pressure load, the transverse response characteristics obtained through various models are presented in Fig.1b. It is noticed that the changes in the initial responses predicted by different models are less. However, with the increase in the response time, the variation of displacement depends on the type of models employed. It is further seen that, like thermal case, HSDT7 and FSDT5 predict similar response except the occurrence of peak amplitudes. Although there is some reduction in the maximum amplitude value predicted by HSDT11a, the response pattern is very close to actual model HSDT13 whereas the response period calculated through the model HSDT11b is less in comparison with those of the complete model. In general, it can be opined that, for the mechanical load, the response predicted by the model having zig-zag variation in the in-plane displacement (HSDT11a) is, qualitatively, similar as those of complete model. It is hoped that the results presented here are useful in dealing with the analysis of thick composite shells under different loading environment.

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